

THE SEARCH FOR
THE QCD CRITICAL
POINT

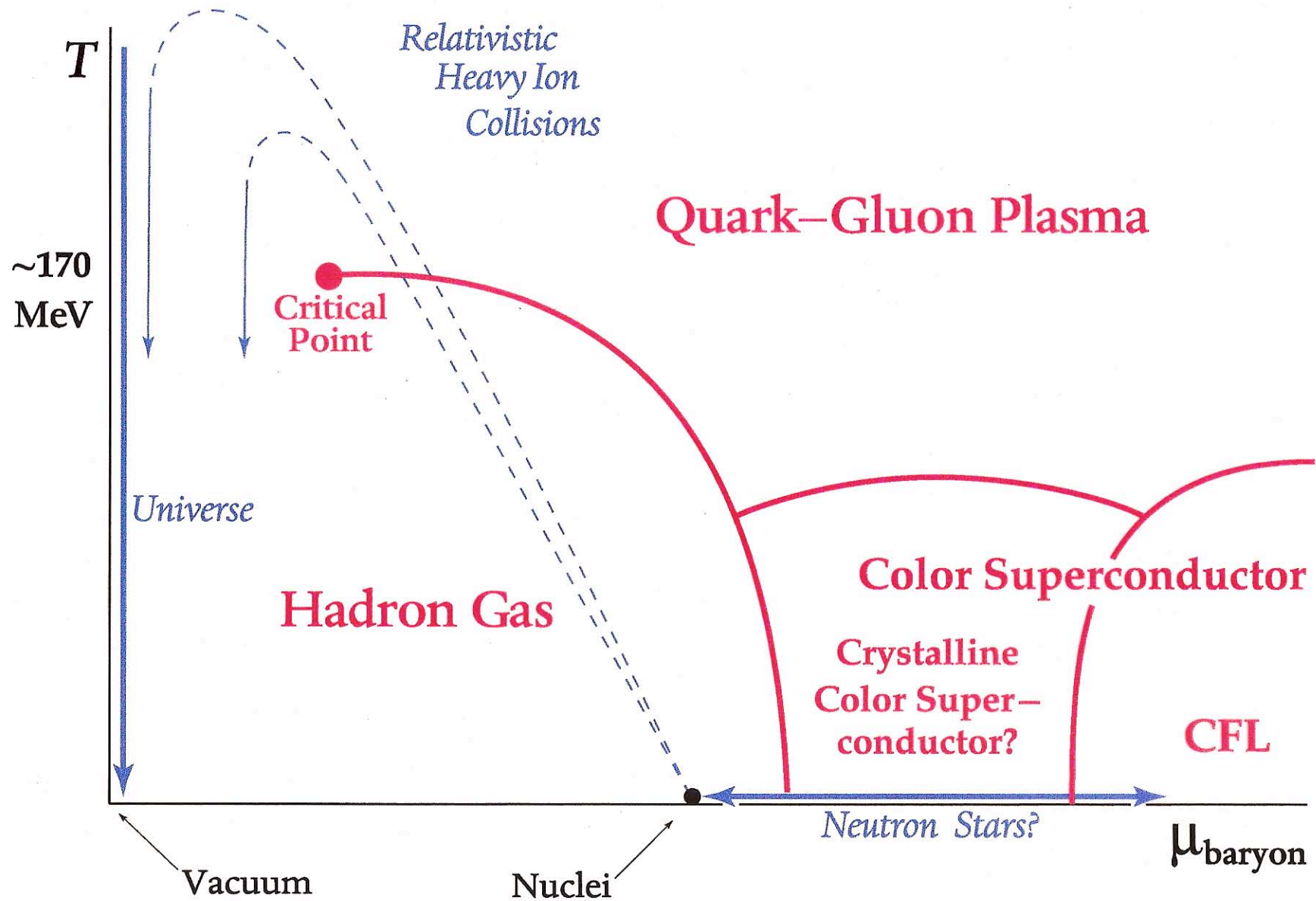
USING LATTICE QCD
CALCULATIONS

AND HEAVY ION COLLISION
EXPERIMENTS

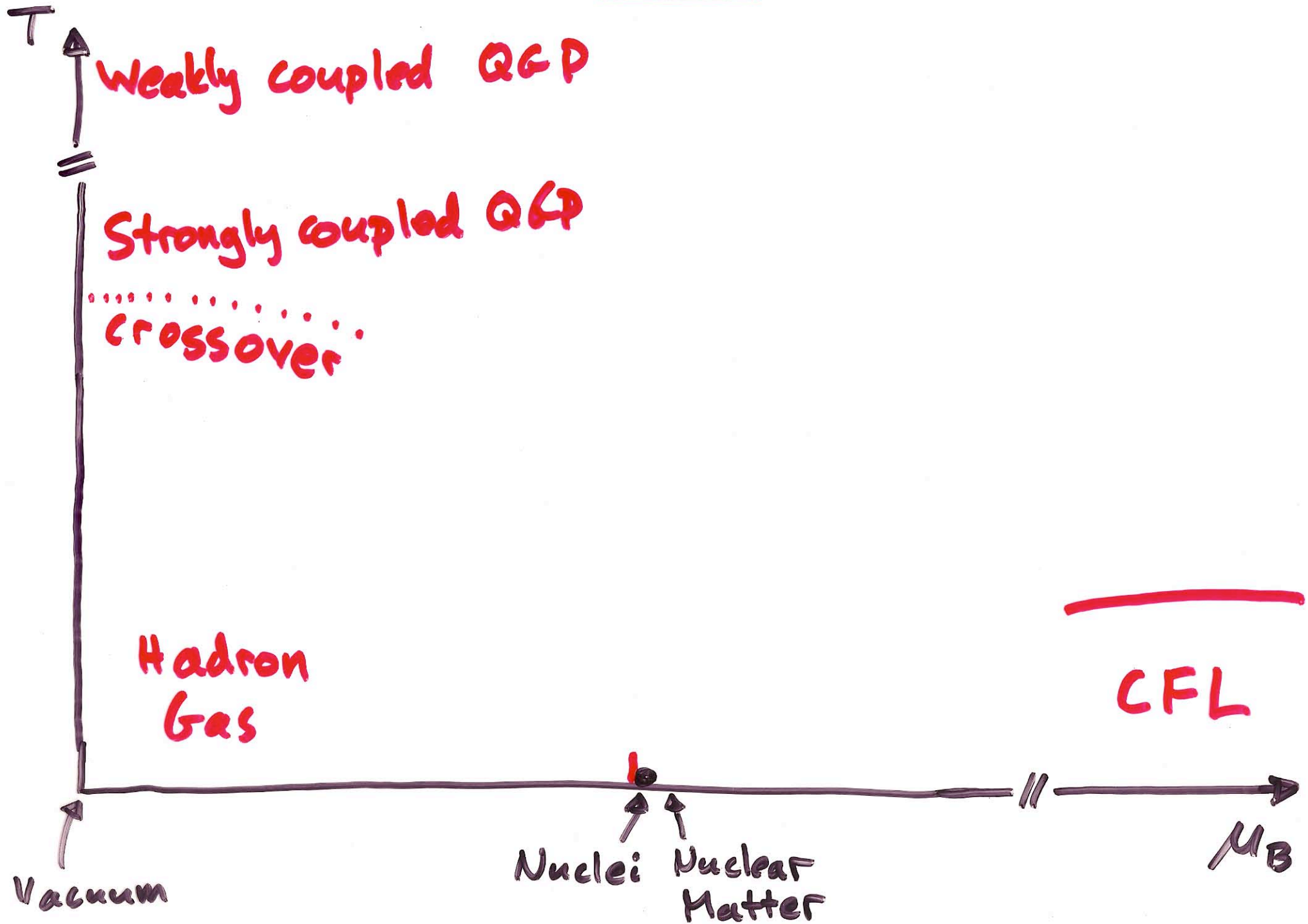
KRISHNA RAJAGO PAZ
(MIT)

INT, Seattle. 8/11/08

EXPLORING *the* PHASES of QCD

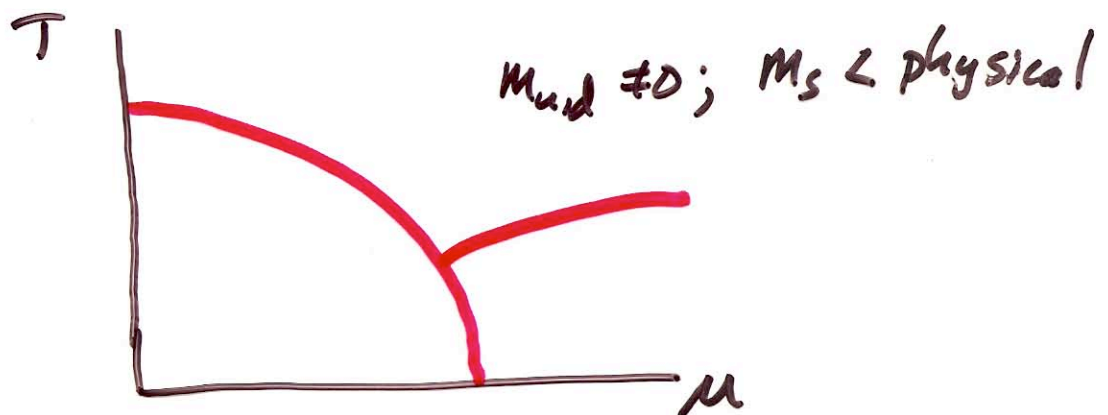
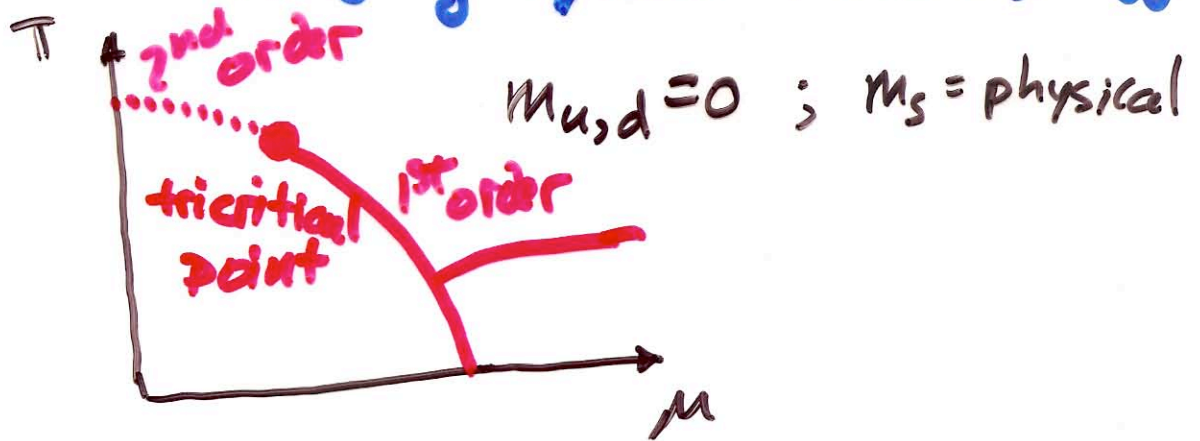


WHAT WE KNOW, SO FAR



WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu=0$ with varying quark masses; suggest:



- Need lattice calculations with $T \neq 0, \mu \neq 0$ to locate it
- Universality class known (Ising)

LOCATING THE CRITICAL POINT...

- either via lattice calculations
- or via experimental detection of its signatures

would add a point and a line to the known

QCD phase diagram.

OUTLINE OF TALK (AND WEEK)

- Lattice calculations
- Experimental signatures and searches

$T \neq 0; \mu \neq 0; \mu/T$ NOT LARGE

- a regime explored by heavy ion collisions
- a regime explored by lattice calculations that rely on smallness of μ/T to keep fermion sign problem under control. [$\mu \neq 0 \rightarrow$ complex Euclidean action \rightarrow sign problem that makes difficulty of standard Monte Carlo $\sim \exp V$.]
- Either method may be used to locate the CRITICAL POINT, a 2nd order point where a line of 1st order transitions ends, if it is located at a μ/T that is not too large....

SEVERAL LATTICE METHODS

① Reweighting Fodor + Katz

Want physics at $(a) \equiv (\mu, T_a)$

Simulate using an ensemble of configurations at $(b) \equiv (0, T_b)$,

and "reweight": lump difference between physics at (b) and (a) into observables.

Difficulty $\sim \exp \left[\frac{|F_b - F_a| V}{T} \right]$

F+K: choose T_b to minimize \S

BUT: still cannot use method at large volumes....

The endpoint is at $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV. As expected, μ_E decreased as we decreased the light quark masses down to their physical values (at approximately three-times larger $m_{u,d}$ the critical point was at $\mu_E = 720$ MeV; see [8]).

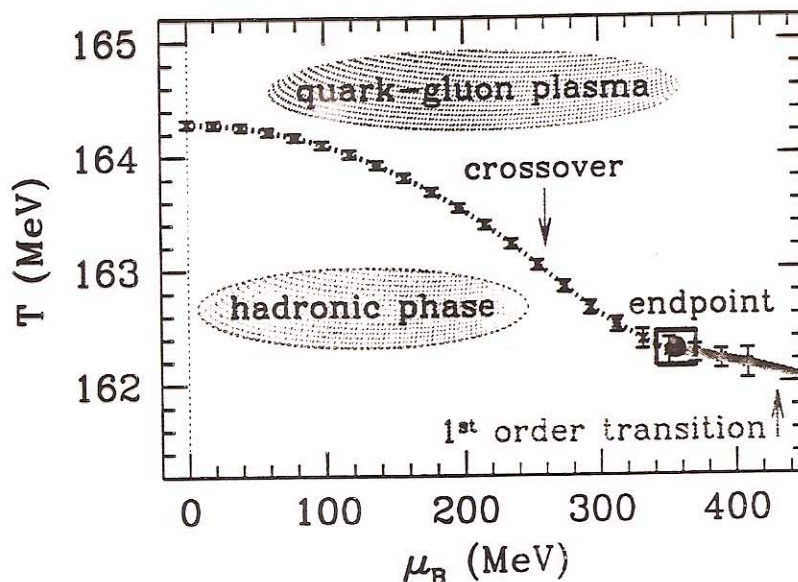


Figure 2: The phase diagram in physical units. Dotted line illustrates the crossover, solid line the first order phase transition. The small square shows the endpoint. The depicted errors originate from the reweighting procedure. Note, that an overall additional error of 1.3% comes from the error of the scale determination at $T=0$. Combining the two sources of uncertainties one obtains $T_E = 162 \pm 2$ MeV and $\mu_E = 360 \pm 40$ MeV.

The above result is a significant improvement on our previous analysis [8] by two means. We increased the physical volume by a factor of three and decreased the light quark masses by a factor of three. Increasing the volumes did not influence the results, which indicates the reliability of the finite volume analysis. Clearly, more work is needed to get the final values. Most importantly one has to extrapolate to the continuum limit.

Fodor, Katz
2004

$$\left. \begin{aligned} \mu_E &= 360 \pm 40 \text{ MeV} \\ \frac{\mu_E}{T_E} &= 2.22 \pm .25 \end{aligned} \right\} \text{statistical errors only}$$

CONCERNS, aka "SYSTEMATIC ISSUES"

- $N_\tau = 4$ (no continuum limit)
- $V = 12^3$, and method must break down for $V \rightarrow \infty$

- $\frac{M_E}{3} \approx \frac{M_\pi}{2}$. This was also

the case in older $F+K$ calculation

at larger M_π . If this is not a

coincidence, it is a problem. ^{Splitterf}

$\mu_q = M_\pi/2$ is where phase quenched
QCD has onset of pion condensation.

- $\frac{M}{T}$ held fixed during reweighting,
not m .

ALL these, except for $V \rightarrow \infty$, are
IMPROVABLE.

② Continue from imaginary μ .
deForcrand + Philipsen
D'Elia + Lombardo et al

Simulate at $\mu = i\mu_I$; calculate

$T_c(\mu_I)$; Taylor expand:

$$= C_0 + C_2 \mu_I^2 + C_4 \mu_I^4 + \dots$$

• valid for $\frac{\mu_I}{T} < \frac{\pi}{3}$

• Good luck ... C_4, C_6, \dots terms all small over this range.

• So, boldly continue:

$$T_c(\mu) = C_0 - C_2 \mu^2 + C_4 \mu^4 \dots$$

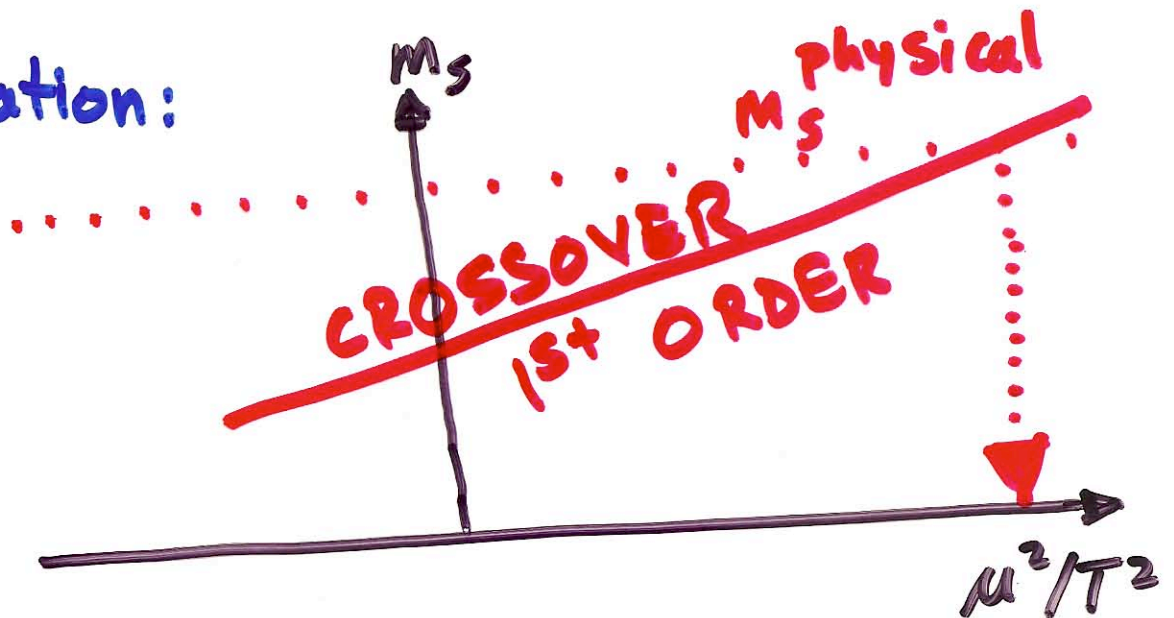
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Curvature of crossover line on phase diagram

CRITICAL POINT ??

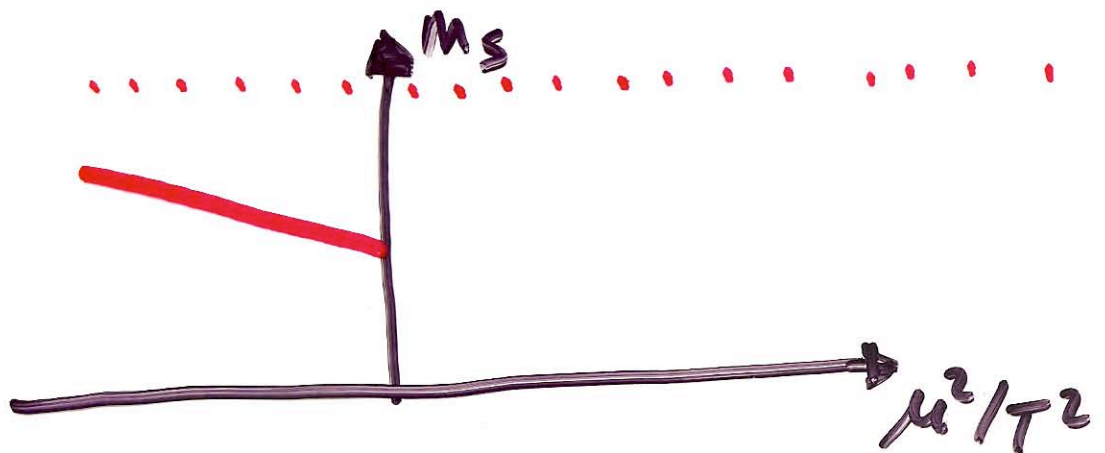
• Calculate

$$\frac{\partial}{\partial \mu^2} \left[M_a \text{ at which transition goes from 1st order to crossover} \right]$$

• Expectation:



• deForcrand + Philipsen find:



• \Rightarrow No CRITICAL POINT
with $\frac{\mu}{T} < \theta(1)$.

CONCERNS, aka "SYSTEMATIC ISSUES"

Let's defer their discussion to after Philippe's talk, but here are two:

- $N_f = 4$

- Staggered fermions with

$N_f = 3$ or $2+1 \dots$

- $\text{Det}^{3/4}$ or $\text{Det}^{1/2} \text{Det}^{1/4}$

- First order phase transition at small m_s originates from 't Hooft $uds\bar{u}\bar{d}\bar{s}$ interaction, in low energy effective theory. Pisarski Wilczek
- do staggered fermions describe this adequately??

Fukushima, Stephanov

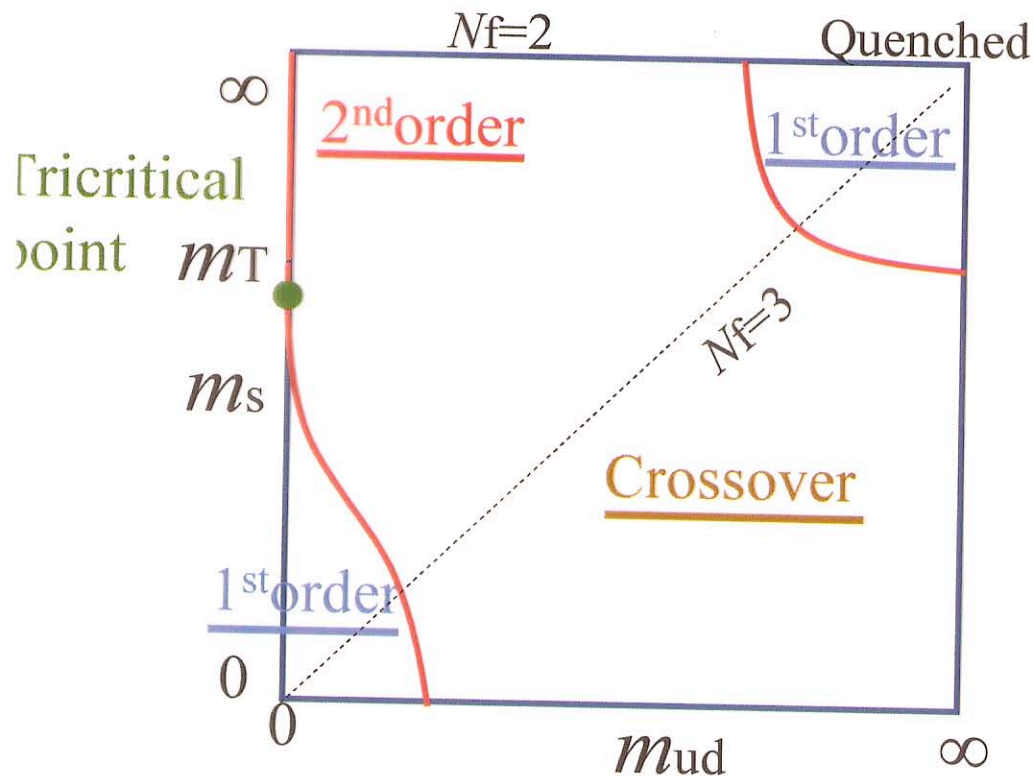
also an issue for F+K

Mean field argument Ejiri

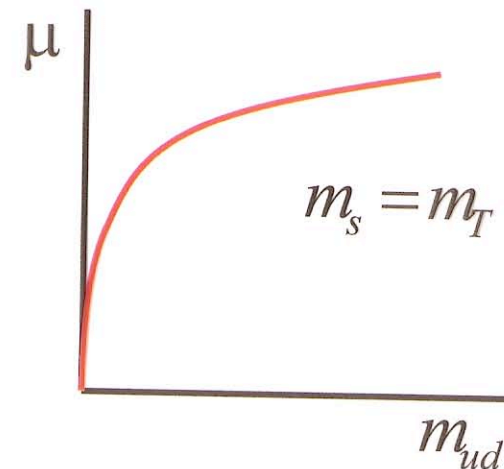
- Sigma model prediction near tri-critical point on the m_s axis.

$$V_{\text{eff}}(\sigma) = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4 + \frac{1}{6}c\sigma^6 - h\sigma$$

Critical point: $\frac{\partial^n V_{\text{eff}}}{\partial \sigma^n} = 0 \quad (n=1,2,3)$



$$b \sim (m_T - m_s) \quad \Rightarrow \quad b \sim \mu^2$$



$$m_{ud}^{\text{crit}} \sim (m_T - m_s)^{5/2}$$



$$m_{ud}^{\text{crit}} \sim \mu^5$$

③ Taylor Expansion of the Pressure.

Bielefeld - Swansea; Gauri Gupta

Calculate the coefficients in:

$$\frac{P}{T^4} = b_0(T) + b_2(T)\mu^2 + b_4(T)\mu^4 + b_6(T)\mu^6 + \dots$$

and hence in:

$$\chi_B \equiv \frac{\partial^2 P}{\partial \mu^2} = c_0(T) + c_2(T)\mu^2 + c_4(T)\mu^4 + c_6(T)\mu^6 + \dots$$

which should diverge at critical point.

Several ways to look for critical point:

- Look for μ at which χ_B peaks
- Do Taylor expansion at varying M_q

and evaluate

$$\frac{\partial}{\partial \mu^2} \left[M_q \text{ at which crossover at } \mu=0 \text{ becomes 1st order} \right]$$

[Defer discussion of these to Karsch.]

- And...

RADIUS OF CONVERGENCE METHOD

Use fact that Taylor expansion must break down at critical point.

Bielefeld Swansea; Gavai Gupta

New results from Gavai + Gupta,
June 2008 & earlier this workshop:

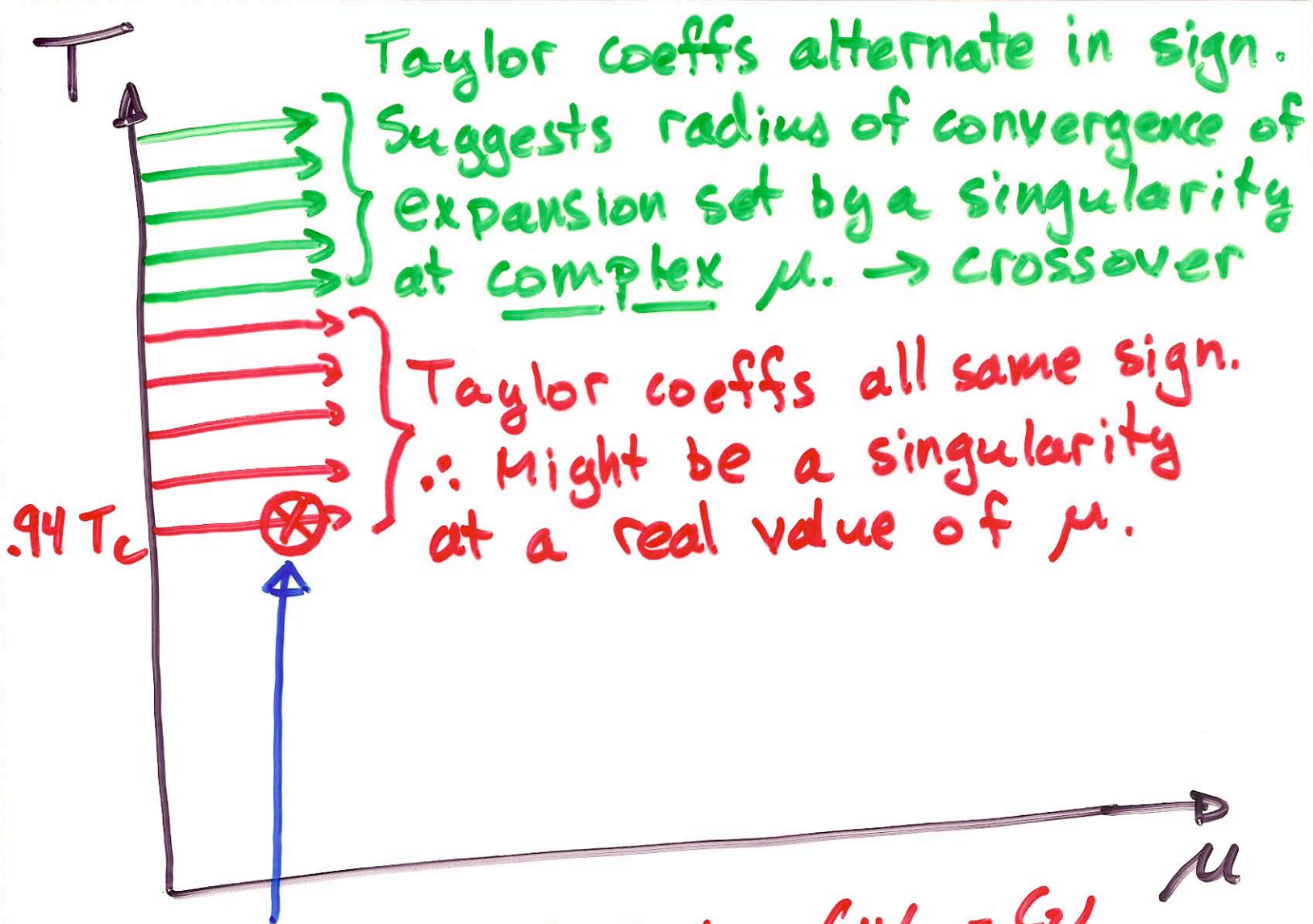
- $N_T = \underline{\underline{6}}$; $V = 24^3$

- $N_f = 2$; $m_\pi = 230 \text{ MeV}$

- staggered fermions, so
maybe not a bad thing
that $N_f = 2$.

- Taylor coefficients $c_0(T)$,
 $c_2(T)$, $c_4(T)$, $c_6(T)$.

[ie up to μ^6 term in P]



At this T , find $c_6/c_4 = c_4/c_2 = c_2/c_0$,
 as would be the case for a pole at real μ . And, as yields a consistent estimate of radius of conv.
 Also, at the same T , coeffs have expected finite size scaling (upon comparing $LT = 2$ and 4).
 I identify this ~~T~~ T as T_E , and this radius of convergence as μ_E .

Gavai and Gupta find:

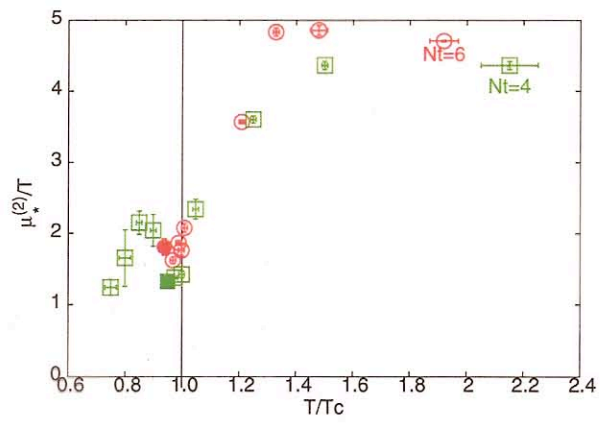
$$\frac{T^E}{T_c} = 0.94 \pm 0.01$$

$$\frac{M^E}{T^E} = 1.8 \pm 0.1$$

Issues:

- $N_T = 6$. "Crawling towards the continuum limit." Gupta
- $N_f = 2 \rightarrow N_f = 2+1$
- What is the best estimator of T^E , ie what combination of criteria, given $C_0(T)$, $C_2(T)$, $C_4(T)$, $C_6(T)$?

Radius of convergence



Lattice spacing dependence quantifies possible systematic errors.

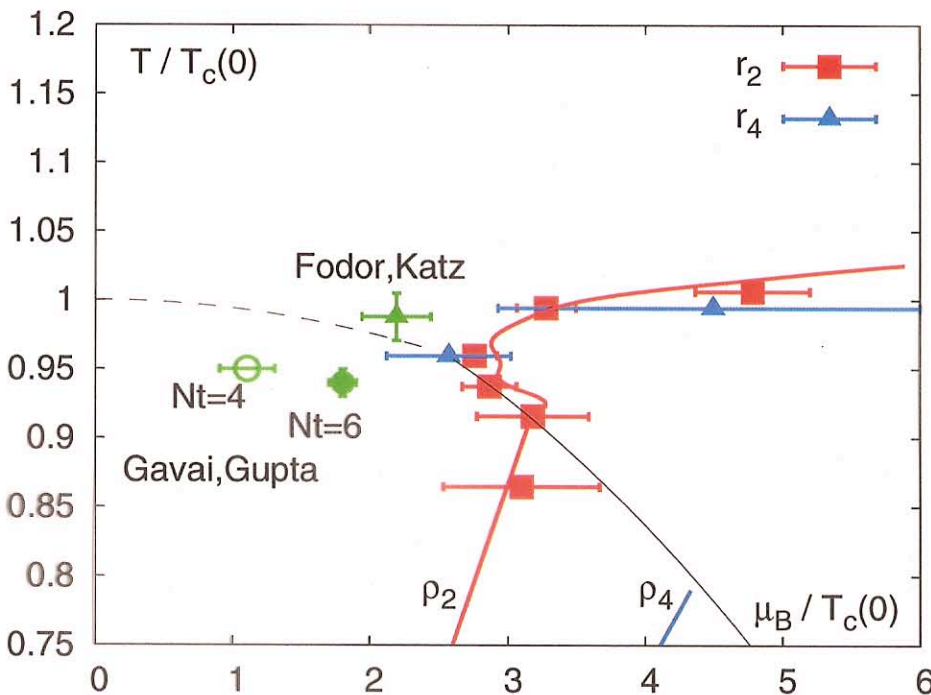
Estimating $T_c(\mu_c)$ and μ_c/T

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $\mathcal{O}(a^2)$ improved staggered action (p4fat3)

● estimator for μ_c :

$$\left(\frac{\mu_c(T)}{T_c(0)} \right)_n \equiv \rho_n = \frac{T}{T_c(0)} \sqrt{\frac{c_n}{c_{n+2}}}$$



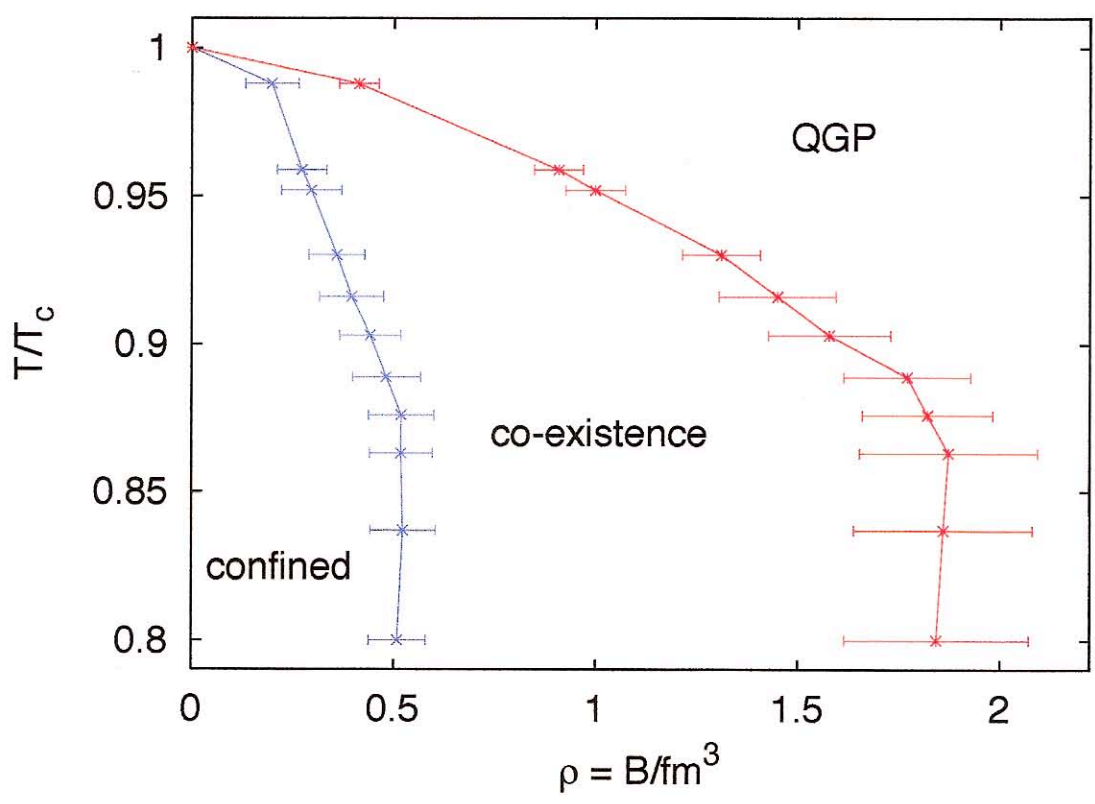
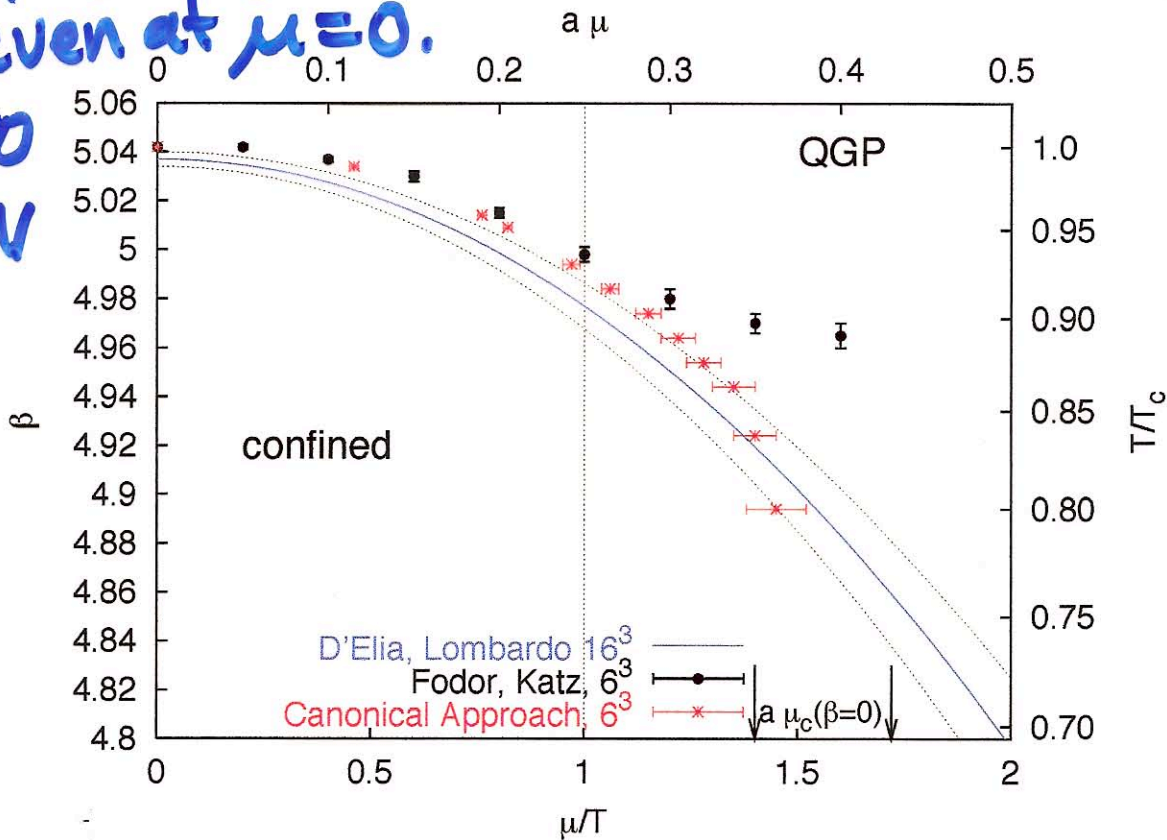
- slight quark mass dependence
- weak cut-off dependence
- $\mathcal{O}(\mu^6)$ requires more statistics

LATTICE CALCULATIONS AT FIXED β_B

$N_s = 4 \rightarrow$ 1st order even at $\mu = 0$.

de Forcrand Kratochvila

$m_\pi = 350$
MeV



Calculation on 6^3 lattice, with $0 < B < 30$. ($V \sim (2\text{fm})^3$)

- Will be very interesting to see what they find with $N_f = 2+1$.
- Determining the location of the critical point this way will have very different "systematic error" relative to calculations relying on $\mu/T < 1$. (ie reweighting ^{Fodor} ^{Katz} or Taylor expansion Ejiri et al, Gouzi Gupta)
- In principle can be pushed to larger μ/T , but remains to be seen how large a V can be reached at a given μ or n_B .

LOCATING THE CRITICAL POINT

Location still uncertain:

$\mu_B^{\text{critical point}}$

$T_c(\mu=0)$

$\sim 2, > \theta(3), \sim 1.7, \sim 2$

Fedor
Katz

Philipsen
deForcrand

Gavai
Gupta


RBC
-31
↓

- gaining confidence will require "crawling towards the continuum limit", and several methods agreeing.
- If $\mu_B^{\text{C.P.}} < 3T$, this \uparrow will happen
- If $\mu_B^{\text{C.P.}} > 3T$, all methods should come to agree on this. But, barring an unforeseen algorithmic breakthrough, unlikely that lattice calculations will locate it with confidence.

In the race between lattice calculations and experimental searches to locate the critical point, the lattice team is running strongly but not yet threatening to end the race.

So, let's turn to experimental searches

HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

- ① Need evidence that at large \sqrt{s} , i.e. small μ , collisions equilibrate well above the crossover. $v_2 @ RHIC$.
- ② Decrease \sqrt{s} , moving freezeout point to larger and larger μ_B .
- ③ Look for signatures:
 - a) Of the critical point itself. Those relying on the long wavelength gaussian fluctuations occurring only near . Rise and then fall as μ_B increases.
 - b) Onset of signatures of non-equilibrium "lumpy" final state expected after cooling through a first order transition.
Mishustin; Dumintra Pasch Stöcker; Randrup;
Koch Majumder Randrup; ...
→ NON Gaussian fluctuations

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (means fluctuations couple to $\pi\pi$) and baryon number. The more effectively equilibrium is maintained, the longer the correlation length ξ gets, the bigger the signatures:

- Gaussian event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ . Fujii Ohtani; Sen Stephano
- Vary μ by varying \sqrt{s} , search for enhancement of these fluctuations in a window in \sqrt{s} , i.e. μ . Stephanov KR Shuryak
- Examples But first:

HOW LARGE CAN ξ GET?

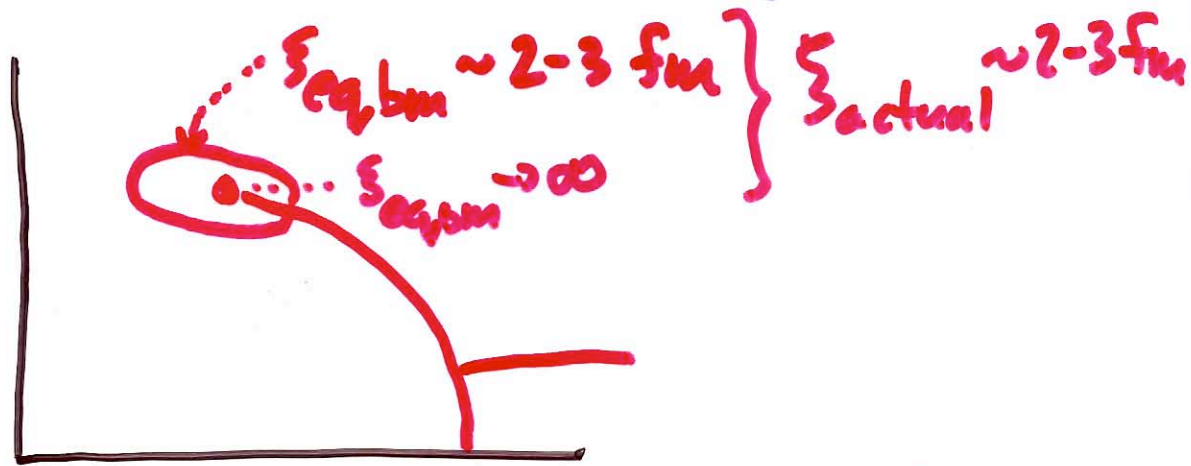
HOW CLOSE TO \bullet NEED WE BE?

• Obviously ξ limited by finite size of system. But, turns out that finite time is a more severe limitation.


Berdnikov KR; Asakawa Nonaka


• Finite time spent in critical region means that even if equilibrium value of ξ is much larger, actual ξ won't grow bigger than 2-3 fm.

• Means no need to hit \bullet precisely.



Signatures will be just as big if you pass anywhere in \circ . No bigger, even if you hit \bullet .

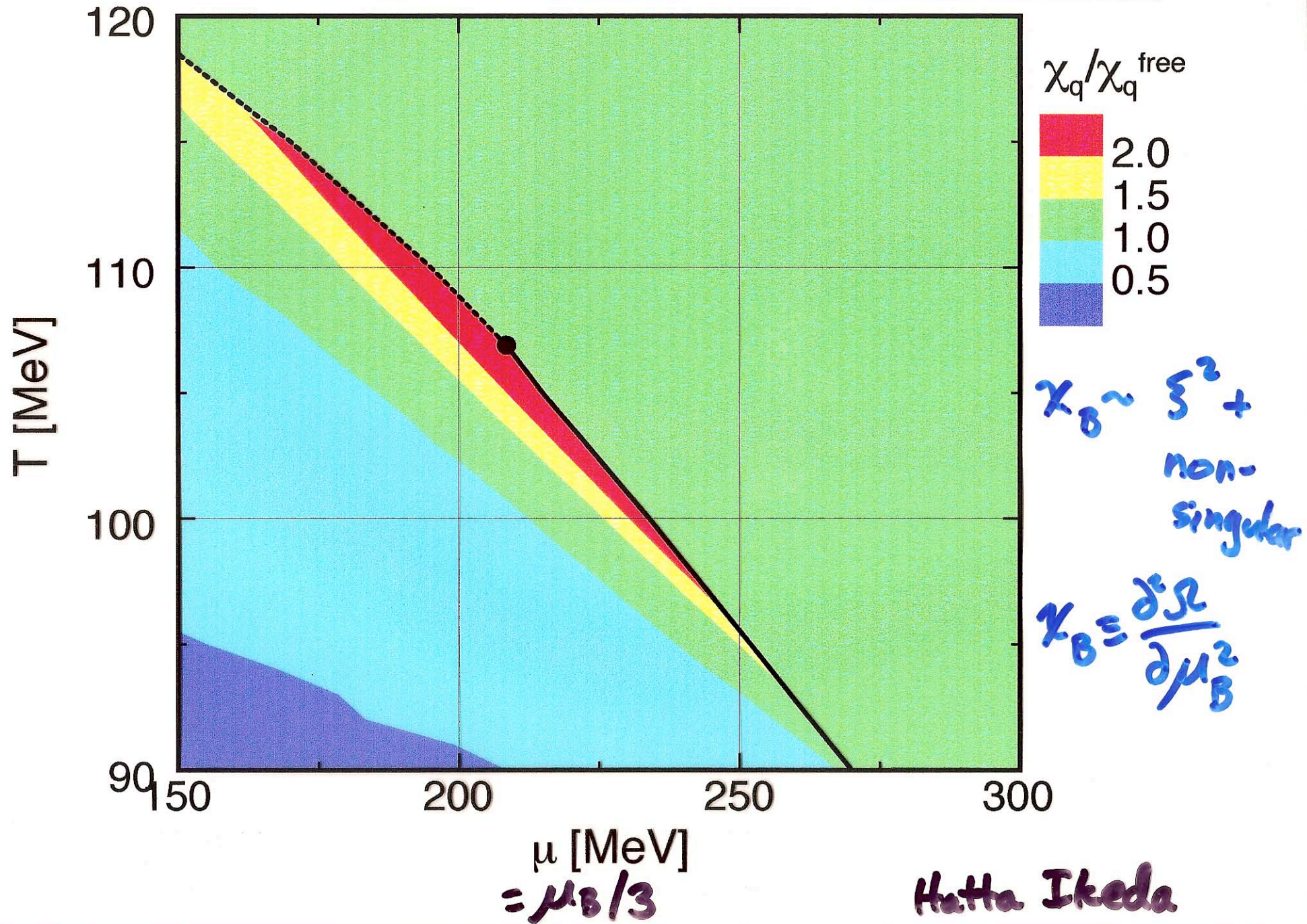
- Hatta + Ikeda calculated "'s" in a model, but did so with contours of χ_B rather than ξ . → Figs.

The robust point is that the extent of these 's in μ_B is not small. Width in μ_B is ~ 100 MeV, an estimate that is both crude and uncertain. Can this be obtained on lattice??

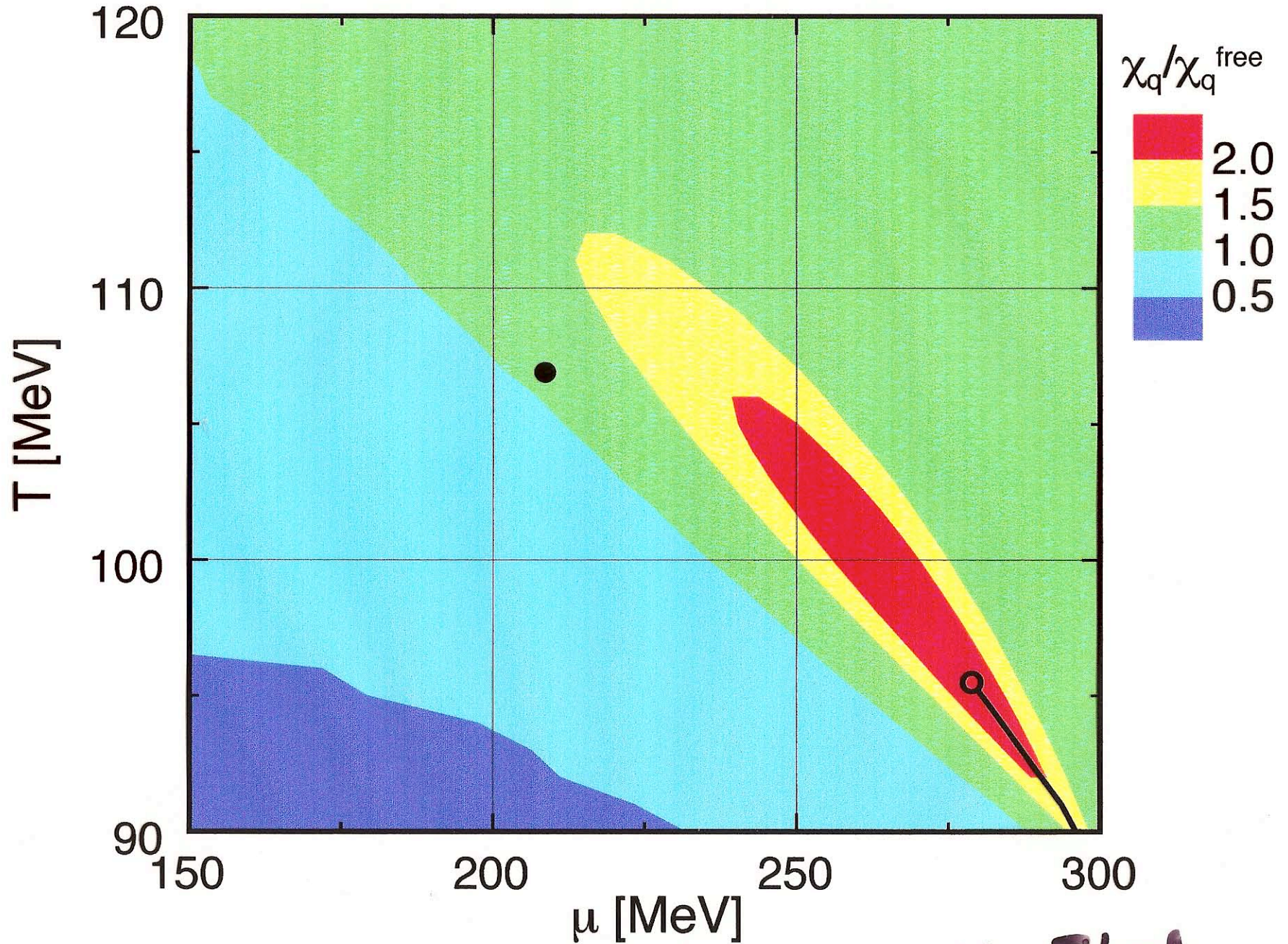
- NB also: since ξ cannot be $> 2-3$ fm, heavy ion collision experiments can never be used to measure the critical exponents of the 2nd order critical point. That's OK: we know it is Ising. What we don't know, and need experiments for, is where it is located.

$m_u = m_d = 0$

MODEL ANALYSIS OF EXTENT OF CRITICAL REGION



$$m_u = m_d = 5 \text{ MeV}$$



$$= \mu_B / 3$$

Hatta Ikeda

SIGNATURES OF CRITICAL POINT

Decreasing \sqrt{s} \rightarrow Increasing μ_B

\sqrt{s} :	200 GeV	12 GeV	5 GeV
$\mu_B^{\text{freezeout}}$:	25 MeV	300 MeV	550 MeV

Vary \sqrt{s} , and hence μ_B , and look for nonmonotonic enhancement (rise and then fall) of Gaussian event-by-event fluctuations of:

- i) Mean p_T of low p_T pions
- ii) proton number
- iii) Particle ratios involving pions and/or protons.

And, also, signatures due to focussing of trajectories:

- iv) elevation of $T_{\text{freezeout}}$
- v) steepening of \bar{p} spectrum

MEAN P_T OF LOW P_T PIONS

Stephanov KR Shuryak

Advantage: directly controlled by long wavelength fluctuations of the chiral order parameter.

Disadvantage: will they survive the late time hadron gas??

Result: NA49 has done a very nice analysis of Pb Pb collisions at $\sqrt{s} = 6.3, 7.6, 8.8, 12.3, 17.3$ and sees no \sqrt{s} dependence \rightarrow Fig

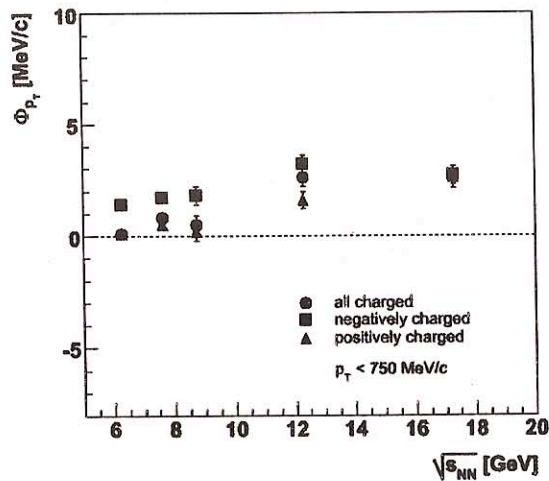
So.....

- try lighter ions, so $T_{freezeout}$ higher, shorter time in hadron gas phase. \rightarrow NA61
- try other observables that are harder to wash out....

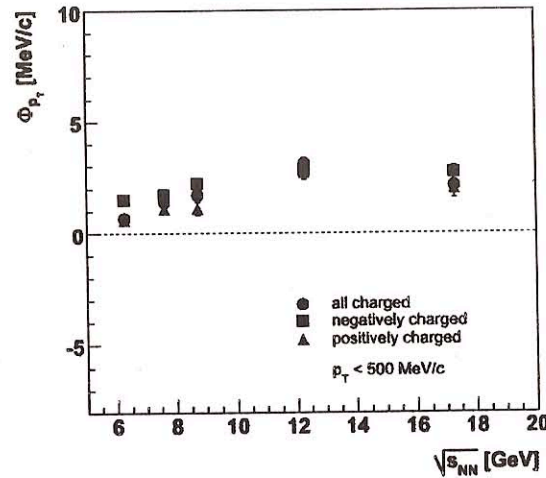
**Fluctuations due to the critical point should be dominated
by fluctuations of pions with $p_T \leq 500$ MeV/c**

M. Stephanov, K. Rajagopal, E. V. Shuryak (Phys. Rev. D60, 114028, 1999):
suggestion to do analysis with several upper p_T cuts

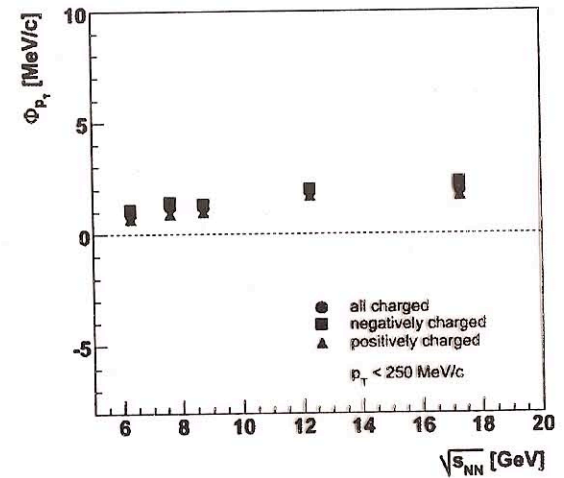
$p_T < 750$ MeV/c



$p_T < 500$ MeV/c



$p_T < 250$ MeV/c



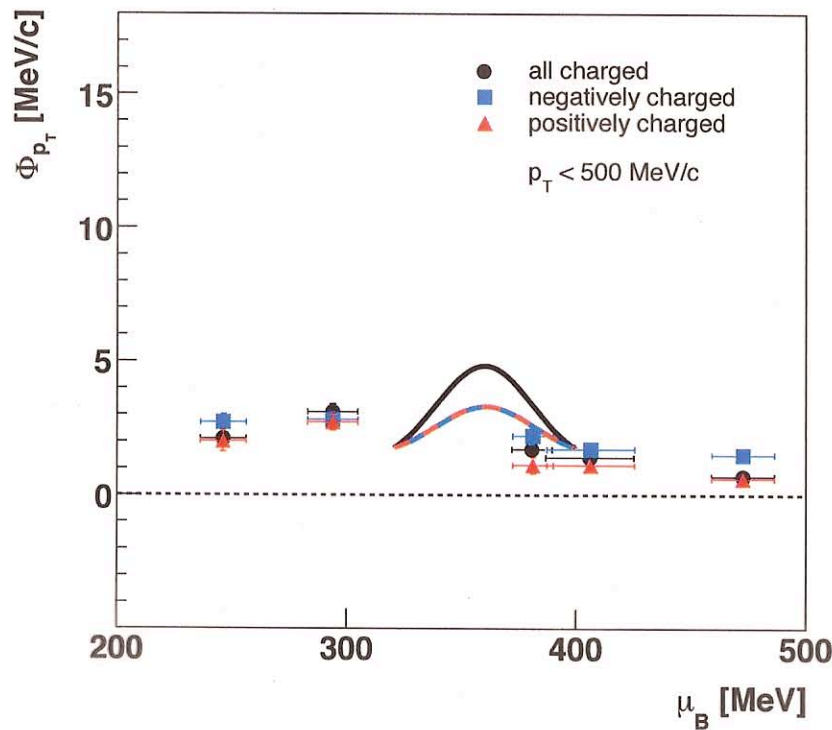
**No significant energy dependence of Φ_{PT} measure
also when low transverse momenta are selected.**

Remark: predicted fluctuations at the critical point should result in $\Phi_{PT} \cong 20$ MeV/c, the effect of limited acceptance of NA49 reduces them to $\Phi_{PT} \cong 10$ MeV/c

NA49 data; slide from K. Grebieszko talk at CPD 2007

- Anticipated effect of critical point in NA49 acceptance:
(large systematic error on prediction)

- $\Delta\Phi_{p_T} \approx 1.5$ MeV/c (for negative/positive particles separately)
- $\Delta\Phi_{p_T} \approx 3$ MeV/c (for all charged particles)



NA49 data:
arXiv:0805.2245 [nucl-ex]

μ_B from hadron gas fit:
F. Becattini et al,
Phys. Rev. C 73 (2006) 044905

Amplitude of effect:
Stephanov, Rajagopal, Shuryak,
Phys.Rev.D60:114028
and private communication

Position of critical point:
Z. Fodor and S. Katz,
JHEP 0404, 050, 2004

Width of critical point:
Y. Hatta and T. Ikeda,
Phys. Rev. D67, 014028, 2003

Onset of deconfinement? No predictions
 Critical point? No signal observed

MULTIPLICITY FLUCTUATIONS

Also coupled to order parameter fluctuations. In fact their effect on multiplicity fluctuations is greater than on P_T fluctuations.

Stephanov KR Shuryak

BUT: large "background", due to impact parameter fluctuations.

Still, nonmonotonic variation with \sqrt{s} would be suggestive.

BARYON, AND PROTON, NUMBER FLUCTUATIONS

Hatta Ikeda; Hatta Stephanov

- seen on the lattice → FIG
- should be looked for in
experimental data

$\frac{\partial^2 \Omega}{\partial \mu_B^2} \rightarrow B\text{-fluctuations}$
 $\sim \xi^2 + \text{nonsingular}$

$\frac{\partial^2 \Omega}{\partial \mu_I^2} \rightarrow \text{no enhanced (u-d)}$
 fluctuations
 $\sim \text{nonsingular}$

Suggests
 $\mu_q \sim 1$
 T
 getting
 close
 to \bullet .

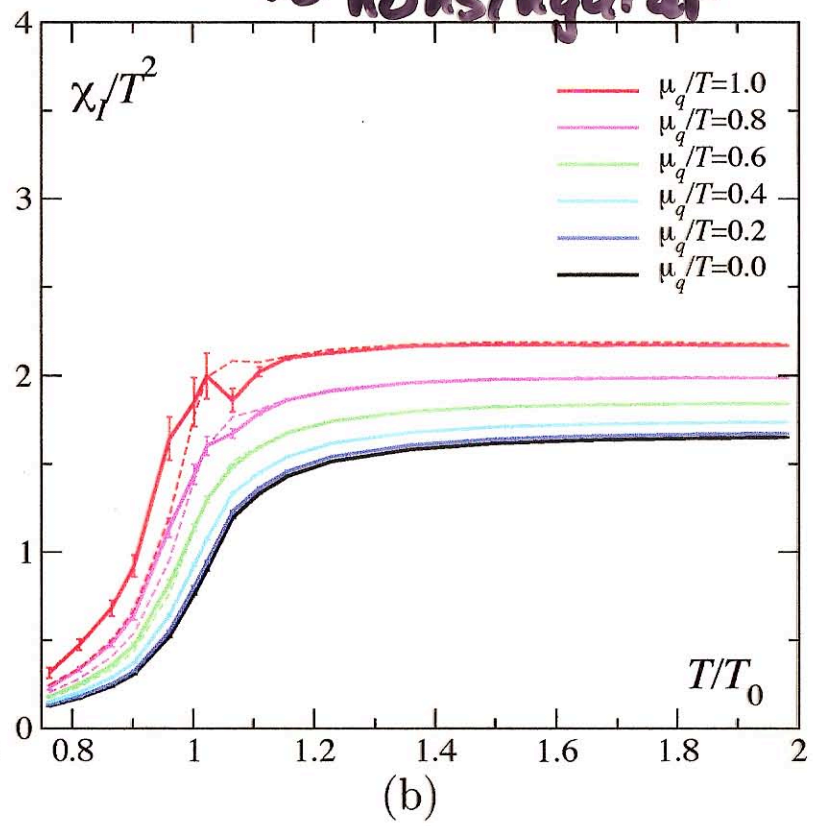
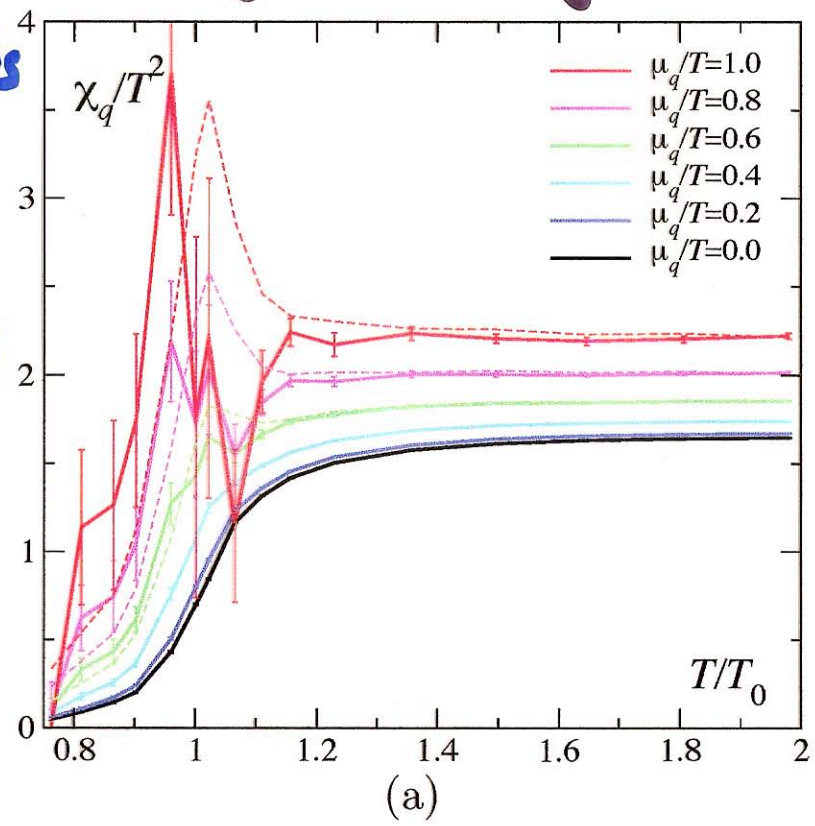


Figure 3.3: The quark number susceptibility χ_q/T^2 (left) and isovector susceptibility χ_I/T^2 (right) as functions of T/T_0 for various μ_q/T ranging from $\mu_q/T = 0$ (lowest curve) rising in steps of 0.2 to $\mu_q/T = 1$, calculated from a Taylor series in 6th order. Also shown as dashed lines are results from a 4th order expansion in μ_q/T .

(Because B fluctuates while isospin does not, proton
 fluctuations \sim B fluctuations)

Hatta Stephanov

Ejiri et al

PARTICLE RATIOS

NA 49

- Originally motivated by peak in $\langle K \rangle / \langle \pi \rangle$ at $\sqrt{s} = 7.6$ GeV.

To better understand this, look at fluctuations of K/π ratio.

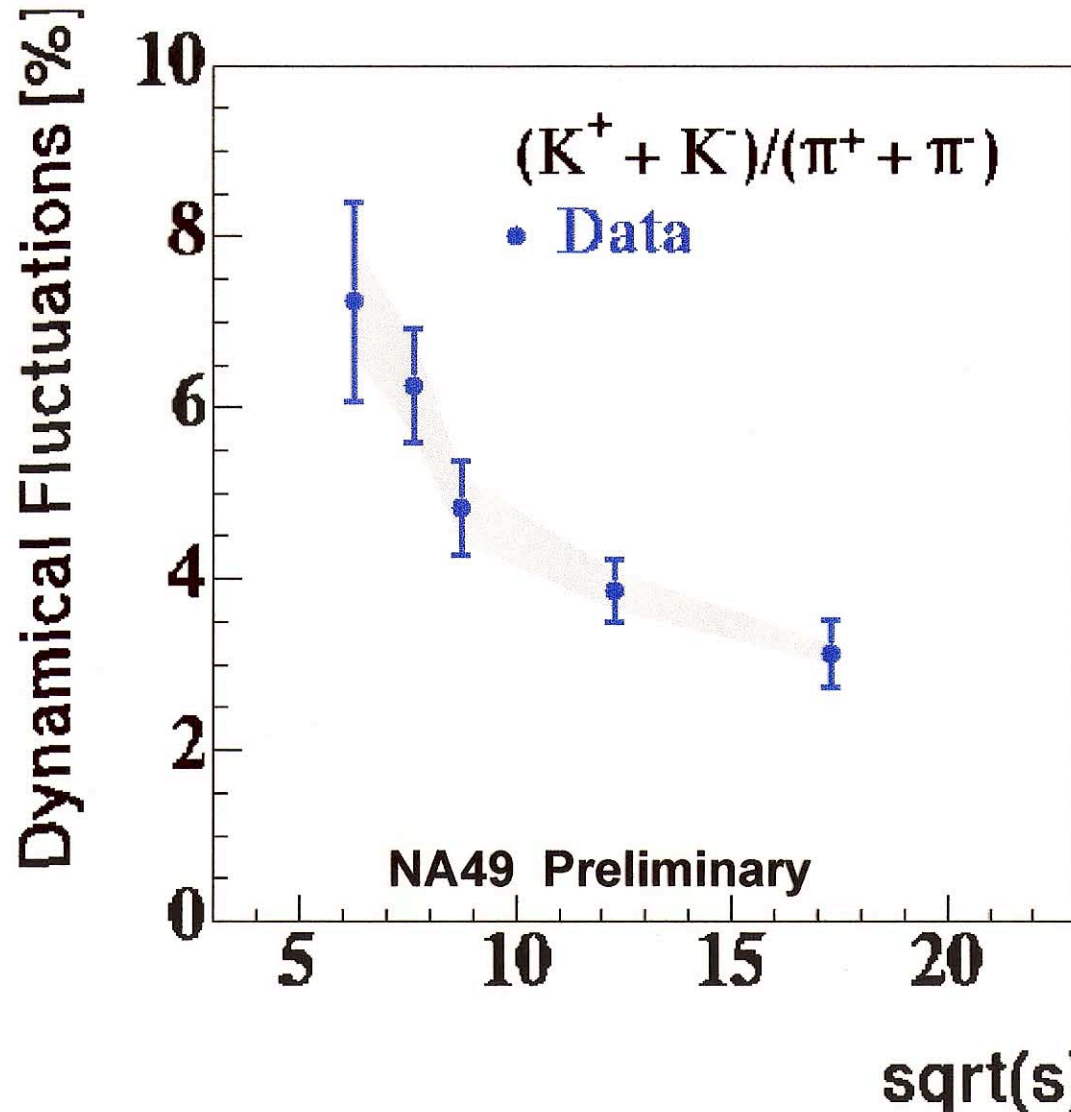
- Now motivated by observation that these fluctuations will better survive the late time hadron gas.

RESULT:

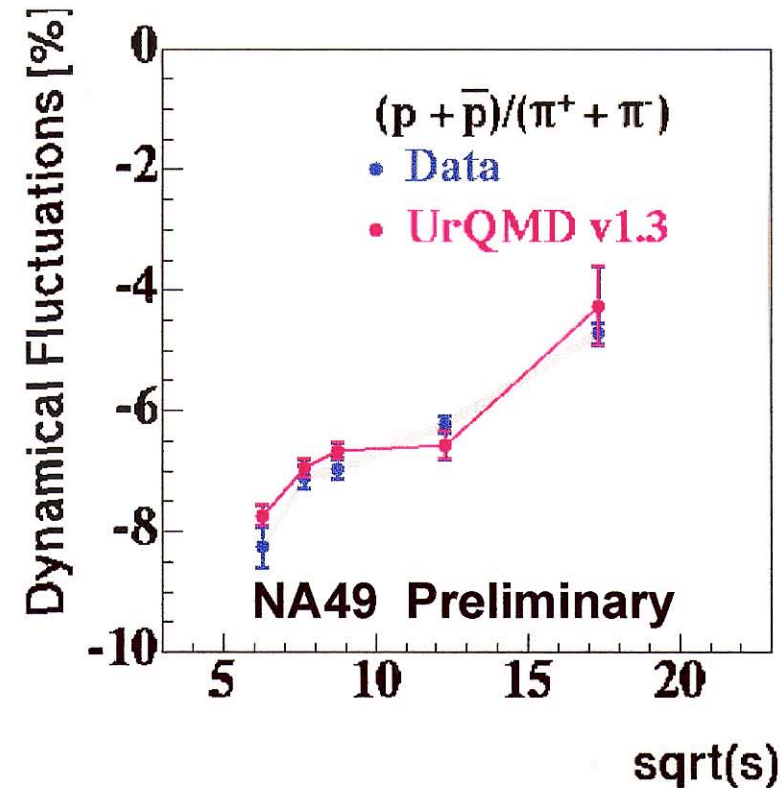
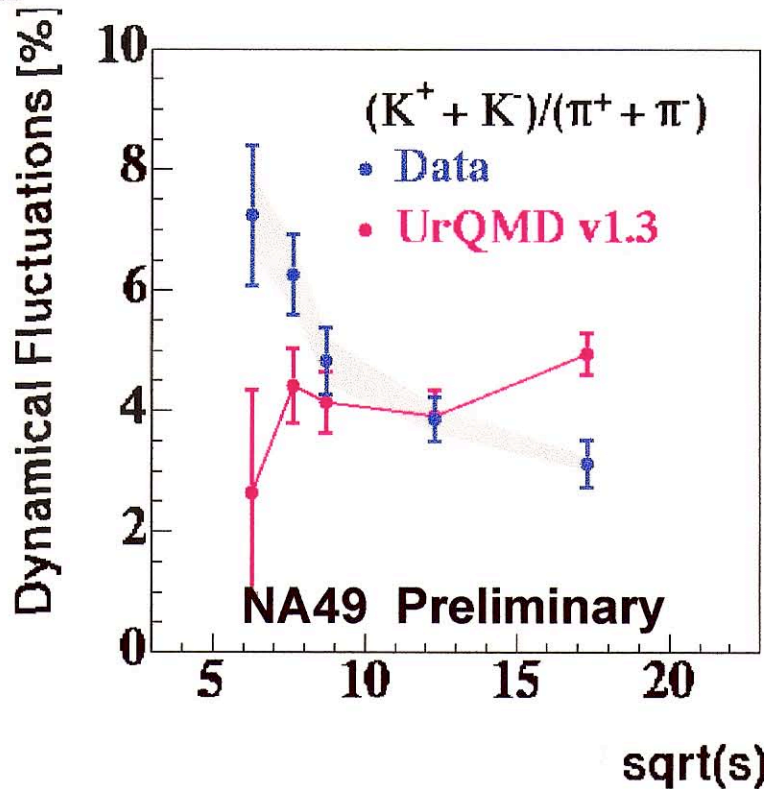
- Large K/π fluctuations at $\mu_B \sim 400 - 450$ MeV

- Why no P/π fluctuations ???

The E-by-E Kaon/Pion Ratio



Increased fluctuation signal at lower beam energies



- K/π fluctuations increase towards lower beam energy
 - Significant enhancement over hadronic cascade model
- p/π fluctuations are negative
 - indicates a strong contribution from resonance decays

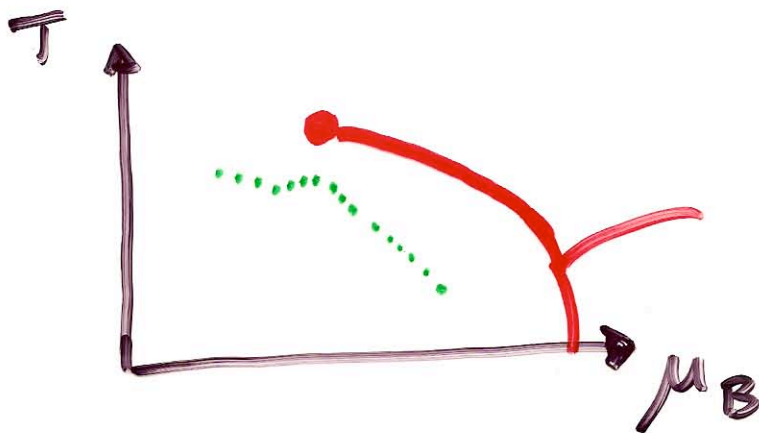
Intriguing....

- Large event-by-event fluctuations in K/π at $\mu_B \sim 400-450$ MeV
- Are the K/π fluctuations dominated by low p_T π ? Apparently not....
- Why no P/π fluctuations ???
- Koch Majumder Randrup suggest the K/π fluctuations due to hadronization of "blobs" left by a first order transition.
 - If so, expect non Gaussian fluctuations (vs. rapidity?)
 - And, expect critical point at lower μ_B .
- At present, IMHO, these data are a very intriguing anomaly that is not well explained.

LINGERING AND FOCUSSING

Isentropic trajectories passing near the critical point on the phase diagram:

- Linger, due to enhanced C_V .
 - energy density, entropy density change at usual rate;
T changes more slowly
 - likely a small effect, since C_V dominated by other modes, not by low P_T modes.
 - in principle, an elevated kinetic freezeout temperature



Stephanov
KR Shuryak

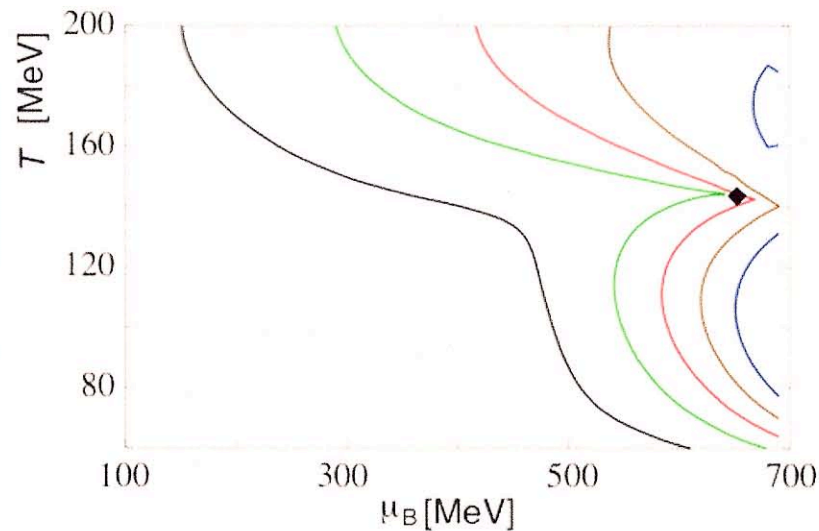
- are Focussed

Stephanov KR Shuryak
Asakawa Nonaka

Focusing Effect

■ Isentropic trajectories on T - μ_B plane

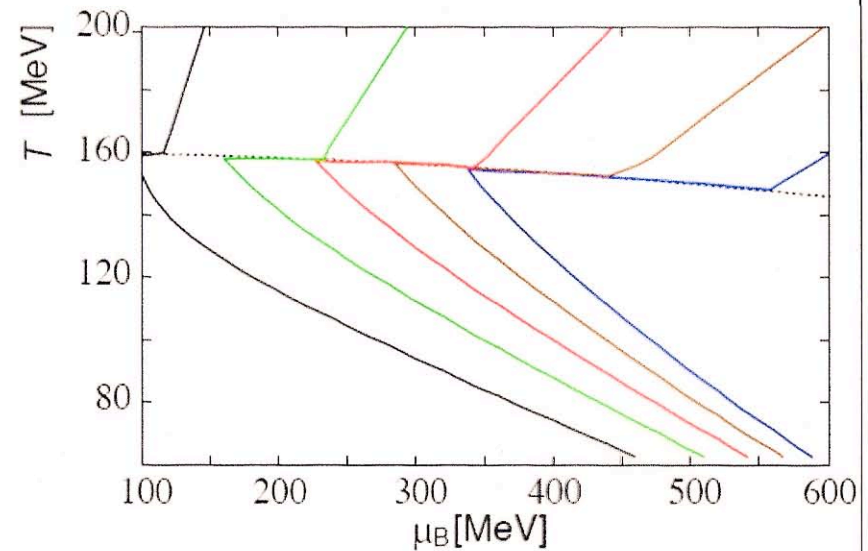
With QCD critical point



Focused

Bag Model +
Excluded Volume Approximation
(No Critical Point)

= Usual Hydro Calculation

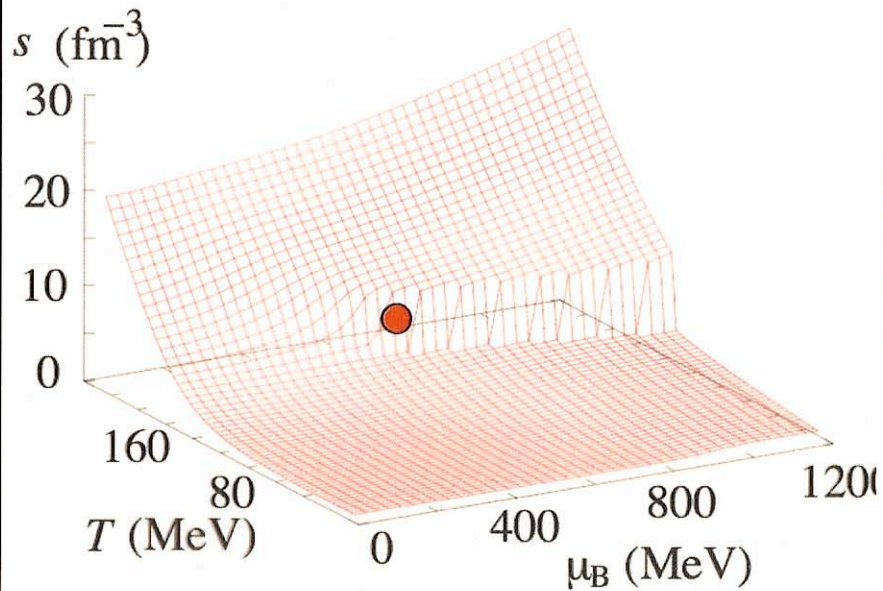


Not Focused

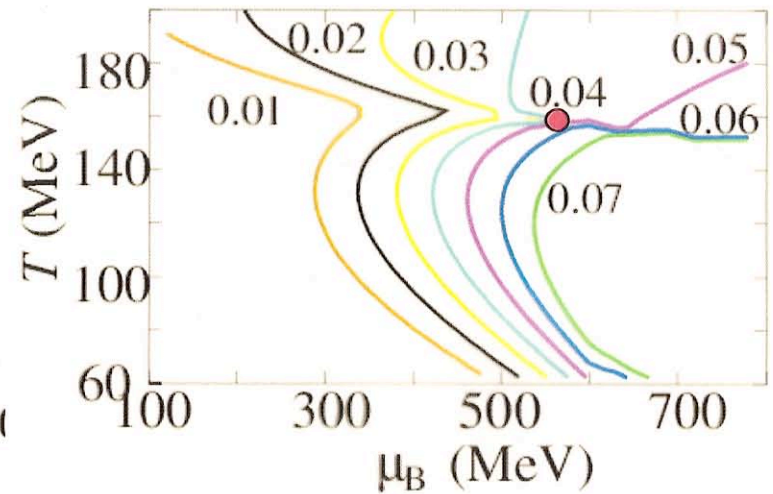
Equation of State

- QCD critical point $(\mu_B, T) = (550, 159)$

entropy density



trajectories



Focussing of trajectories that pass near the critical point:

- is another argument that one need not take very small steps in μ_B
- has observable consequences that do not involve event-by-event fluctuations.

Asakawa Bass Muller Nonaka

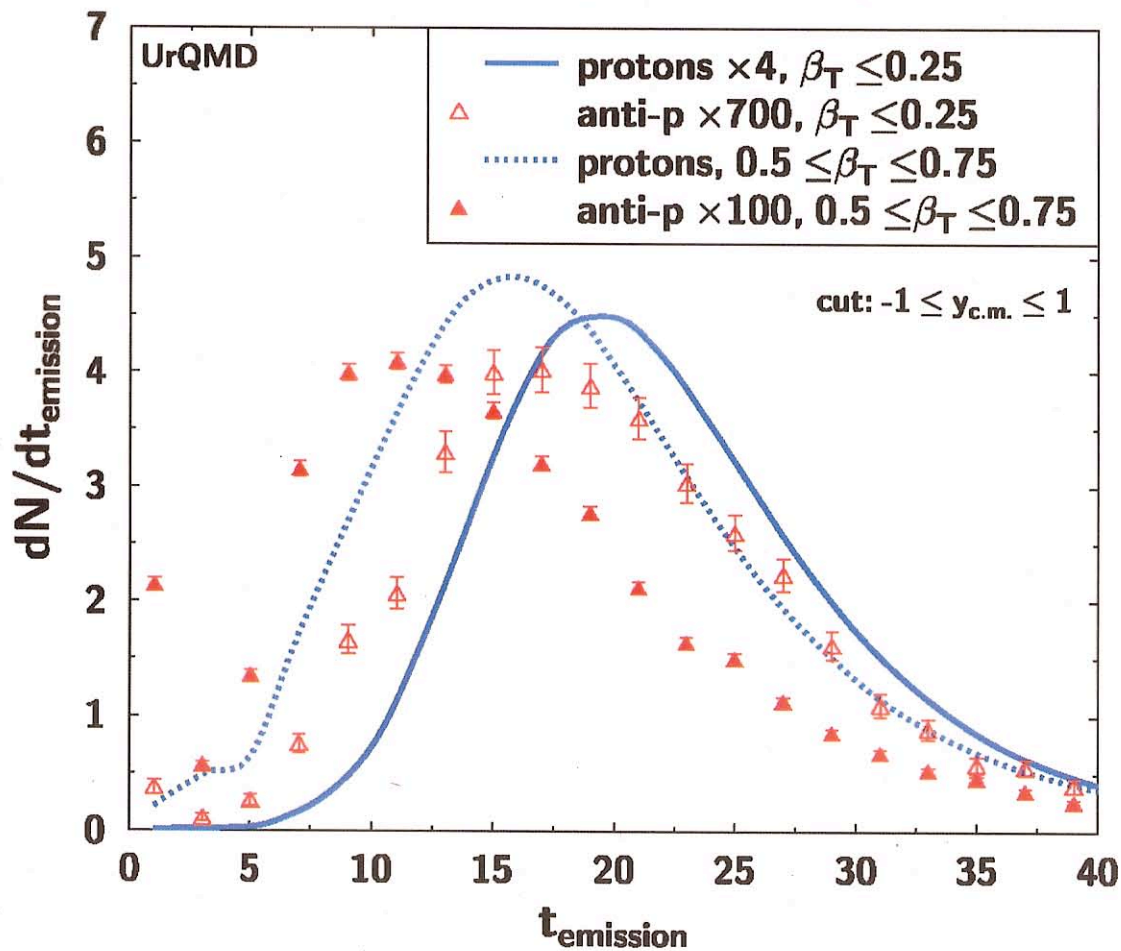
Two key ideas:

- Higher P_T p & \bar{p} freezeout earlier

- Only for trajectories near critical point, higher P_T p & \bar{p} freezeout at higher μ/T
 $\Rightarrow \bar{p}/p$ ratio drops w P_T .

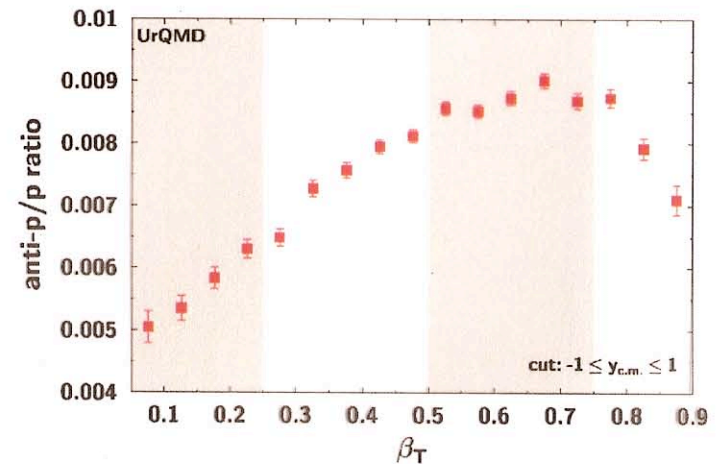
Emission Time Distribution

Au+Au, $E_{\text{lab}}=40$ GeV/A

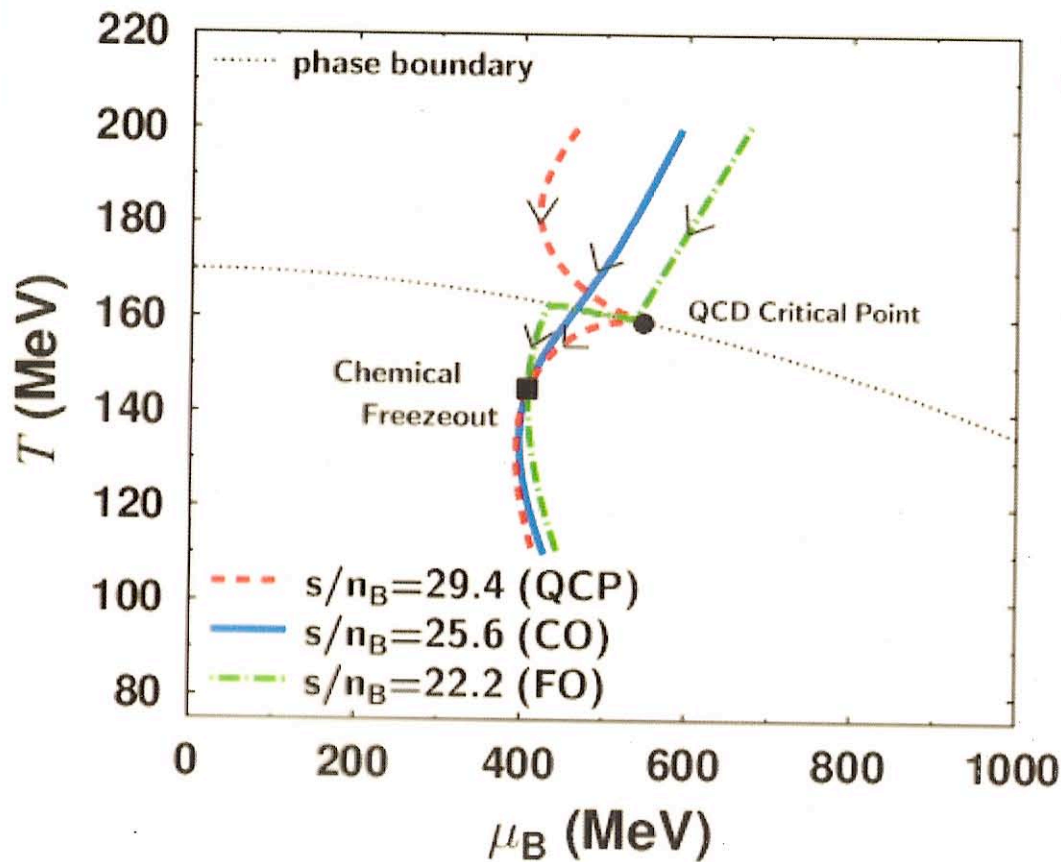


Emission Time

- Larger β_T , earlier emission
- No CEP effect (UrQMD)



Isentropic Trajectories



• Hadronization occurs from the phase boundary and chemical freezeout point

{

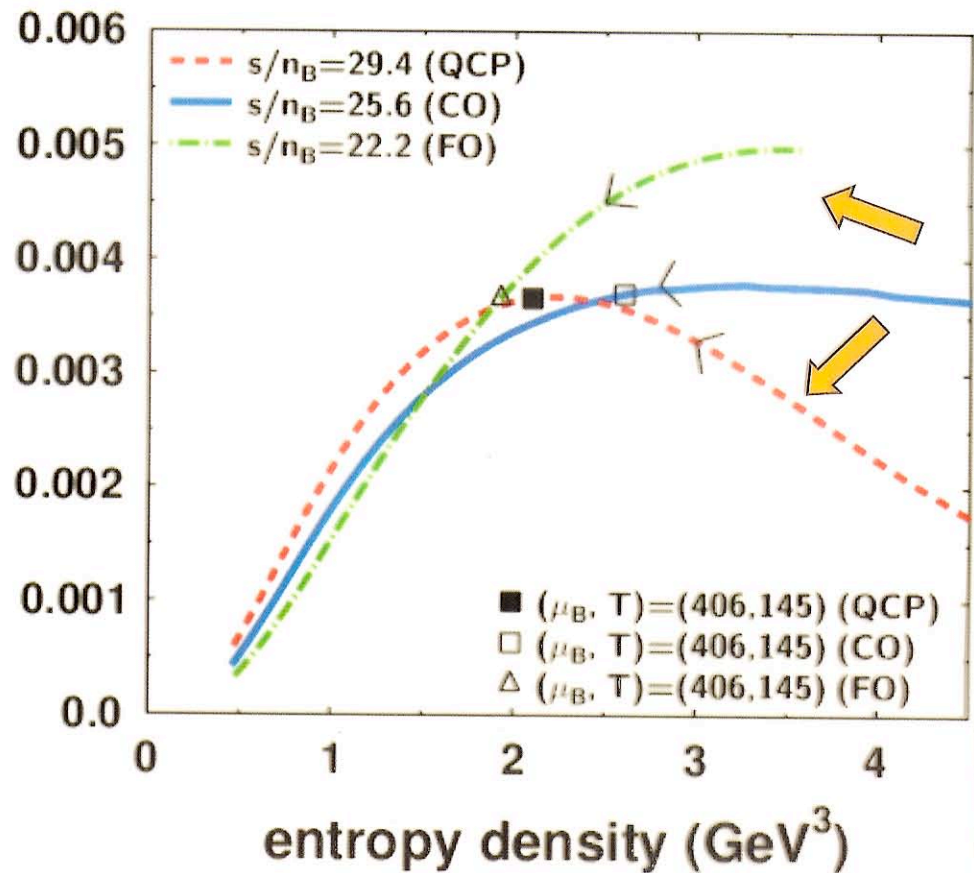
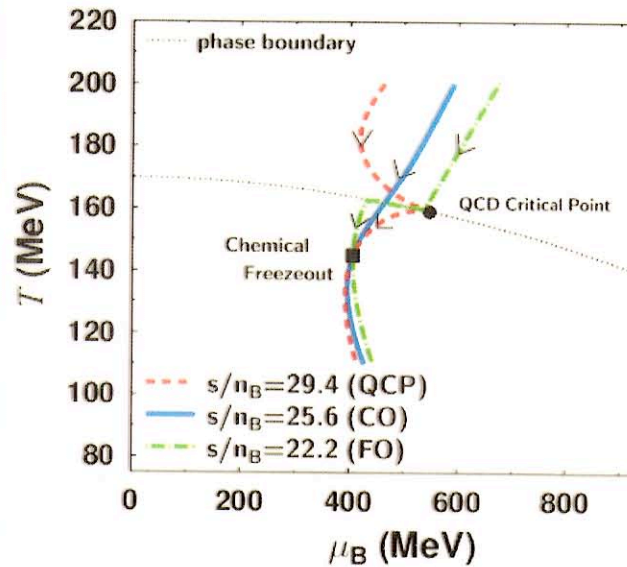
- FO, CO \nearrow
- QCP \rightarrow

 $\frac{\mu_B}{T}$

$\rightarrow \bar{p}/p$ ratio

$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

Signature of QCP

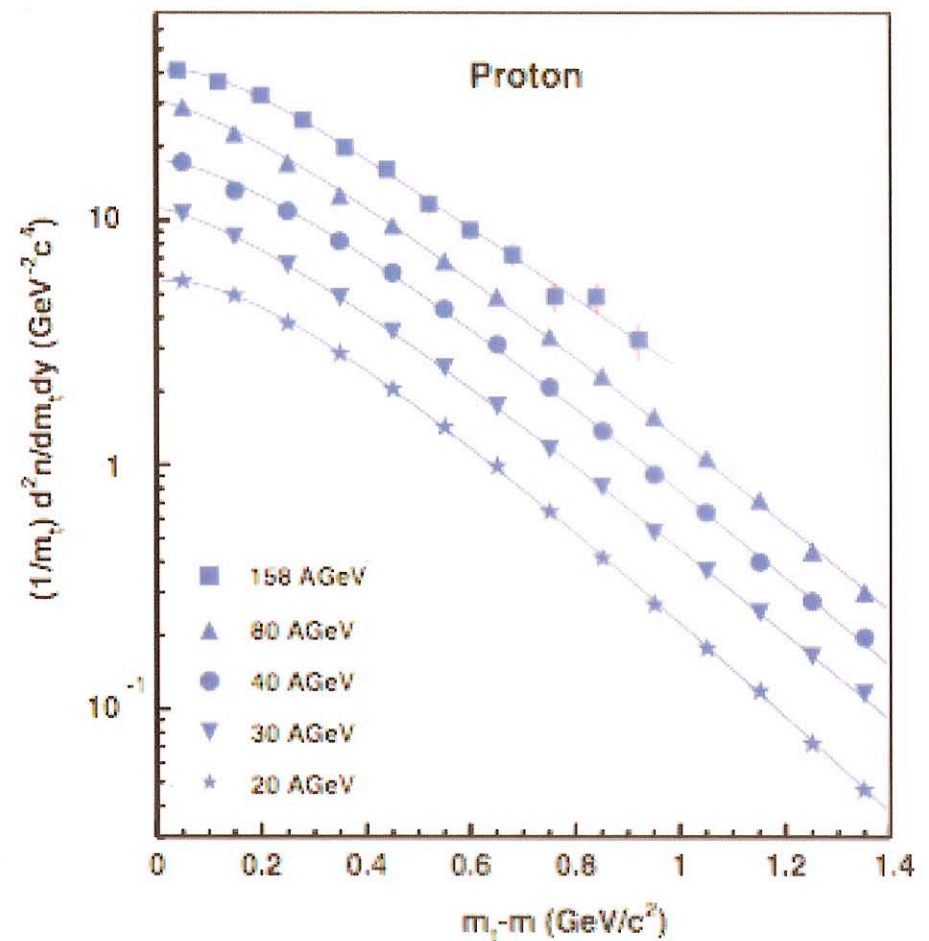
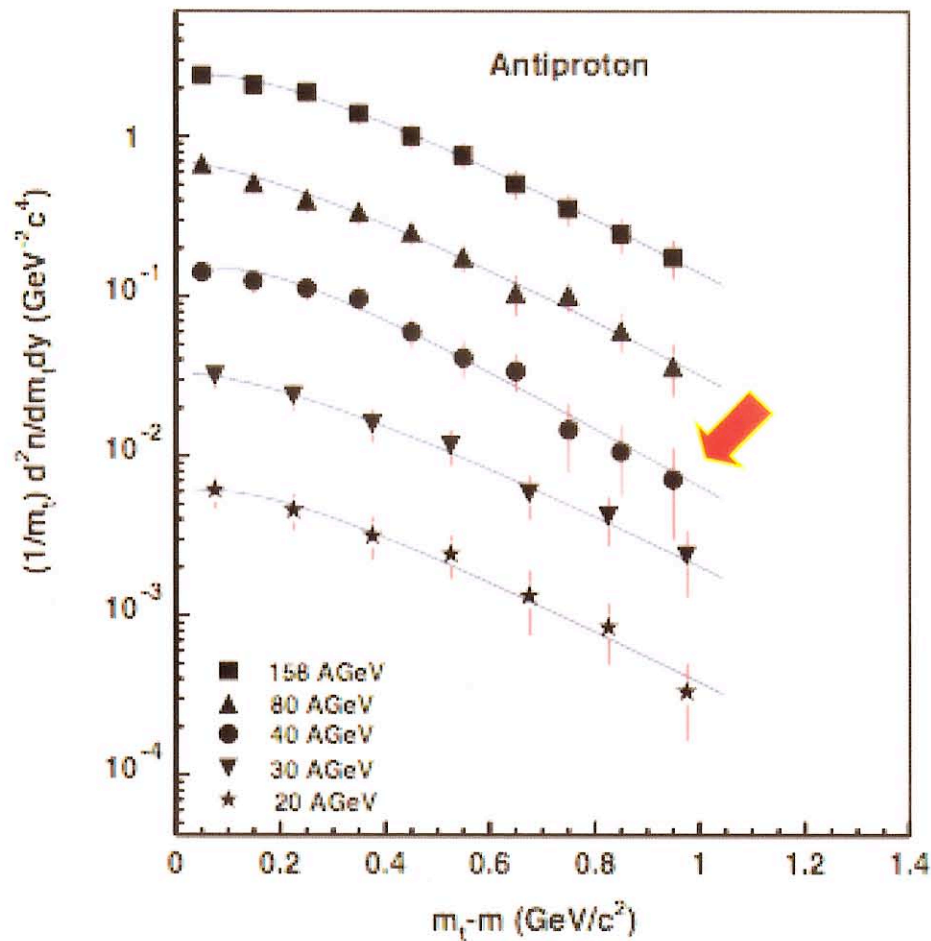


$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

- decreases (FO,CO)
- increases (QCP)

with QCP
steeper \bar{p} spectra at high P_T

Effect on Spectra ?

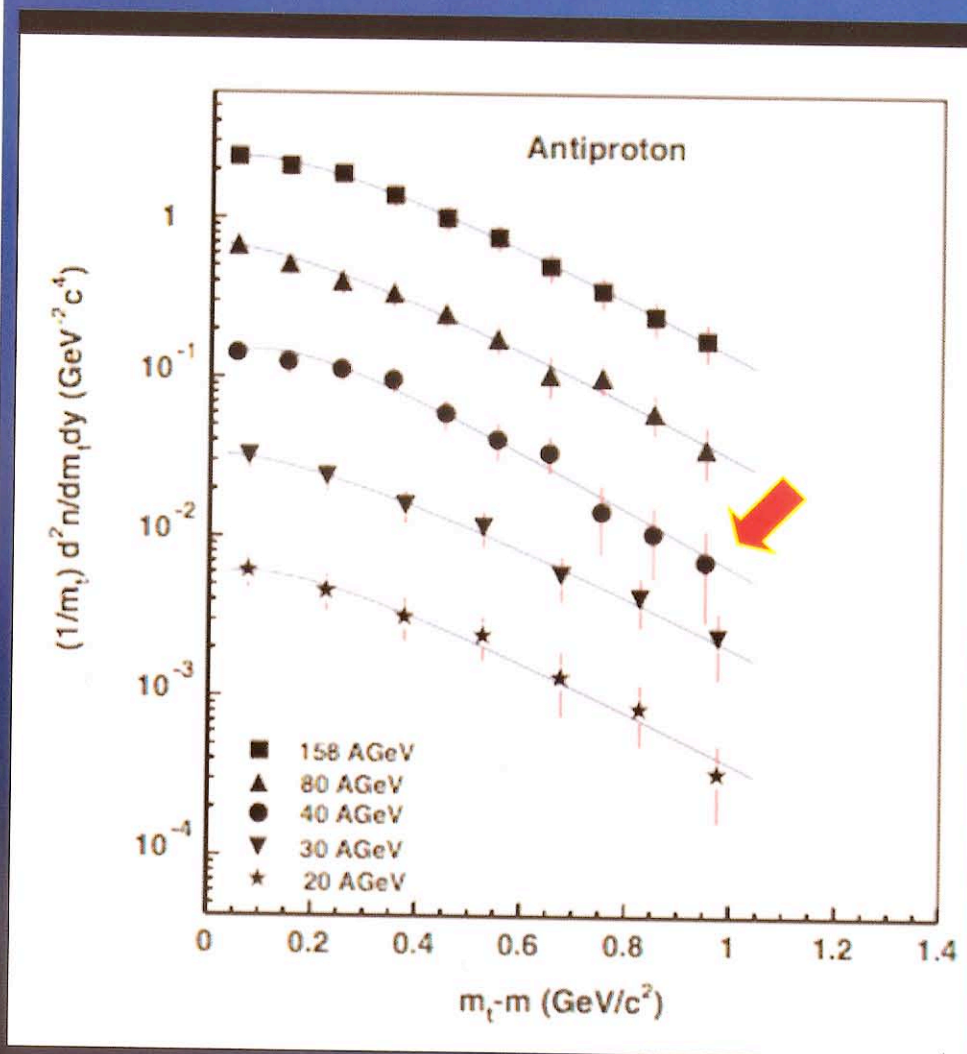


steeper \bar{p} spectra at high P_T

NA49, PRC73, 044910(2006)

M. Asakawa (Osaka University)

Result of One Temperature Fit



NA49, PRC73, 044910(2006)

	E_{beam} (A GeV)	dn/dy	T (MeV)	$\langle m_t \rangle - m$ (MeV/c ²)
\bar{p}	158	1.66 ± 0.17	291 ± 15	384 ± 19
	80	0.87 ± 0.07	283 ± 30	385 ± 41
	30	0.16 ± 0.02	290 ± 45	395 ± 60
	20	0.06 ± 0.01	279 ± 64	394 ± 60
	158	29.6 ± 0.9	308 ± 9	413 ± 13
p	80	30.1 ± 1.0	260 ± 11	364 ± 16
	40	41.3 ± 1.1	257 ± 11	367 ± 16
	30	42.1 ± 2.0	265 ± 10	362 ± 14
	20	46.1 ± 2.1	249 ± 9	352 ± 13

- Only one experimental result for \bar{p} slope
- Still error bar is large

- A newly proposed signature, qualitatively distinct from the fluctuation signatures.
- Deserves careful scrutiny by theorists and experimentalists alike.
- Plotting \bar{P}/p ratio vs P_T and looking for nonmonotonic \sqrt{s} dependence of this plot could be instructive
- Error bars still likely too large to get intrigued... But, lets push on this and see...

How is the experimental team doing in the race?

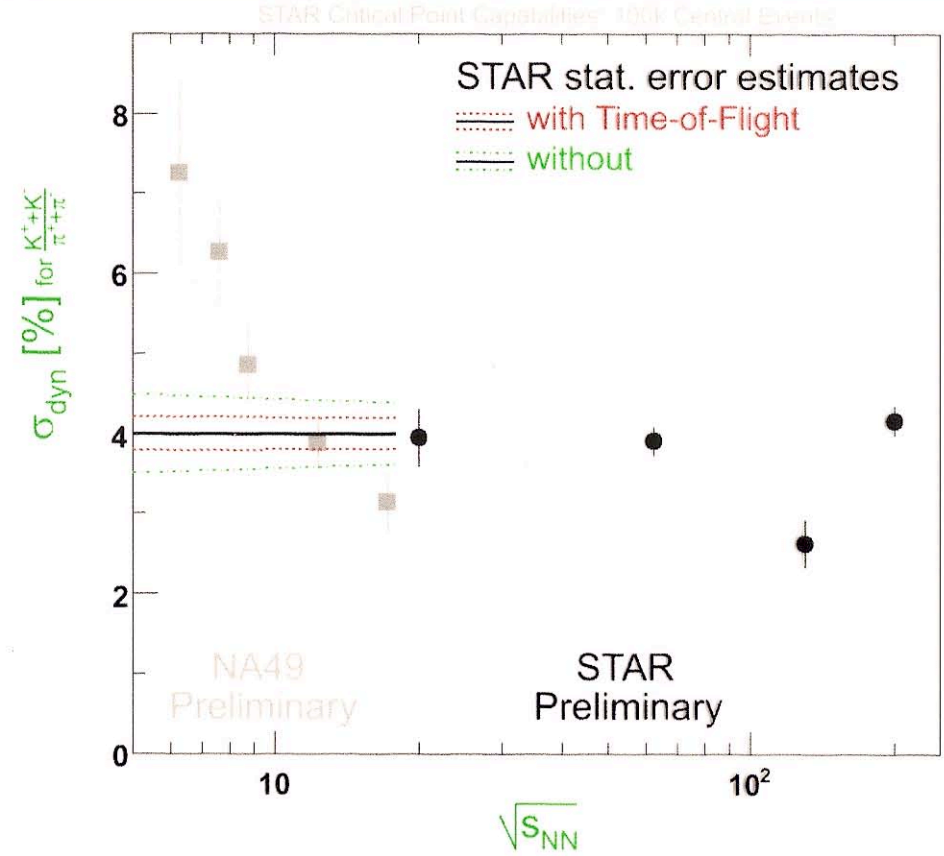
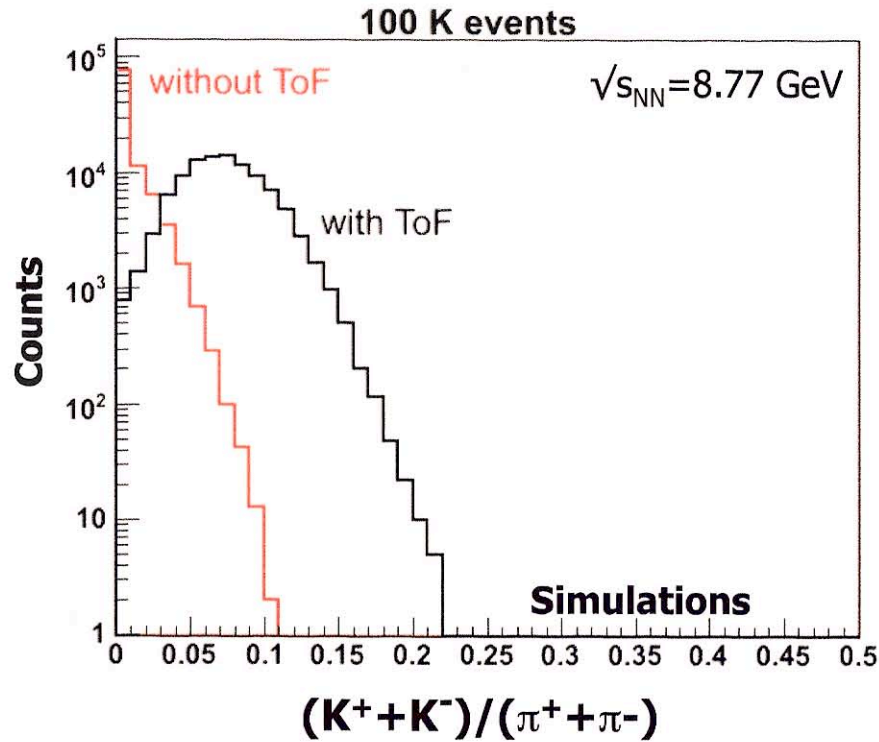
- intriguing anomaly in K/π fluctuation
- newly proposed signature in \bar{P}/P vs. P_T as function of \sqrt{s} .
- new experiments to come
 - NA61. Lighter ions \rightarrow shorter duration hadron gas phase
 - CRITRHIC
 - \rightarrow last few slides
 - CBM@FAIR
 - Best case is NA61/critRHIC discover critical point, making it possible for CBM to look for non Gaussian fluctuations from 1st order transition.

CAN RHIC FIND THE CRITICAL POINT?

what I learned at a March 2006 workshop with this title:

- Advantages of using a collider vs. fixed target machine to study event-by-event fluctuations at varying \sqrt{s} :
 - ~ same acceptance
 - same detectors
 - less change in track densityas \sqrt{s} changes
- With 10^6 min bias events per energy, STAR with its TOF upgrade can reduce statistical and systematic errors on K/π fluctuations each by factor of 4.
- No show stoppers on the accelerator side

K/ π measure with ToF



With ToF can improve:

- momentum range
- purity

Au+Au 100k central $\sqrt{s_{NN}}=8.77$ GeV
statistical errors:

- without ToF $\approx \pm 11\%$ (relative)
- with ToF $\approx \pm 5\%$ (relative)

WHAT RANGE OF \sqrt{s} , ie μ_B

- RHIC should, and can, explore $\mu_B < 500 \text{ MeV}$
- Want to test NA49 observation of K/π fluctuations at $\mu_B \sim 400-450 \text{ MeV}$
- If $\mu_B < 3T_c \sim 500 \text{ MeV}$, plausibly the different lattice calculations will converge as each improves. If $\mu_B > 500 \text{ MeV}$, quantitative comparison with theory will be hard.
- If $\mu_B > 500 \text{ MeV}$, also tough to find experimentally. (Low $T_{\text{freezeout}}$ equilibration???)
- A scan with steps $\lesssim 100 \text{ MeV}$ apart in μ_B should allow to make discoveries.
- In the vicinity of a discovery, will want μ_B 's spaced by $\sim 50 \text{ MeV}$.

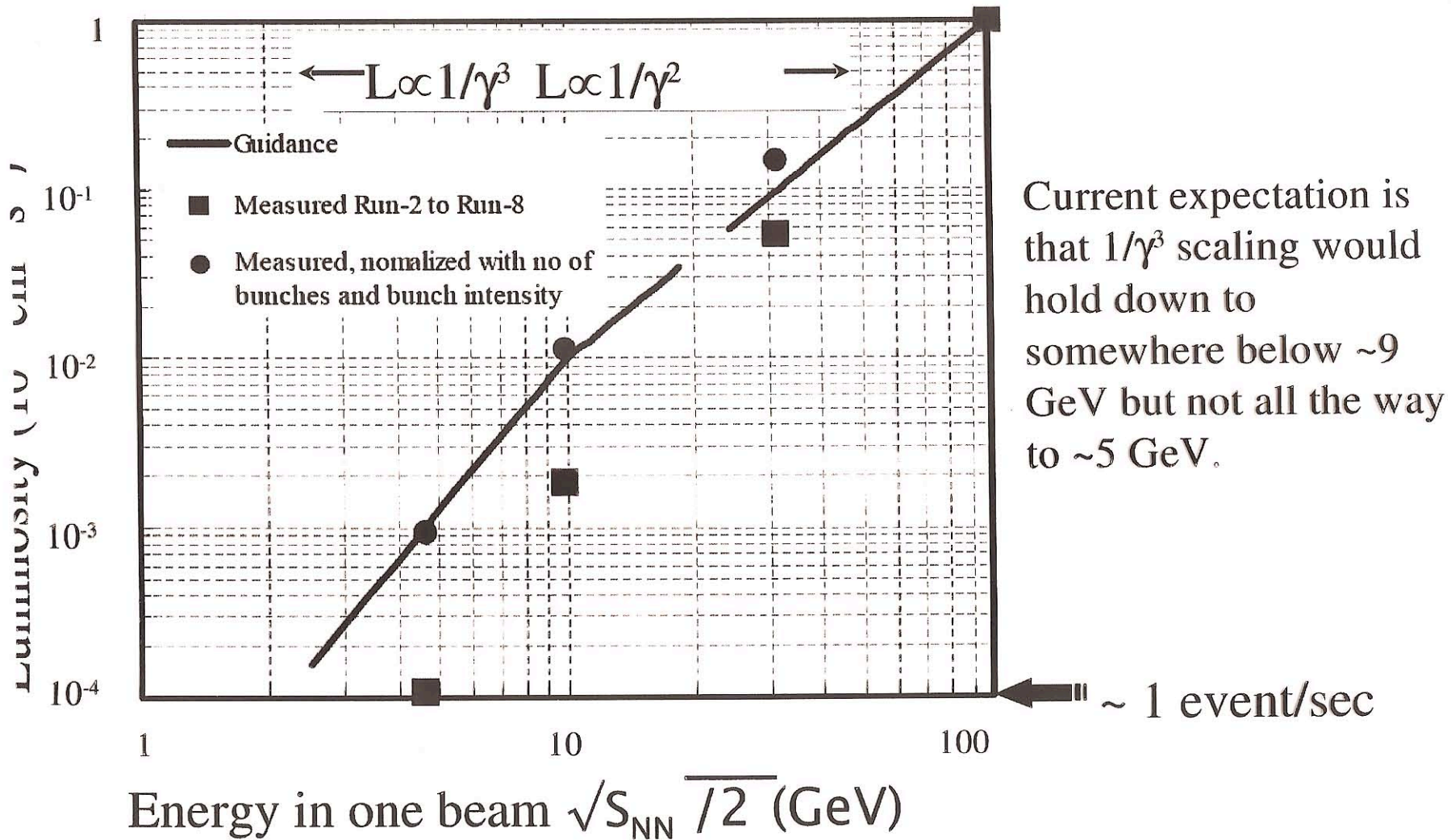
A "STRAW MAN" CHOICE OF ENERGIES

	\sqrt{s} (AGeV)	μ_B (MeV) [*]	10 hr days per 10^6 events [†]
largest K/ π fluctuations	5	550	20
	6.27	480	9
	7.62	425	5
	9.4	365	3
	12.3	300	1
	18	220	0.4
	24	170	0.2
	36	120	0.1
done	60	75	
	130	40	
	200	25	

* from Cleymans et al's 2005 empirical fit to compilation of data

† from Roser's "guidance" luminosity vs. \sqrt{s} curve

Actual Luminosity Scaling With Energy



Current expectation is that $1/\gamma^3$ scaling would hold down to somewhere below ~9 GeV but not all the way to ~5 GeV.

je courtesy of T. Roser

WHAT NEED BE MEASURED AT EACH ENERGY

- Enough $\langle \text{particle ratios} \rangle$ to first evaluate μ_B . You have to know where on the phase diagram you are freezing out.
- Event-by-event fluctuations in:
 - $\langle p_T \rangle$, with equal or smaller error bars as in NA49 data
 - $\langle K \rangle / \langle \pi \rangle$ and $\langle p \rangle / \langle \pi \rangle$ with smaller error bars than in NA49 data
 - All fluctuation analyses done for $p_T < p_T^{\text{cut}}$ for several choices of p_T^{cut} down to 500 MeV
- \bar{p}/p vs. p_T

CAN WE DISCOVER THE QCD CRITICAL POINT AT RHIC?

YES, IF:

- Accelerator & detector capabilities permit measurement of the event-by-event fluctuations of the hadronic observables I described, at a sequence of energies like that I described
- Nature is kind, and puts $\mu_B^0 < 500 \text{ MeV}$

IF YES:

- The landmark discovered. Our map of the QCD phase diagram then anchored by experiment.
- Assuming reasonable progress in lattice QCD, quantitative comparison between theory & experiment for μ_B^0
- FAIR (and RHIC) can study the first order phase transition.