RC Circuits

1 The Circuit

Suppose we have an electrical circuit like the one shown in Figure **??**. It

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|       \___/                        ||          |
|  power source     resistor      capacitor      |
|                                                |
|________________________________________________|
```

Figure 1: A resistor/capacitor (RC) circuit.

has a resistor, a capacitor, and a voltage source: it's an RC circuit. This is not a course in electromagnetism or in circuits, but I will use words from that subject. Let's pretend we understand what they mean! Current flows around the circuit. (Confusingly, if the current flows to the right, the actual electrons flow to the left, because they are negatively charged. It's confusing. Sorry, I didn't invent this.) The current is measured in "amperes" and is denoted by I . (I don't know what language has a word for current starts with an I!) In this "series" circuit, the current is the same everywhere but it may vary with time.

Let's say the positive direction in the circuit is clockwise (to the right over the top, for digital clock users). So if current is flowing counterclockwise along the wire, an ammeter would give a negative reading. The system is powered by a variable power source, which creates a "voltage increase" across it. This what makes current move. Write V(t) for the voltage *increase* from the bottom to the top of the source. Write V_R and V_C for the voltage *drops* across resistor and capacitor.

2 Kirchhoff's Voltage Law

"Kirchhoff's voltage law" (KVL) states that the total voltage change around a circuit loop is 0, i.e. $V(t) = V_R(t) + V_C(t)$

Here $V_R(t)$ and $V_C(t)$ are the voltage drop across R and C and $V(t)$ is the voltage gain across the power source. The graph in Figure **??** illustrates this. There is a relationship between the voltage drop across each circuit

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            |                           |
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|        \___/                          ||          |
|                                                   |
|___________________________________________________|
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Figure 2: Voltage in an RC circuit.

element and the current flowing through it. The relationship is different for resistors and capacitors:

Resistor: $V_R(t) = RI(t)$ for a constant R, the "resistance"
Capacitor: $V_C'(t) = \frac{1}{C}I(t)$ for a constant C, the "capacitance"

So:

- The voltage drop across the resistor is proportional to the current flowing through it. High resistance means big voltage drop.
- The voltage drop across the capacitor is proportional to the *integral* of the current; it results from a buildup of charge on the two plates of the capacitor. High capacitance means lots of space for the charge. A very large capacitor is like no capacitor at all.

To relate these, differentiate KVL:

$$V'(t) = V_R'(t) + V_C'(t) = RI'(t) + (1/C)I(t)$$

This is a first order linear differential equation for $I(t)$. In standard form:

$$RI'(t) + (1/C)I(t) = V'(t).$$

3 Block Diagram

The circuit is the system and it is represented by the left hand side. The input signal is V, and the voltage increase across the power source.

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                        I(0)
                         |
                         |
                         |
                         V
                   --------------
                  |              |
 --------------> |   Circuit    | -------------->
      V(t)        |______________|       I(t)
```

Figure 3: Block diagram for the RC circuit of Figure **??**.

The *derivative* of the input signal is what appears on the right of the equation. Note this well – the right hand side is derived from what we call the input. In general, what constitutes the input and output signals is a matter of interpretation of the equation, not of the equation itself.

The system response is the current.