

Complex Arithmetic Examples

In the following we let $z = 2 + 3i$ and $w = 4 + 5i$.

1 Real and imaginary parts

$$\operatorname{Re}(z) = 2, \quad \operatorname{Re}(w) = 4, \quad \operatorname{Im}(z) = 3, \quad \operatorname{Im}(w) = 5.$$

Note: the imaginary part does not include i .

2 Addition and Subtraction

$$\begin{aligned} z + w &= (2 + 3i) + (4 + 5i) = 6 + 8i \\ z - w &= (2 + 3i) - (4 + 5i) = -2 - 2i. \end{aligned}$$

2.1 Multiplication

$$z \cdot w = (2 + 3i)(4 + 5i) = 8 - 15 + i(10 + 12) = -7 + 22i.$$

3 Complex Conjugate and Magnitude

$$\begin{aligned} \bar{z} &= \overline{2 + 3i} = 2 - 3i \\ |z| &= \sqrt{4 + 9} = \sqrt{13} \\ z + \bar{z} &= 2 + 3i + 2 - 3i = 4 = 2 \operatorname{Re}(z) \\ z \cdot \bar{z} &= (2 + 3i)(2 - 3i) = 4 + 9 = 13 = |z|^2 \end{aligned}$$

4 Division

Multiply numerator and denominator by the complex conjugate of the denominator:

$$\frac{w}{z} = \frac{4+5i}{2+3i} = \frac{4+5i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{8+15+i(-12+10)}{13} = \frac{23}{13} - \frac{2}{13}i.$$