1 Sinusoidal Functions

We will see that sinusoidal functions play a big role in our study of ODEs. Here we will collect the terms and formulas we will need.

A sinusoidal function (or sinusoidal oscillation or signal) is one that can be wrtten in the form

$$f(t) = A\cos(\omega t - \phi) \tag{1}$$

The function $f(t)$ is a cosine function which has been *amplified* by A, *shifted* by ϕ, and *compressed* by ω.

$|A|$ is its *amplitude*: how high its graph rises over the t-axis at its maximum points;

ϕ is its *phase lag*: the value of ωt for which the graph is at its maximum (if $\phi = 0$, the graph has the position of $\cos\omega t$; if $\phi = \pi/2$, it has the position of $\sin\omega t$);

ϕ/ω is its *time delay* or *time lag*: how far to the right on the t-axis the graph of $\cos\omega t$ has been moved to make the graph of 1; (to see this, write $A\cos(\omega t - \phi) = A\cos(\omega(t - \phi/\omega)))$

ω is its *angular frequency*: the number of complete oscillations it makes in a time interval of length 2π; that is, the number of radians per unit time;

$\omega/2\pi$ (usually written ν) is its *frequency*: the number of complete oscillations the graph makes in a time interval of length 1; that is, the number of cycles per unit time;

$P = 2\pi/\omega = 1/\nu$ is its *period*, the t-interval required for one complete oscillation.

Here are the instructions for building the graph of 1 from the graph of $\cos t$. First *amplify*, or vertically stretch, the graph by a factor of A; then *shift* the result to the right by ϕ units; and finally *compress* it horizontally by a factor of ω.

graph

One can also write 1 as

$$f(t) = A\cos\left(\omega(t - t_0)\right)$$

where $wt_0 = \phi$, or

$$t_0 = \frac{\phi}{2\pi} P \tag{2}$$

t_0 is the *time lag*. It is measured in the same units as t, and represents the amount of time $f(t)$ lags behind the compressed cosine signal $\cos \omega t$. Equation 2 expresses the fact that t_0 makes up the same faction of the period P as the phase lag ϕ does of the period of the cosine function.