1 General solution to $\dot{y} + ky = q(t)$

The majority of this course is focused on constant coefficient linear equations.Here, we will solve the first-order linear constant coefficient equation

$$\dot{y} + ky = q(t) \tag{1}$$

where k is a constant. This is easy using the integrating factor.

$$u(t) = e^{\int k dt} = e^{kt}$$

from session 5. We get the solution

$$y = e^{-kt}\left(\int e^{kt}q(t)dt + c\right) \tag{2}$$

$$= e^{-kt}\int e^{kt}q(t)dt + ce^{-kt} \tag{3}$$

If $k > 0$ (so, the system models exponential decay), then the term ce^{-kt} goes to 0 as t goes to ∞. Because it goes to 0 we call ce^{-kt} the *transient*. The term $e^{-kt}\int e^{kt}q(t)dt$ is called the *steady-state* or *long-term* solution.

The value of c in 2 is determined by the initial value $y(0)$ of the solution. If $k > 0$ then no matter what the initial value, i.e. for any value of c, every solution goes asymptotically to the steady-state.

Graphically, this means that all solutions approach the steady-state as $t \to \infty$.

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Since all the solutions approach each other, there is a vagueness in choosing one to be the one we call steady-state. In fact, we can choose any one to be the steady-state solution. Generally we just choose the simplest-looking solution.

2 Input-Response

In session 4 we itroduced the notions of system, input, and response. In the case of equation 1 we have:

$\dot{y} + ky =$