

Exercises on matrix spaces; rank 1; small world graphs

Problem 11.1: [Optional] (3.5 #41. *Introduction to Linear Algebra: Strang*) Write the 3 by 3 identity matrix as a combination of the other five permutation matrices. Then show that those five matrices are linearly independent. (Assume a combination gives $c_1P_1 + \cdots + c_5P_5 = 0$ and check entries to prove c_i is zero.) The five permutation matrices are a basis for the subspace of three by three matrices with row and column sums all equal.

Problem 11.2: (3.6 #31.) \mathbf{M} is the space of three by three matrices. Multiply each matrix X in \mathbf{M} by:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Notice that $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

- Which matrices X lead to $AX = 0$?
- Which matrices have the form AX for some matrix X ?
- Part (a) finds the “nullspace” of the operation AX and part (b) finds the “column space.” What are the dimensions of those two subspaces of \mathbf{M} ? Why do the dimensions add to $(n - r) + r = 9$?