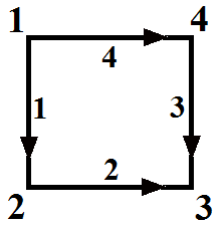
Exercises on graphs, networks, and incidence matrices

Problem 12.1: (8.2 #1. *Introduction to Linear Algebra:* Strang) Write down the four by four incidence matrix A for the square graph, shown below. (Hint: the first row has -1 in column 1 and +1 in column 2.) What vectors (x_1, x_2, x_3, x_4) are in the nullspace of A? How do you know that (1,0,0,0) is not in the row space of A?



Problem 12.2: (8.2 #7.) Continuing with the network from problem one, suppose the conductance matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Multiply matrices to find A^TCA. For $\mathbf{f} = (1, 0, -1, 0)$, find a solution to $A^TCA\mathbf{x} = \mathbf{f}$. Write the potentials $\mathbf{x}$ and currents $\mathbf{y} = -CA\mathbf{x}$ on the square graph (see above) for this current source $\mathbf{f}$ going into node 1 and out from node 3.