**Exercises on orthogonal vectors and subspaces**

**Problem 16.1:** (4.1 #7. *Introduction to Linear Algebra:* Strang) For every system of $m$ equations with no solution, there are numbers $y_1, ..., y_m$ that multiply the equations so they add up to $0 = 1$. This is called *Fredholm's Alternative*:

> Exactly one of these problems has a solution:
> $A\mathbf{x} = \mathbf{b}$ OR $A^T\mathbf{y} = \mathbf{0}$ with $\mathbf{y}^T\mathbf{b} = 1$.

If $\mathbf{b}$ is not in the column space of $A$ it is not orthogonal to the nullspace of $A^T$. Multiply the equations $x_1 - x_2 = 1$, $x_2 - x_3 = 1$ and $x_1 - x_3 = 1$ by numbers $y_1$, $y_2$ and $y_3$ chosen so that the equations add up to $0 = 1$.

**Problem 16.2:** (4.1#32.) Suppose I give you four nonzero vectors $\mathbf{r}$, $\mathbf{n}$, $\mathbf{c}$ and $\mathbf{l}$ in $\mathbb{R}^2$.

a) What are the conditions for those to be bases for the four fundamental subspaces $C(A^T)$, $N(A)$, $C(A)$, and $N(A^T)$ of a 2 by 2 matrix?

b) What is one possible matrix $A$?