

Exercises on orthogonal matrices and Gram-Schmidt

Problem 17.1: (4.4 #10.b *Introduction to Linear Algebra*: Strang)

Orthonormal vectors are automatically linearly independent.

Matrix Proof: Show that $Q\mathbf{x} = \mathbf{0}$ implies $\mathbf{x} = \mathbf{0}$. Since Q may be rectangular, you can use Q^T but not Q^{-1} .

Problem 17.2: (4.4 #18) Given the vectors \mathbf{a} , \mathbf{b} and \mathbf{c} listed below, use the Gram-Schmidt process to find orthogonal vectors \mathbf{A} , \mathbf{B} , and \mathbf{C} that span the same space.

$$\mathbf{a} = (1, -1, 0, 0), \mathbf{b} = (0, 1, -1, 0), \mathbf{c} = (0, 0, 1, -1).$$

Show that $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ and $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ are bases for the space of vectors perpendicular to $\mathbf{d} = (1, 1, 1, 1)$.