Exercises on Cramer's rule, inverse matrix, and volume

Problem 20.1: (5.3 #8. *Introduction to Linear Algebra:* Strang) Suppose

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}.$$

Find its cofactor matrix C and multiply AC^T to find $\det(A)$.

$$C = \begin{bmatrix} 6 & -3 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \text{ and } AC^T = \underline{\qquad}.$$

If you change $a_{1,3} = 4$ to 100, why is $\det(A)$ unchanged?

Problem 20.2: (5.3 #28.) Spherical coordinates ρ, ϕ, θ satisfy

$$x = \rho \sin\phi \cos\theta, \; y = \rho \sin\phi \sin\theta \text{ and } z = \rho \cos\phi.$$

Find the three by three matrix of partial derivatives:

$$\begin{bmatrix} \partial x/\partial\rho & \partial x/\partial\phi & \partial x/\partial\theta \\ \partial y/\partial\rho & \partial y/\partial\phi & \partial y/\partial\theta \\ \partial z/\partial\rho & \partial z/\partial\phi & \partial z/\partial\theta \end{bmatrix}.$$

Simplify its determinant to $J = \rho^2 \sin\phi$. In spherical coordinates,

$$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

is the volume of an infinitesimal "coordinate box."