

Exercises on eigenvalues and eigenvectors

Problem 21.1: (6.1 #19. *Introduction to Linear Algebra: Strang*) A three by three matrix B is known to have eigenvalues 0, 1 and 2. This information is enough to find three of these (give the answers where possible):

- a) The rank of B
- b) The determinant of $B^T B$
- c) The eigenvalues of $B^T B$
- d) The eigenvalues of $(B^2 + I)^{-1}$

Problem 21.2: (6.1 #29.) Find the eigenvalues of A , B , and C when

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$