

### Exercises on differential equations and $e^{At}$

**Problem 23.1:** (6.3 #14.a *Introduction to Linear Algebra*: Strang) The matrix in this question is skew-symmetric ( $A^T = -A$ ):

$$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \mathbf{u} \quad \text{or} \quad \begin{aligned} u_1' &= cu_2 - bu_3 \\ u_2' &= au_3 - cu_1 \\ u_3' &= bu_1 - au_2. \end{aligned}$$

Find the derivative of  $\|\mathbf{u}(t)\|^2$  using the definition:

$$\|\mathbf{u}(t)\|^2 = u_1^2 + u_2^2 + u_3^2.$$

What does this tell you about the rate of change of the length of  $\mathbf{u}$ ? What does this tell you about the range of values of  $\mathbf{u}(t)$ ?

**Problem 23.2:** (6.3 #24.) Write  $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$  as  $S\Lambda S^{-1}$ . Multiply  $Se^{\Lambda t}S^{-1}$  to find the matrix exponential  $e^{At}$ . Check your work by evaluating  $e^{At}$  and the derivative of  $e^{At}$  when  $t = 0$ .