Exercises on symmetric matrices and positive definiteness

Problem 25.1: (6.4 #10. *Introduction to Linear Algebra:* Strang) Here is a quick "proof" that the eigenvalues of all real matrices are real:

False Proof: $A\mathbf{x} = \lambda\mathbf{x}$ gives $\mathbf{x}^T A\mathbf{x} = \lambda\mathbf{x}^T\mathbf{x}$ so $\lambda = \dfrac{\mathbf{x}^T A\mathbf{x}}{\mathbf{x}^T\mathbf{x}}$ is real.

There is a hidden assumption in this proof which is not justified. Find the flaw by testing each step on the 90° rotation matrix:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

with $\lambda = i$ and $\mathbf{x} = (i, 1)$.

Problem 25.2: (6.5 #32.) A *group* of nonsingular matrices includes AB and A^{-1} if it includes A and B. "Products and inverses stay in the group." Which of these are groups?

a) Positive definite symmetric matrices A.

b) Orthogonal matrices Q.

c) All exponentials e^{tA} of a fixed matrix A.

d) Matrices D with determinant 1.