

Exercises on positive definite matrices and minima

Problem 27.1: (6.5 #33. *Introduction to Linear Algebra*: Strang) When A and B are symmetric positive definite, AB might not even be symmetric, but its eigenvalues are still positive. Start from $AB\mathbf{x} = \lambda\mathbf{x}$ and take dot products with $B\mathbf{x}$. Then prove $\lambda > 0$.

Problem 27.2: Find the quadratic form associated with the matrix $\begin{bmatrix} 1 & 5 \\ 7 & 9 \end{bmatrix}$. Is this function $f(x, y)$ always positive, always negative, or sometimes positive and sometimes negative?