Exercises on singular value decomposition

Problem 29.1: (Based on 6.7 #4. *Introduction to Linear Algebra:* Strang)
Verify that if we compute the singular value decomposition $A = U\Sigma V^T$ of
the Fibonacci matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$,

$$\Sigma = \begin{bmatrix} \frac{1+\sqrt{5}}{2} & 0 \\ 0 & \frac{\sqrt{5}-1}{2} \end{bmatrix}.$$

Problem 29.2: (6.7 #11.) Suppose A has orthogonal columns $\mathbf{w}_1$, $\mathbf{w}_2$, ...,
$\mathbf{w}_n$ of lengths σ_1, σ_2, ..., σ_n. Calculate $A^T A$. What are U, Σ, and V in the
SVD?