Exercises on solving $A\mathbf{x} = \mathbf{b}$ and row reduced form R

Problem 8.1: (3.4 #13.(a,b,d) *Introduction to Linear Algebra:* Strang) Explain why these are all false:

a) The complete solution is any linear combination of $\mathbf{x}_p$ and $\mathbf{x}_n$.

b) The system $A\mathbf{x} = \mathbf{b}$ has at most one particular solution.

c) If A is invertible there is no solution $\mathbf{x}_n$ in the nullspace.

Problem 8.2: (3.4 #28.) Let

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}.$$

Use Gauss-Jordan elimination to reduce the matrices $[U \;\; 0]$ and $[U \;\; \mathbf{c}]$ to $[R \;\; 0]$ and $[R \;\; \mathbf{d}]$. Solve $R\mathbf{x} = \mathbf{0}$ and $R\mathbf{x} = \mathbf{d}$.

Check your work by plugging your values into the equations $U\mathbf{x} = \mathbf{0}$ and $U\mathbf{x} = \mathbf{c}$.

Problem 8.3: (3.4 #36.) Suppose $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{b}$ have the same (complete) solutions for every $\mathbf{b}$. Is it true that $A = C$?