

Exercises on multiplication and inverse matrices

Problem 3.1: Add AB to AC and compare with $A(B + C)$:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix}$$

Solution: We first add AB to AC :

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}, \quad AC = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix}$$
$$\longrightarrow AB + AC = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & 12 \\ 20 & 24 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 23 & 24 \end{bmatrix}.$$

We then compute $A(B + C)$:

$$B + C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix}$$
$$\longrightarrow A(B + C) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 23 & 24 \end{bmatrix} = AB + AC.$$

Therefore, $AB + AC = A(B + C)$.

Problem 3.2: (2.5 #24. *Introduction to Linear Algebra: Strang*) Use Gauss-Jordan elimination on $[U \ I]$ to find the upper triangular U^{-1} :

$$UU^{-1} = I \quad \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ x_1 & x_2 & x_3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Solution: Row reduce $[U \ I]$ to get $[I \ U^{-1}]$ as follows (here, $R_i =$ row i)

$$\begin{aligned}
\begin{bmatrix} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} &\longrightarrow \begin{matrix} (R_1 = R_1 - aR_2) \\ (R_2 = R_2 - cR_2) \end{matrix} \begin{bmatrix} 1 & 0 & b-ac & 1 & -a & 0 \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \\
\longrightarrow &\begin{matrix} (R_1 = R_1 - (b-ac)R_3) \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = [I \ L^{-1}]
\end{aligned}$$