APPROXIMATIONS (CONT'D) $T' = T\left(1-\frac{v^2}{c^2}\right)^{1/2} \approx T\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$ $\frac{\Delta T}{T} = \frac{1}{2}\frac{v^2}{c^2}$ $\Delta T = T' - T$

Error fraction is proportional to $\frac{v^2}{c^2}$ with factor $\frac{1}{2}$.

2BII1: $\Delta v \approx a\Delta h$

QUADRATIC APPROX

(Use these when linear is not enough)

$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$

$xneq0$

why $\frac{1}{2}f''(0)$?

$f(x) = a + bx + cx^2$

$f'(x) = b + 2cx$

$f''(x) = 2c$

recover a, b, c

$f(0) = a$

$f'(0) = b$

$f''(0) = 2c$

$\sin x \approx x$

$\cos x \approx 1 - \frac{1}{2}x^2$

$e^x \approx 1 + x + \frac{1}{2}x^2$

(x near 0)

$\ln(1 + x) \approx x - \frac{1}{2}x^2$

$(1 + x)^r \approx 1 + rx + \frac{r(r-1)}{2}x^2$

$a_k \left(1+\frac{1}{k}\right)^k \approx e_{k\to\infty}$
as $k \to \infty$:
$\ln a_k = k(\ln(1+\frac{1}{k})) \to 1$
$\approx k\left(\frac{1}{k}\right)$
$\ln(1+x) \approx x(x = \frac{1}{k} \approx 0)$

rate of convergence
$\underbrace{(\ln a_k)}_{texthowbig?} \quad -1 \to 0$
use quadratic approximation.

Find quadratic approx.

$x \approx 0$

$$e^{-3x}\,(1+x)^{-1/2} \approx \left(1 + (-3x) + \frac{(-3x)^2}{2}\right)\left(1 - \tfrac{1}{2}x + \tfrac{1}{2}\underbrace{\left(\frac{-1}{2}\right)}_{r}\underbrace{\left(\frac{-3}{2}\right)}_{r-1}x^2\right)$$

$$\approx 1 - \underbrace{3x}_{\frac{1}{100}} - \underbrace{\frac{1}{2}x}_{\frac{1}{100}} + \underbrace{\frac{3}{2}x^2}_{\frac{1}{(100)^2}} + \tfrac{9}{2}x^2 - \tfrac{3}{8}x^2$$

$$= 1 - \tfrac{7}{2}x + \tfrac{51}{8}x^2$$

(drop x^3, x^4, etc. terms of size about $\frac{1}{100^3}$)

f	at $x = 0$
$\ln(1+x)$	0
$f' = \frac{1}{1+x}$	1
$f'' = -\frac{1}{(1+x)}^2$	-1

f	at $x = 0$
$(1+x)^r$	1
$r(1+x)^{r-1}$	r
$r(r-1)(1+x)^{r-2}$	$r(r-1)$

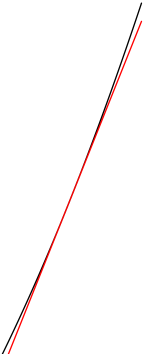
CURVE SKETCHING

GOAL: Draw graph of f using f',f'' positive/negative

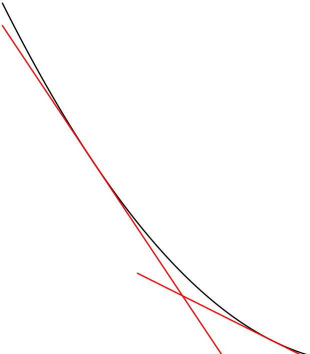
WARNING:

Don't abandon your precalc skills!
common sense!

$f' > 0 \rightarrow f$ is increasing



$f'' > 0 \rightarrow f'$ is increasing



Ex 1. $f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2$

$\qquad = 3(1-x)(1+x)$

$f'(x) = 0 \to (1-x)(1+x) = 0 \to$

$x = \pm 1$

$-1 < x < 1 \to f'(x) > 0$

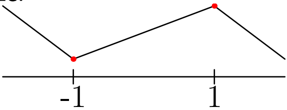
f increasing.

$1 < x \to f'(x) < 0$

f decreasing.

$x < -1$ also decreasing.

schematic:

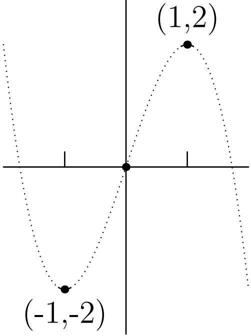


turning points

Defn If $f'(x_0) = 0$ we call
x_0 a *critical point*
$y_0 = f(x_0)$ is a *critical value*

Plot critpts/vals

$f(1) = 3 \cdot 1 - 1^3 = 2$



f is odd. $f(0) = 0$

Ends $x \to +\infty$
$f(x) = 3x - x^3 \approx x^3 \to -\infty$
$x \to +\infty$
$f(x) \to +\infty$ if $x \to -\infty$

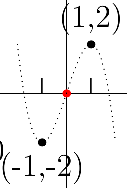
$f''(x) = (3 - 3x^2)' = -6x$
$f''(x) < 0$ if $x > 0$ (concave down)
$f''(x) > 0$ if $x < 0$ (concave up)

Plot critpts/vals

$f(1) = 3 \cdot 1 - 1^3 = 2$

$f(-1) = 3(-1) - (-1)^3 = -2$

$f(x) = 3x - x^3$ ($f(0) = 0$)



inflection point: $f''(0) = 0$

f is odd. $f(0) = 0$