MATH 100 SAMPLE MIDTERM SOLUTIONS

These solutions may contain errors. If you find an error, please report it by email. The first student to report each error will get half a point of extra credit added to their final grade.

(1) (10 pts) Evaluate $x^2 + 3x$ when $x = -8$.

$$8^2 + 3 \cdot 8 = 64 + 24 = 88.$$

(2) (10 pts) Simplify: $4(5y - 3) - (6y + 3)$.

$$4(5y - 3) - (6y + 3) = 20y - 12 - 6y - 3 = 14y - 15$$

(3) (10 pts) Simplify: $(49x^2y^4)^{-1/2}$.

$$(49x^2y^4)^{-1/2} = \frac{1}{(49x^2y^4)^{1/2}} = \frac{1}{\sqrt{(49x^2y^4)}} = \frac{1}{\sqrt{49}\sqrt{x^2}\sqrt{y^4}} = \frac{1}{7xy^2}$$

(4) (10 pts) Simplify: $4(1 - t^2) + 2t(t + 1)$.

$$4(1 - t^2) + 2t(t + 1) = 4 - 4t^2 + 2t^2 + 2t = -2t^2 + 2t + 4$$

(5) (10 pts) Factor completely: $x^3 + 3x^2 + 2x$.

$$x^3 + 3x^2 + 2x = x(x^2 + 3x + 2) = x(x + 2)(x + 1)$$

(6) Consider the rational equation $\frac{2}{x-1} + 4 = \frac{14}{x-1}$.

a) (5 pts) What value or values of x make the denominator zero?

$$x = 1$$

b) (5 pts) Solve the equation for x.

$$\frac{2}{x-1} + 4 = \frac{14}{x-1}$$

$$(x-1)(\frac{2}{x-1} + 4) = (x-1)\frac{14}{x-1}$$

$$2 + 4(x-1) = 14$$

$$2 + 4x - 4 = 14$$

$$-2 + 4x = 14$$

$$4x = 16$$

$$x = 4$$

Check: $2/3 + 4 = 14/3$?

(7) (10 pts) Solve for x: $2x^2 + 5x + 3 = 0$.

$$2x^2 + 5x + 3 = 0$$
$$(2x + 3)(x + 1) = 0$$
$$2x + 3 = 0 \text{ or } x + 1 = 0$$
$$x = -3/2 \text{ or } x = -1$$

Check: $2(-3/2)^2 + 5(-3/2) + 3 = 0$? $2(-1)^2 + 5(-1) + 3 = 0$?

(8) (10 pts) Find the equation of the line connecting the points $(1, 4)$ and $(3, 7)$.

$$y - y_1 = m(x - x_1) \text{ and } m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7 - 4}{3 - 1} = \frac{3}{2}$$
$$y - 4 = \frac{3}{2}(x - 1)$$
$$y - 4 = \frac{3}{2}x - \frac{3}{2}$$
$$y = \frac{3}{2}x + \frac{5}{2}$$

Check: $7 = \frac{3}{2}(3) + \frac{5}{2}$?

(9) The graph of the equation $y = 5x + 3$ is a line.

a) (5 pts) Find the x- and y-intercepts of that line.

$$0 = 5x + 3 \text{ when } x = -\frac{3}{5}. \; y = 3 \text{ when } x = 0.$$

The x-intercept is at $(-\frac{3}{5}, 0)$. The y-intercept is at $(0, 3)$.

b) (5 pts) Give the equation of the line perpendicular to that line that passes through the point (5, 10).

$$\text{The slope of the perpendicular will be } -\frac{1}{5}.$$
$$y - y_1 = m(x - x_1) \text{ so } y - 10 = -\frac{1}{5}(x - 5)$$
$$y - 10 = -\frac{1}{5}x + 1$$
$$y = -\frac{1}{5}x + 11$$

Check by graphing (if you're using a graphing calculator, make sure distances along your x-axis match those along your y-axis.)

(10) The graph of the function $f(x) = x^3 - 3x^2 + 2x$ is shown below.

a) (5 pts) For approximately what value(s) of x does $f(x) = 1$?

Sketch the horizontal line $y = 1$. This intersects the graph at about $(2.3, 1)$, so $f(x) = 1$ when x is approximately 2.3.

b) (5 pts) Is the function f even, odd or neither? Justify your answer.

$$f(-x) = (-x)^3 - 3(-x)^2 + 2(-x) = -x^3 - 3x^2 - 2x.$$

This does not equal $f(x)$ or $-f(x)$, so f is neither even nor odd.

Bonus (5 pts) Simplify:

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2}{x^2}\frac{1}{(x+h)^2}-\frac{1}{x^2}\frac{(x+h)^2}{(x+h)^2}}{h}$$

$$=\frac{\frac{x^2}{x^2(x+h)^2}-\frac{(x+h)^2}{x^2(x+h)^2}}{h}=\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$$

$$=\frac{\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}}{h}=\frac{\frac{-2xh-h^2}{x^2(x+h)^2}}{\frac{h}{1}}$$

$$=\frac{h(-2x-h)}{x^2(x+h)^2}\frac{1}{h}=\frac{-2x-h}{x^2(x+h)^2}$$