

MA 100 SAMPLE MIDTERM

Be sure to write your name on your blue book! Please try to show your work and give justifications for your answers. It is permitted to use calculators on the midterm. Try not to spend too much time on any single problem; if you get stuck on a problem leave a partial answer and move on to the next.

- (1) (5 pts) What is the slope of the line described by the equation $y = 3x + 5$?

The slope is 3.

- (2) (10 pts) Factor the polynomial $x^2 - 9$.

$$x^2 - 9 = (x - 3)(x + 3)$$

- (3) (10 pts) Find all solutions of the equation $x^3 + 4x^2 + 4x = 0$.

$$\begin{aligned}x^3 + 4x^2 + 4x &= 0 \\x(x^2 + 4x + 4) &= 0 \\x(x + 2)(x + 2) &= 0 \\x = 0 \text{ or } (x + 2) &= 0 \\x = 0 \text{ or } x &= -2\end{aligned}$$

- (4) (5 pts) Check your solutions to the problem above in the original equation.

$$\begin{aligned}0^3 + 4 \cdot 0^2 + 4 \cdot 0 &= 0 \\(-2)^3 + 4 \cdot (-2)^2 + 4 \cdot (-2) &= -8 + 4 \cdot 4 + (-8) = 0\end{aligned}$$

- (5) (10 pts) Solve $|x - 2| > 3$.

Method 1: The distance from x to 2 is greater than 3, so x must be to the left of 1 or to the right of 5. $x < 1$ or $5 < x$.

Method 2:

$$\begin{aligned}|x - 2| &> 3 \\(x - 2) < -3 \text{ or } (x - 2) &> 3 \\(x - 2) + 2 < -3 + 2 \text{ or } (x - 2) + 2 &> 3 + 2 \\x < -1 \text{ or } x &> 5\end{aligned}$$

- (6) (10 pts) Simplify $\frac{x\sqrt{y}}{x^{-2}y^2}$.

$$\frac{x\sqrt{y}}{x^{-2}y^2} = \frac{xy^{1/2}}{x^{-2}y^2} = \frac{x}{x^{-2}} \cdot \frac{y^{1/2}}{y^2} = x^{1-(-2)} \cdot y^{(1/2)-2} = x^{-3}y^{-3/2}.$$

- (7) (10 pts) Sketch the graph of the square root function. What is the domain of this function?

See page 55 graph (d). The domain is all non-negative numbers; i.e. $x \geq 0$.

- (8) (10 pts) Find the y -intercept(s) (if any) of the graph of the equation $(x - 1)^2 + y^2 = 1$.

Method 1: The y -intercepts of the graph of $(x - 1)^2 + y^2 = 1$ are at the solution points for which $x = 0$.

$$(0 - 1)^2 + y^2 = 1$$

Solve for y :

$$1 + y^2 = 1$$

$$y^2 = 0$$

$$y = 0$$

The graph touches the y -axis where $y = 0$; i.e. the y -intercept is at the point $(0,0)$.

Method 2: This is the equation of a circle with center $(1, 0)$ and radius 1. If you draw such a circle you see it touches the y -axis only at point $(0, 0)$.

- (9) (15 pts) Sketch the graph of the function $f(x) = x^2 - 2$.

First construct a table of values: you will find that $f(2) = f(-2) = 2$, $f(-1) = f(1) = -1$ and $f(0) = -2$. Plotting the points $(x, f(x))$ you will get a graph similar to (e) on page 55 but with y -intercept $(0,-2)$ and x -intercepts $(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$.

- (10) (15 pts) Are the equations $y = \sqrt{x}$ and $x = y^2$ equivalent? Why or why not?

The equations are not equivalent because $(4, -2)$ is a solution to $x = y^2$ but not to $y = \sqrt{x}$. (If we replace y by -2 and x by 4 in $y = \sqrt{x}$ we get $-2 = \sqrt{4}$ which is false. The square root of a positive number is always positive.)