MATH 130 SAMPLE FINAL

For full credit, please show your work and give justifications for your answers. You may use your calculator and both sides of an 8 1/2 by 11 sheet of notes on the midterm. You may not use a cell phone or computer. Try not to spend too much time on any single problem; if you get stuck on a problem leave a partial answer and move on to the next. If you have time left over at the end of the exam, try to check your work.

(1) Determine the truth value of the following statement, assuming that the domain of discourse D is the set of all real numbers. Justify your answer.

$$\exists x : x^2 > x$$

(2) Prove by cases: If $d = \max\{d_1, d_2\}$ and $x \geq d$ then $x \geq d_1$ and $x \geq d_2$.

(3) Prove: $\sum_{i=1}^{n}(2i-1) = n^2$.
In other words, prove that the sum of the first n odd integers is n^2. Hint: use induction.

(4) If $S_i = \{1, i, 2i, 3i, ...\}$ is the set of positive multiples of the integer i, describe the set:

$$\bigcap_{i=1}^{4} S_i$$

(5) Find the inverse of the function described below, or explain why it has none.

$$f = \{(1,3), (1,4), (2,5), (2,6), (3,7)\}$$

(6) Is the matrix below the matrix of an equivalence relation?

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(7) Write the decimal number 23 in binary form.

(8) Find $\text{lcm}(110, 143)$.

(9) How many different ways are there to select 5 cards from a aset of 13 distinct cards if the order in which the cards are selected doesn't matter? (In other words, how many different poker hands are diamond flushes?)

(10) Explain why the graph shown on the left below does not contain a Hamiltonian cycle.

(11) Find the shortest path from A to D in the graph shown on the right above, based on the weights of the edges of the graph.