MATH 130 SAMPLE MIDTERM

For full credit, please show your work and give justifications for your answers. You may use your calculator and one side of an 8 1/2 by 11 sheet of notes on the midterm. You may not use a cell phone or computer. Try not to spend too much time on any single problem; if you get stuck on a problem leave a partial answer and move on to the next. If you have time left over at the end of the exam, try to check your work.

(1) Is the statement $p \wedge (\neg q)$ logically equivalent to $\neg((\neg p) \vee q)$? Justify your answer.

(2) Prove or disprove: $\exists x, x! < 2^x$.

(3) State the principle of mathematical induction or list the three steps required in a valid proof by induction.

(4) Let $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$.
a) What is $A \cup B$?
b) What is $\overline{B}$?

(5) A function f from $X = \{1, 2, 3\}$ to $Y = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is described by $f(x) = x^2$. Write f as a set of ordered pairs – in other words, as a subset of $X \times Y$.

(6) Consider the sequence $\{s_i\}_{i=5}^{10}$, $s_i = i - 4$.
a) What numbers are in this sequence?
b) Compute $\displaystyle\prod_{i=5}^{10}(i - 4)$.

(7) Let R be the (binary) relation on $X = \{1, 2, 3, 4\}$ defined by $R = \{(1,1), (2,2), (1,2), (2,1), (2,3), (3,2), (4,4)\}$.
a) Draw the matrix of relation R.
b) Is R a symmetric relation? How can you tell?

(8) Suppose $S = \{\{1, 3\}, \{2, 4\}, \{1, 5\}\}$ and $X = \{1, 2, 3, 4, 5\}$. Is S a partition of X? Why or why not?

(9) Define an equivalence relation R on $X = \{2, 4, 6, 8, 10\}$ by aRb if $a \bmod 3 = b \bmod 3$. Is $(2, 6) \in R$? Explain.

(10) Find the GCD and LCM of 220 and 1400.