

Claim $p^2 = 2$ has no solutions for $p \in \mathbb{Q}$

Pf Suppose $\exists p \in \mathbb{Q}$ s.t. $p^2 = 2$ Then $p = \frac{a}{b}$, $a, b \in \mathbb{N}$ Furthermore, assume a and b have no common factors.

$$\left(\frac{a}{b}\right)^2 = 2$$

$$a^2 = 2b^2$$

$$2|a \Rightarrow a = 2k, k \in \mathbb{N}$$

$$(2k)^2 = 2b^2 \Rightarrow 2k^2 = b^2$$

$$2|b$$

So $2|b$ and $2|a$, violating the assumption they share no common factors