Factorized, Resummed, and Gapped Angularity Distributions

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Introduction

Motivation

Angularities are certain class of Event Shapes, controlled by a continuous parameter a, which varies the sensitivity of the observable to narrower or wider jets

Examples of other Event Shapes: thrust, jet masses, jet broadening, C-parameter

Event Shapes are used for:

- Tests of PQCD
- Extractions of α_s Becher, Schwartz, 08 Hoang, Mateu, Stewart et al., 09
- Substructure of Jets (Angularities) Almeida et al., 08

Introduction

Definition

Large class of Event Shapes can be written in the form:

$$e(X) = \frac{1}{Q} \sum_{i \in X} \left| \mathbf{p}_\perp^i \right| f_e(\eta_i)$$

Examples:

thrust $f_{1\text{-}T}(\eta) = e^{-|\eta|}$ Brandt, Peyrou, Sosnowski, Wroblewski, 64 Farhi, 77

jet broadening $f_B(\eta) = 1$ Catani, Turnock, Webber, 92

C-parameter $f_C(\eta) = 3/\mathrm{Cosh}(\eta)$ Ellis, Ross, Terrano, 81

and relatively newly introduced...

Angularities $f_{Ta}(\eta) = e^{-|\eta|(1-a)}$ Berger, Kucs, Sterman, 03

Introduction

Factorization of Event Shapes

- Factorization Theorems in QCD

Angularities have been calculated to NLL/LO
a<1

↔ Collins, Soper, Sterman,...
Berger, Kucs, Sterman, 03 (I, II)

- Factorization Theorems in SCET

Bauer, Manohar, Wise, 02
Bauer, Lee, Manohar, Wise, 03
Lee, Sterman, 07
Becher, Schwartz, 08
Bauer, Fleming, Lee, Sterman, 08
Fleming, Hoang, Mantry, Stewart, 07 (I,II)

We calculate Angularities in SCET to NLL/NLO: a<1

Hornig, Lee, GO, 09

Introduction

Factorization of Angularities

$$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$$

Bauer, Fleming, Lee, Sterman, arXiv:0801.4569

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{1}{2Q^2} \sum_X \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x} \sum_{i=V,A} L^i_{\mu\nu} \langle 0|\, j_i^{\mu\dagger}(x)\, |X\rangle \langle X|\, j_i^{\nu}(0)\, |0\rangle\, \delta(e - e(X))$$

$$C \times \bar{\chi}_n\ \overline{Y}_n \Gamma_i^{\mu} Y_n\, \chi_{\bar{n}} \qquad \hat{e} = \hat{e}_n + \hat{e}_{\bar{n}} + \hat{e}_s$$

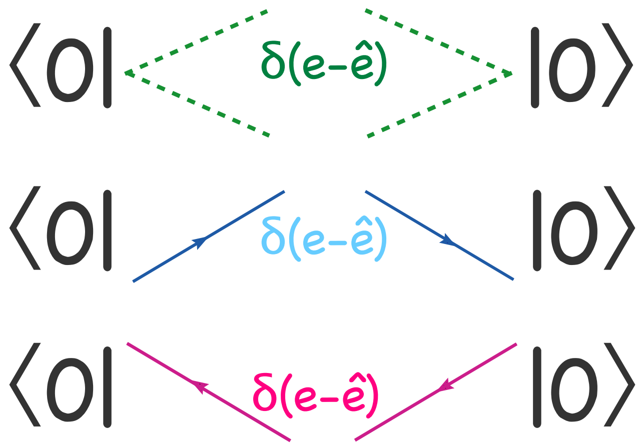
$$\Downarrow$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H(Q) \int de_n\, J_n(e_n)\, de_{\bar{n}}\, J_{\bar{n}}(e_{\bar{n}})\, de_s\, S(e_s)\, \delta(e - e_n - e_{\bar{n}} - e_s)$$

$$H(Q;\mu) = |C_{n\bar{n}}(Qn/2, -Q\bar{n}/2;\mu)|^2 \qquad |C|^2$$

$$S(e_s;\mu) = \frac{1}{N_C} \mathrm{Tr}\, \langle 0|\, \overline{Y}^{\dagger}_{\bar{n}}(0) Y^{\dagger}_n(0) \delta(e_s - \hat{e}_s) Y_n(0) \overline{Y}_{\bar{n}}(0)\, |0\rangle$$

$$J_n(e_n;\mu) = \int \frac{\mathrm{d}l^+}{2\pi} \frac{1}{N_C} \mathrm{Tr} \int \mathrm{d}^4x\, \mathrm{e}^{il\cdot x} \langle 0|\, \chi_{n,Q}(x)_\alpha \delta(e_n - \hat{e}_n) \bar{\chi}_{n,Q}(0)_\beta\, |0\rangle$$



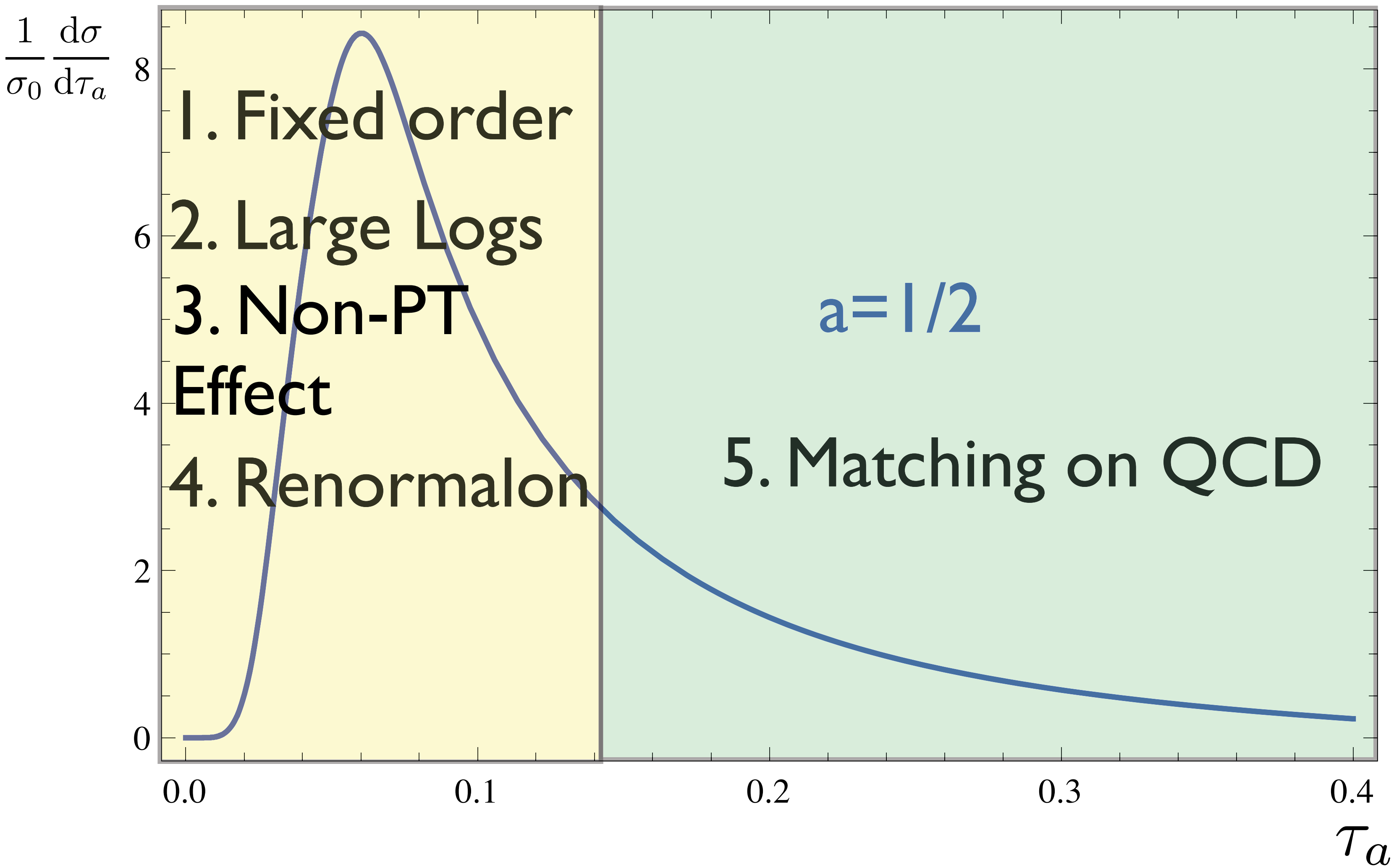
Introduction

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

$\mu_s = Q\tau_a$

Outline

Peak | **Tail**

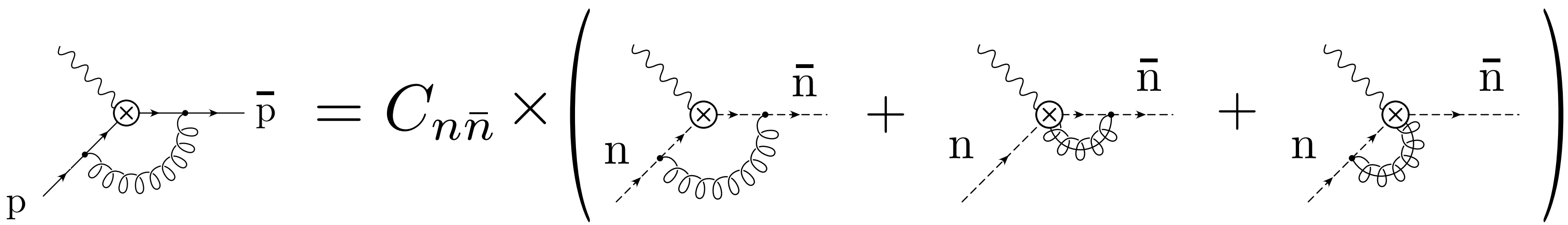


1. Fixed order
2. Large Logs
3. Non-PT Effect
4. Renormalon
5. Matching on QCD

Fixed Order Calculations

Diagrams $\sigma \sim H * J \otimes \bar{J} \otimes S$

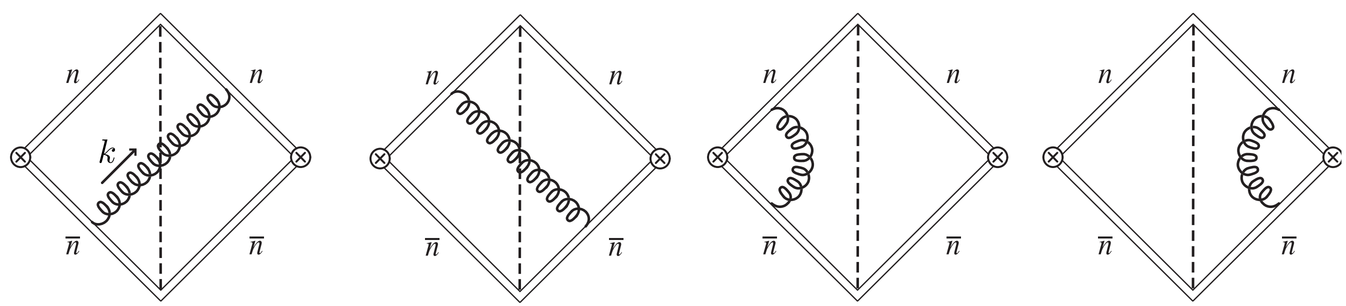
Hard Function



$H^{bare} = Z_H H^{ren}$

Soft Function

$S(\tau^s_a;\mu) = \mathrm{Disc}_{\{\tau a\}}$



Jet Function

$F^{bare} = Z_F \otimes F^{ren}$

$J(\tau^n_a;\mu) = \mathrm{Disc}_{\{\tau a\}}$

Fixed Order Calculations

Cutting Rules

- Standard Soft and Jet functions are of the form:

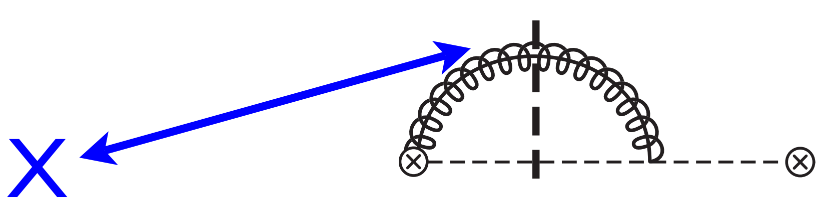
$$\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|\phi(x)\phi^\dagger(0)|0\rangle = \mathrm{Disc}\left[\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|\mathrm{T}\,\phi(x)\phi^\dagger(0)|0\rangle\right]$$

- In our case of Angularities we get a more general form:

$$\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|\phi(x)\,\delta(\tau_a-\hat{\tau}_a)\,\phi^\dagger(0)|0\rangle \equiv \mathrm{Disc}_{\tau_a}\left[\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|\mathrm{T}\phi(x)\phi^\dagger(0)|0\rangle\right]$$

- There exists a method to relate weighted cross-sections to the ordinary discontinuity of the time-ordered product of operators   Ore, Sterman, 1980, **Nucl.Phys.B165:93**

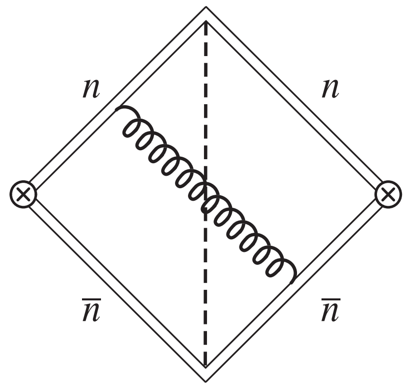
- For simplicity we choose instead to modify the cutting rules to calculate the "τ_a"-discontinuity

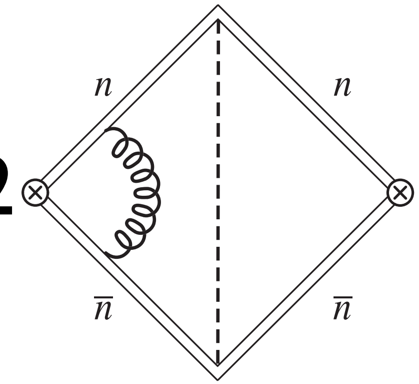


$\times\ \delta(\tau_a-\tau_a(X))$

Fixed Order Calculations

Soft Function

$S(\tau^s_a;\mu)= 2$ 

$\delta_R + 2$ 

δ_V

$\delta_R=\theta(k^- - k^+)\,\delta(\tau^s_a - |k^+|^{1-a/2}|k^-|^{a/2}/Q) + (k^- \leftrightarrow k^+)$ $\qquad$ $\delta_V=\delta(\tau^s_a)$

- For all $a<1$ the Soft Function should stay IR safe
- Both Real and Virtual diagrams contain IR divergences
- The Virtual diagram doesn't know about a, while the Real diagram does!
- How does IR Cancel???

Hornig, Lee, GO, **arXiv:0901.1897**

Fixed Order Calculations

Soft Function

- Soft function can be written in the form:

$$S(\tau^s_a)=A\delta(\tau^s_a)+[B(\tau^s_a)]_+$$

Real+Virtual ↔ $A\delta(\tau^s_a)$; $[B(\tau^s_a)]_+$ ↔ Real

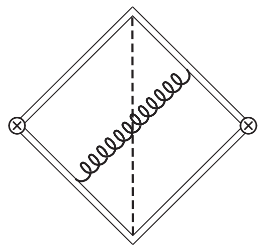
- Contribution of the Real graph to the plus function ($[B(\tau^s_a)]_+$) is regulated by non-zero τ^s_a and is IR finite
- Cancellation between IR divergences of Real and Virtual graphs occurs in the delta-function part(A)
- The delta-function part can be isolated by integrating over τ^s_a: $A=\int d\tau^s_a\, S(\tau^s_a)=A_R+A_V$

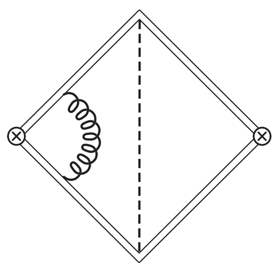
Fixed Order Calculations

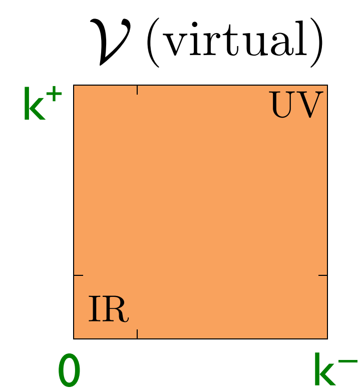
Soft Function

$$\int d\tau^s_a\, S(\tau^s_a) = A_R + A_V$$

$$(k^+k^-)^{-1-\epsilon}$$

$$A_R = \int_0^1 d\tau^s_a \Big\{ 2$$  $$\Big\}\, \delta_R = N \int_R dk^+ dk^- F(k^+,k^-)$$

$$A_V = \int_0^1 d\tau^s_a \Big\{ 2$$  $$\Big\}\, \delta(\tau^s_a) = -N \int_V dk^+ dk^- F(k^+,k^-)$$



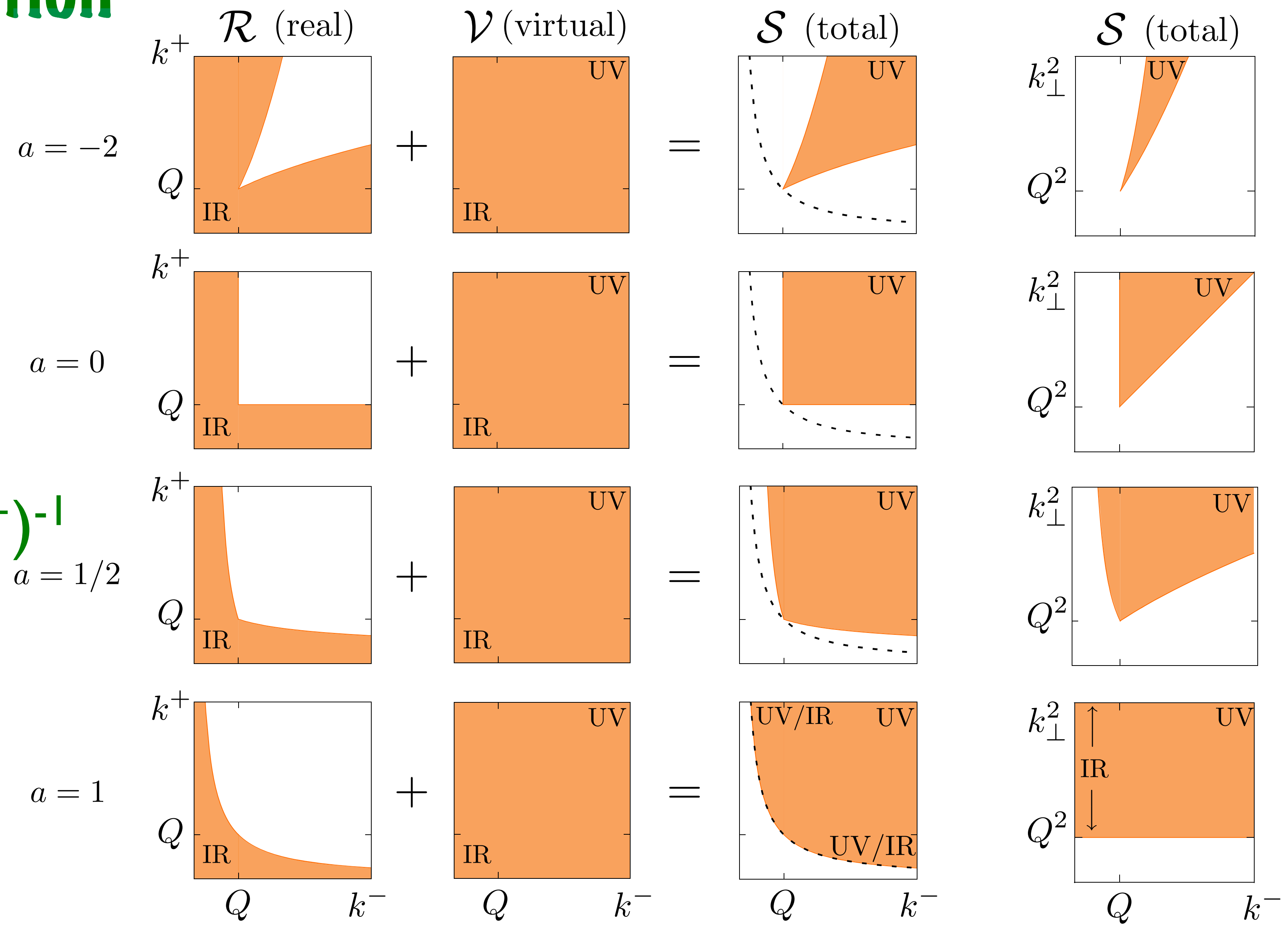
- Virtual Region V is always equal to the first quadrant in the k^+k^- plane
- Thus adding the Virtual graph is equivalent to inversion of the region from the Real graph R

Fixed Order Calculations

Soft Function

$F(k^+,k^-) = (k^+k^-)^{-1-\epsilon}$

$F(k^2_\perp,k^-) = (k^2_\perp)^{-1-\epsilon}\,(k^-)^{-1}$



Fixed Order Calculations

Soft Function

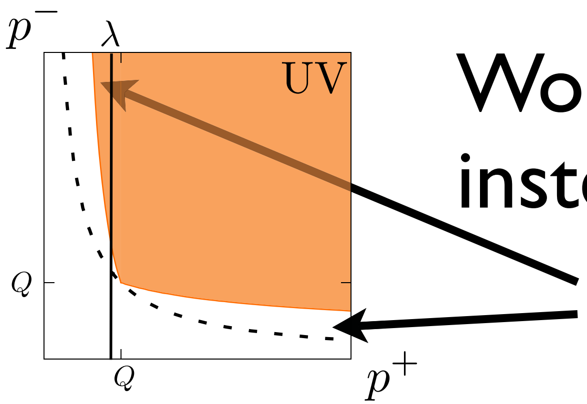
- Soft Function is IR safe for all $a<1$
- The IR safety of the Soft Function breaks down at $a=1$
- We verified these facts without use of explicit IR regulators by analyzing the regions of integration in Virtual and Real diagrams

Fixed Order Calculations

Soft Function

- Using IR regulators in practice is a complicated business
- For Example off-shellness in SCET doesn't regulate all the IR Divergence, but only part of it. The remaining is done by Dim Reg
- Another Example when one needs to be careful is the IR Cutoff Regulator that has been used for the Soft Function: $\frac{1}{l^-} \to \frac{1}{l^- + \lambda}$

Chay, Kim, Kim, Lee, hep-ph/0412110



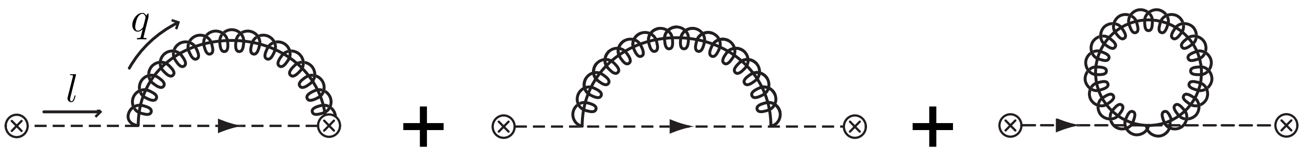
Works for a≤0, BUT regulates UV instead of IR for 0<a<1

$$\frac{1}{\epsilon_{\rm uv}} \ln \lambda = \frac{1}{\epsilon_{\rm uv}^2}$$

Fixed Order Calculations

Jet Function

$J(\tau^n{}_a;\mu) = \mathrm{Disc}_{\{\tau a\}}\ 2$



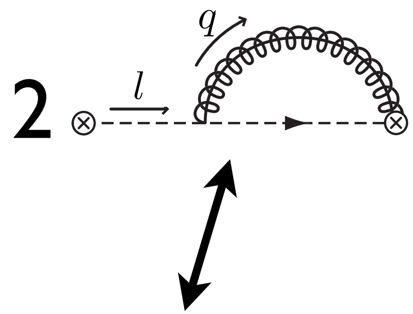
$\delta_R = \delta_R(\tau^n{}_a, q, l^+) = \delta(\tau^n{}_a - [(q^-)^{a/2}(q^+)^{1-a/2} + (Q - q^-)^{a/2}(l^+ - q^+)^{1-a/2}]/Q)$

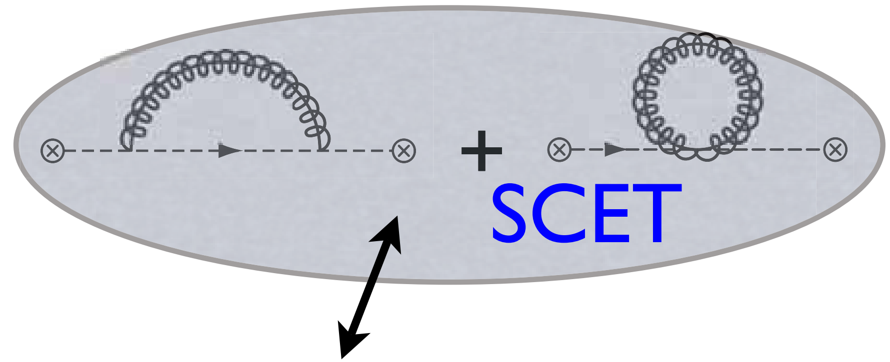
$\delta_V = \delta_V(\tau^n{}_a, l^+) = \delta(\tau^n{}_a - (l^+/Q)^{1-a/2})$

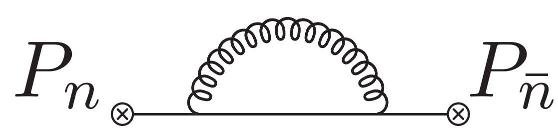
- Note that for the Thrust case(a=0): $\delta_R = \delta_V$ and is independent of loop momenta "q"
- Thus for the Thrust Jet Function, one can take the Imaginary part of the usual Collinear Jet Function times δ_V
- $\delta_R \equiv \delta_V + (\delta_R - \delta_V)$

Fixed Order Calculations

Jet Function

$J(\tau^n{}_a;\mu) = \mathrm{Disc}_{\{\tau a\}}\ 2$ 

$+$ 



Needs a zero bin subtraction in order to avoid double counting with soft modes

Manohar, Stewart, hep-ph/060500

Becher, Neubert, hep-ph/0603140

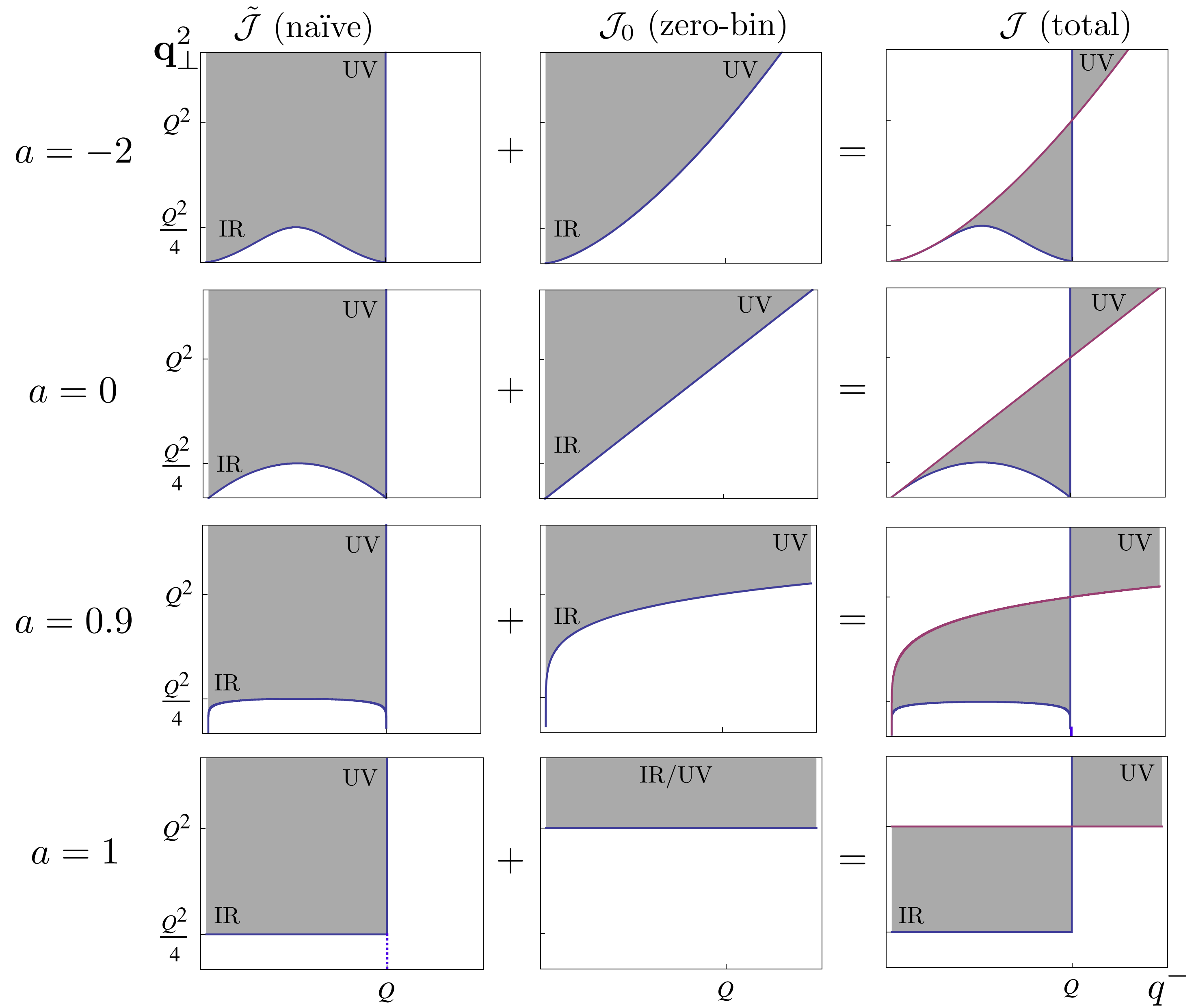
Bauer, Cata, GO, arXiv:0809.1099

- The QCD-like diagrams are manifestly IR finite $a<2$
- The non-trivial cancellation of IR Divergences occurs in the first diagram between Virtual and Real cuts.

Fixed Order Calculations

Jet Function

$F(q^2_\perp, q^-) = (q^2_\perp)^{-1-\epsilon} \, (q^-)^{-1}$



Fixed Order Calculations

Jet Function

- Jet Function is IR safe for all $a<1$
- The IR safety breaks down at $a=1$ as for the Soft Function
- Thus the Jet Function in the naive factorization theorem can no longer be calculated perturbatively
- We verified these facts without use of explicit IR regulators which makes the calculation much nicer

Fixed Order Calculations

Results for Soft Function and Jet Function

$$S_a^{\rm PT}(\tau_a^s;\mu) = \delta(\tau_a^s)\left[1 - \frac{\alpha_s C_F}{\pi(1-a)}\left(\frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{12}\right)\right] + \frac{2\alpha_s C_F}{\pi(1-a)}\left[\frac{\theta(\tau_a^s)}{\tau_a^s}\ln\frac{\mu^2}{(Q\tau_a^s)^2}\right]_+$$

$$\mu_S = Q\tau_a$$

$$J_a^n(\tau_a^n;\mu) = \delta(\tau_a^n)\left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{1-a/2}{2(1-a)}\ln^2\frac{\mu^2}{Q^2} + \frac{3}{4}\ln\frac{\mu^2}{Q^2} + f(a)\right]\right\} - \frac{\alpha_s C_F}{\pi}\left[\left(\frac{3}{4}\frac{1}{1-a/2} + \frac{2}{1-a}\ln\frac{\mu}{Q(\tau_a^n)^{1/(2-a)}}\right)\left(\frac{\theta(\tau_a^n)}{\tau_a^n}\right)\right]_+$$

$$\mu_J = Q\tau_a^{\frac{1}{2-a}}$$

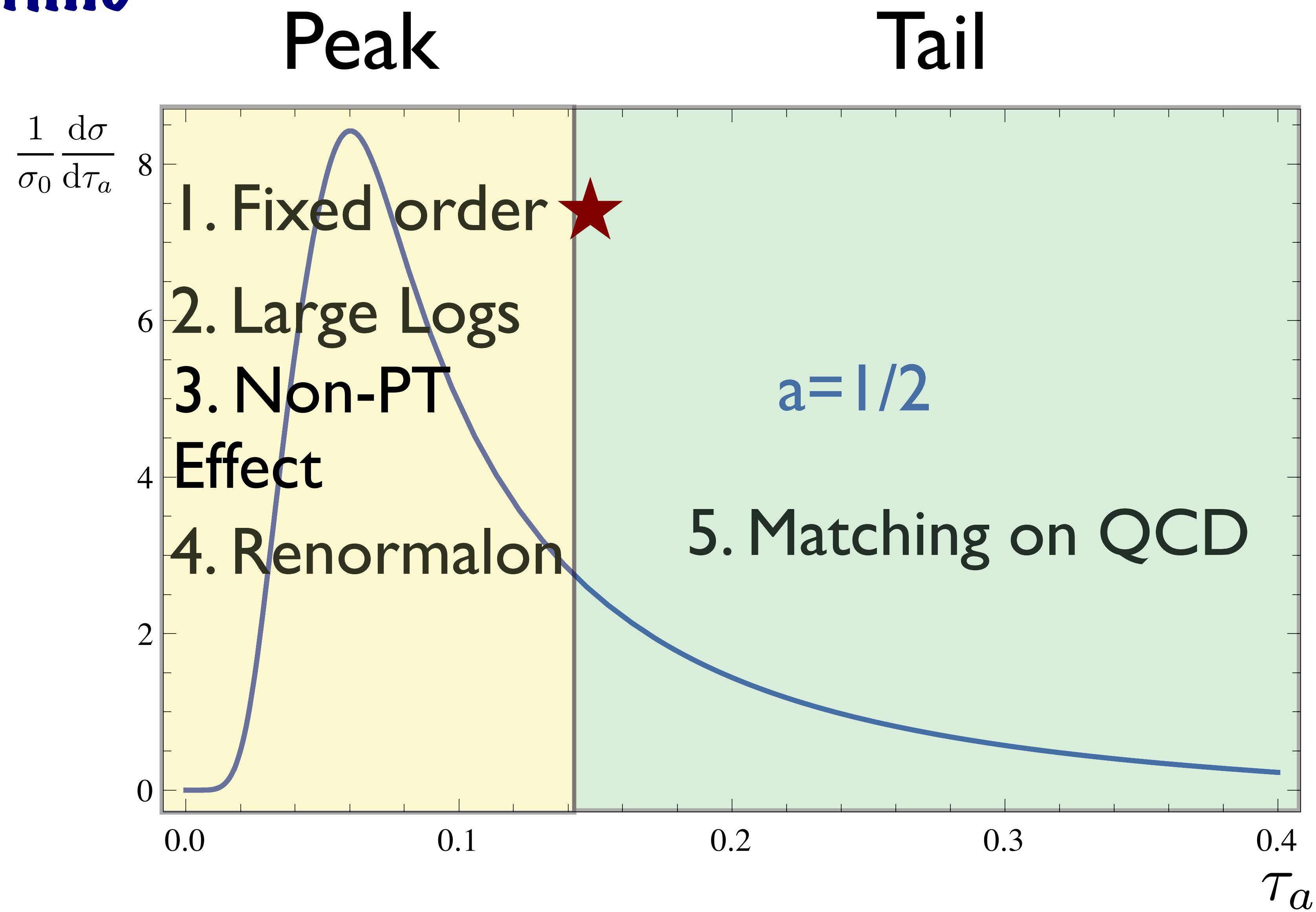
For a<1 the Soft scale is below the Jet scale: $\mu_s < \mu_j$

For a=1 the Jet and Soft Scales coincide

Outline

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

$\mu_s = Q\tau_a$



RG Evolution

Form of RGE

Since we are renormalizing distributions J(τ_a), S(τ_a), the RG Equation is a convolution:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} F(\mu) = \gamma_F(\mu) \otimes F(\mu)$$

$$\gamma_F(\tau;\mu) = -2\Gamma_F[\alpha_s]\left(\frac{1}{j_F}\left[\frac{\theta(\tau)}{\tau}\right]_+ - \ln\frac{\mu}{Q}\delta(\tau)\right) + \gamma_F[\alpha_s]\delta(\tau)$$

The History of solving this RGE is long...

Standard Case: $j_{soft}=1$, $j_{jet}=2$ — Korchemsky, Marchesini, 93; Balzereit, Mannel, Kilian, 98; Neubert, 05; Becher, Neubert, Pecjak, 07;

j arbitrary ⟷ Fleming,Hoang,Mantry,Stewart, 07

Angularities: $j_{jet}=2\text{-}a$ Hornig, Lee,GO

RG Evolution

RG Solution

$$F(\mu) = U_F(\mu, \mu_0) \otimes F(\mu_0)$$

$$\mu_S = Q\tau_a$$

$$S_a(\tau_a;\mu) = \frac{e^{K_S+\gamma_E\omega_S}}{\Gamma(-\omega_S)}\left(\frac{\mu_0}{Q}\right)^{j_S\omega_S} \times \Bigg[\Bigg\{1 - \frac{\alpha_s(\mu_0)C_F}{2\pi}\frac{1}{1-a}\Bigg(\ln^2\frac{\mu_0^2}{(Q\tau_a)^2} + 4H(-1-\omega_S)\ln\frac{\mu_0^2}{(Q\tau_a)^2} + \frac{\pi^2}{2} + 4\Big[[H(-1-\omega_S)]^2 - \psi^{(1)}(-\omega_S)\Big]\Bigg)\Bigg\}\left(\frac{\theta(\tau_a)}{\tau_a^{1+\omega_S}}\right)\Bigg]_+,$$

$$J_a^n(\tau_a;\mu) = \frac{e^{K_J+\gamma_E\omega_J}}{\Gamma(-\omega_J)}\left(\frac{\mu_0}{Q}\right)^{j_J\omega_J} \times \Bigg[\Bigg\{1 + \frac{\alpha_s(\mu_0)C_F}{4\pi}\Bigg(\frac{2-a}{1-a}\ln^2\frac{\mu_0^2}{Q^2\tau_a^{\frac{2}{2-a}}} + \left(3 + \frac{4H(-1-\omega_J)}{1-a}\right)\ln\frac{\mu_0^2}{Q^2\tau_a^{\frac{2}{2-a}}} + 4f(a) + \frac{4}{(1-a)(2-a)}\left[\frac{\pi^2}{6} + [H(-1-\omega_J)]^2 - \psi^{(1)}(-\omega_J)\right]\Bigg)\Bigg\}\left(\frac{\theta(\tau_a)}{\tau_a^{1+\omega_J}}\right)\Bigg]_+,$$

$$\mu_J = Q\tau_a^{\frac{1}{2-a}}$$

RG Evolution

Counting of the Logarithms

$$\alpha L \propto 1$$

α^{-1} 1 1 α α α^2

LO

NLO

$$\frac{d\sigma}{d\tau_a} \propto \exp[\alpha L^2 + \alpha L + \alpha^2 L^2 + \alpha^2 L + \alpha^3 L^2 + \alpha^3 L + \ldots]\{1 + \alpha + \ldots\}$$

1 loop 2 loops 3 loops

- This Log counting leads to consistent combination of resummation and fixed order: NLL/LO, N^2LL/NLO
- However, we will still keep all info we have: NLL/NLO
- The Consistency Relations show that all one needs to go to N^2LL/NLO is the Two Loop Non-Cusp part of the Anomolous Dimension of the Soft Function!!!

RG Evolution

Full Distribution at NLL/NLO

$\tau_a = \tau_J + \bar{\tau}_J + \tau_s$

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

$S \sim (1 + \alpha_s(\mu_s) \ln(\mu_s / Q\tau_s) + \ldots)$

One is tempted to chose the scale $\mu_s = Q\tau_s$, in order to minimize logs in the soft function

But!!! Then we end up integrating over spurious Landau Pole

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_a} = H(Q;\mu)\int_0^1 \mathrm{d}\tau_n \mathrm{d}\tau_{\bar{n}} \mathrm{d}\tau_s \delta(\tau_a - \tau_n - \tau_{\bar{n}} - \tau_s) J_n(\tau_n) J_{\bar{n}}(\tau_{\bar{n}}) S(\tau_s)$$

RG Evolution

Full Distribution at NLL/NLO

$\tau_a = \tau_J + \bar{\tau}_J + \tau_s$

$$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$$

Instead, keep the scales fixed and do the convolution integral analytically, then the final answer will contain only logs of $(\mu_s / Q\tau_a)$

Thus, the Effective Theory allows us to avoid Spurious Landau Poles!!!

Same is true for DIS and Drell-Yan:

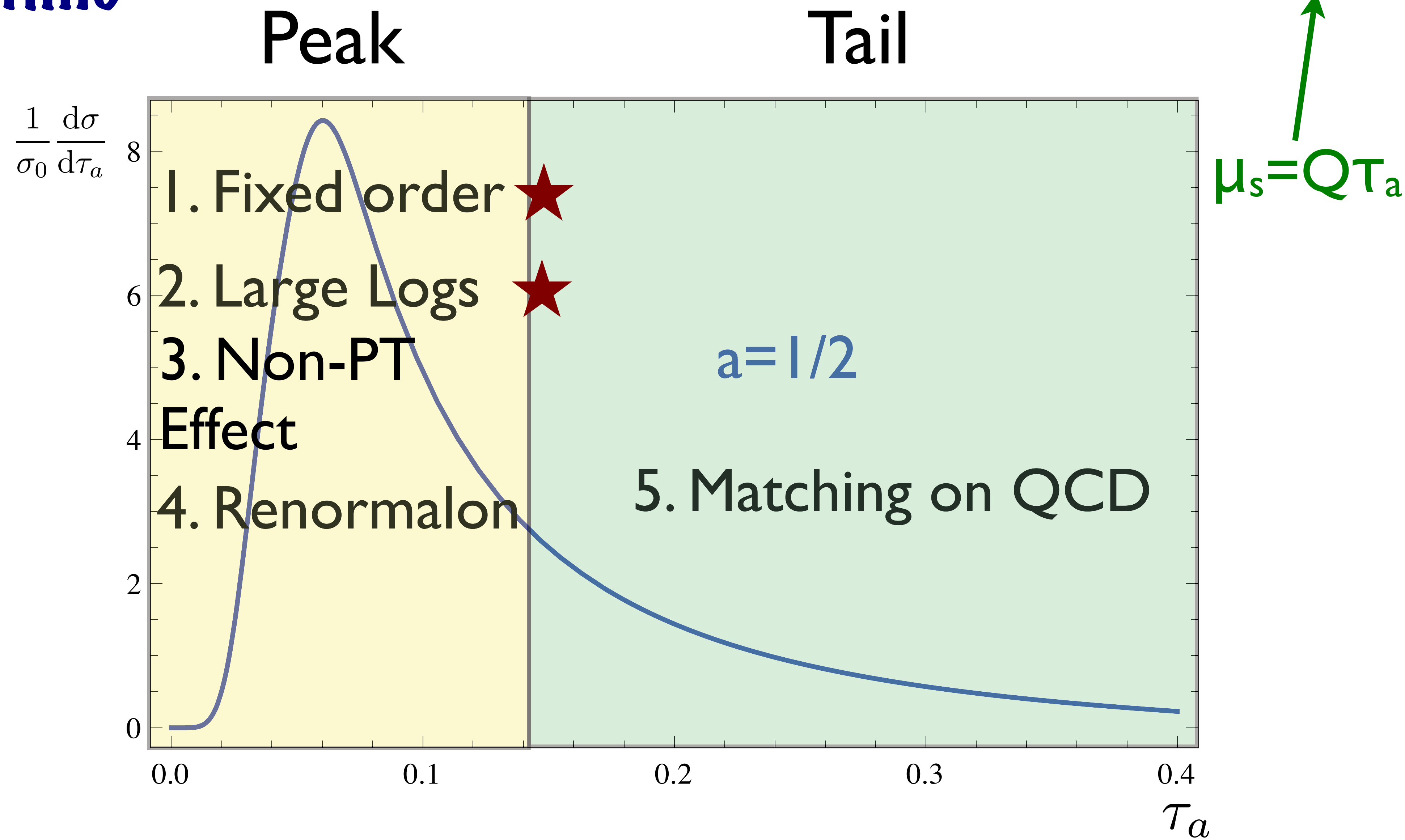
Manohar, hep-ph/0309176

Becher, Neubert, hep-ph/0605050

Becher, Neubert, Pecjak, hep-ph/0607228

Outline

$\sigma(\tau_a)=H * \bar{J} \otimes J \otimes S$



Non-Perturbative Model Function

•Soft function as convolution of Perturbative and Non-Perturbative parts:

$$S_a(\tau_a;\mu) = \int \mathrm{d}\tau_a'\, S_a^{\mathrm{PT}}(\tau_a - \tau_a';\mu)\, f_a^{\exp}\left(\tau_a' - \frac{2\Delta_a}{Q}\right)$$

Korchemsky, Tafat, hep-ph/0007005
Hoang, Stewart, arXiv:0709.3519
Ligeti, Stewart, Tackmann, arXiv:0807.1926

•For the thrust case(a=0) we adopt the appropriate model function that has been constructed

•Scaling of the model function with a we find from universality of the first moment of the Soft Function:

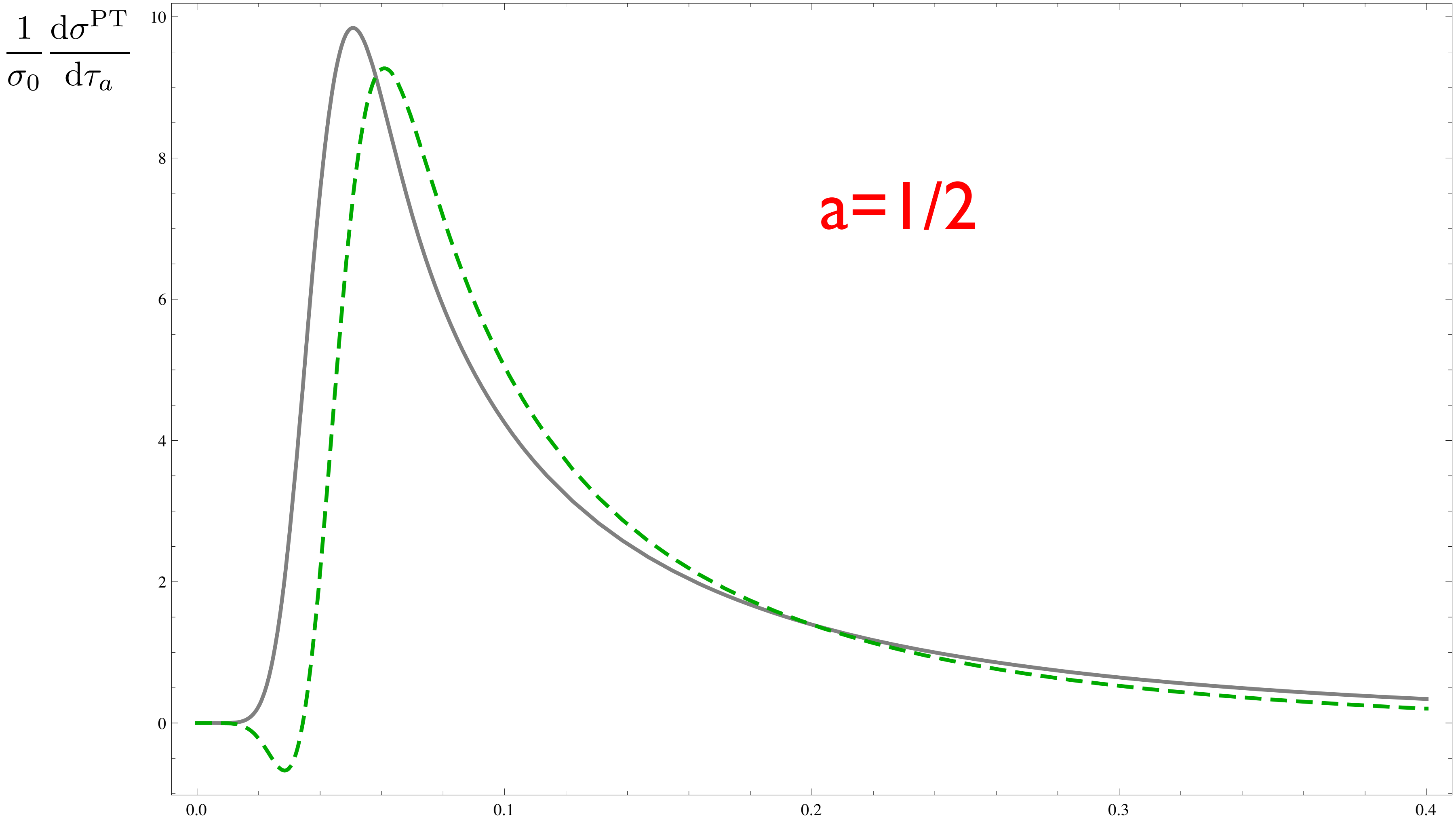
$$\int \mathrm{d}\tau_a \tau_a S(\tau_a,\mu) = \frac{2}{1-a}\frac{A(\mu)}{Q}$$

Lee, Sterman, hep-ph/0611061

$$\Delta_a = \frac{\Delta_0}{1-a}$$

$$f_a^{\exp[1]} = \frac{1}{1-a} f_0^{\exp[1]}$$

Non-Perturbative Model Function



NLL/LO with gapped soft function

NLL/NLO distribution with gapped soft function

Non-Perturbative Model Function

Renormalon Subtracted Gapped Soft Function

$$S_a(\tau_a;\mu) = \int \mathrm{d}\tau_a' \, S_a^{\mathrm{PT}}(\tau_a - \tau_a';\mu)\, f_a^{\exp}\left(\tau_a' - \frac{2\Delta_a}{Q}\right)$$

- The idea is to make a suitable shift in the gap parameter:

$$\Delta_a = \bar{\Delta}_a(\mu) + \delta_a(\mu)$$

- Define $\delta_a(\mu)$ in terms of $S_a^{PT}(\mu)$ in order to cancel the renormalon ambiguity

- We chose to use the "Position-Mass Scheme"

Jain, Scimemi, Stewart, arXiv:0801.0743

$$S_a(\tau_a;\mu) = \int \mathrm{d}\tau_a' \left[S_a^{\mathrm{PT}}(\tau_a - \tau_a';\mu)\, f_a^{\exp}\left(\tau_a' - \frac{2\bar{\Delta}_a(\mu)}{Q}\right)\right] - \frac{2\delta_a^1(\mu)}{Q}\frac{\mathrm{d}}{\mathrm{d}\tau_a} f_a^{\exp}\left(\tau_a - \frac{2\bar{\Delta}_a(\mu)}{Q}\right)$$

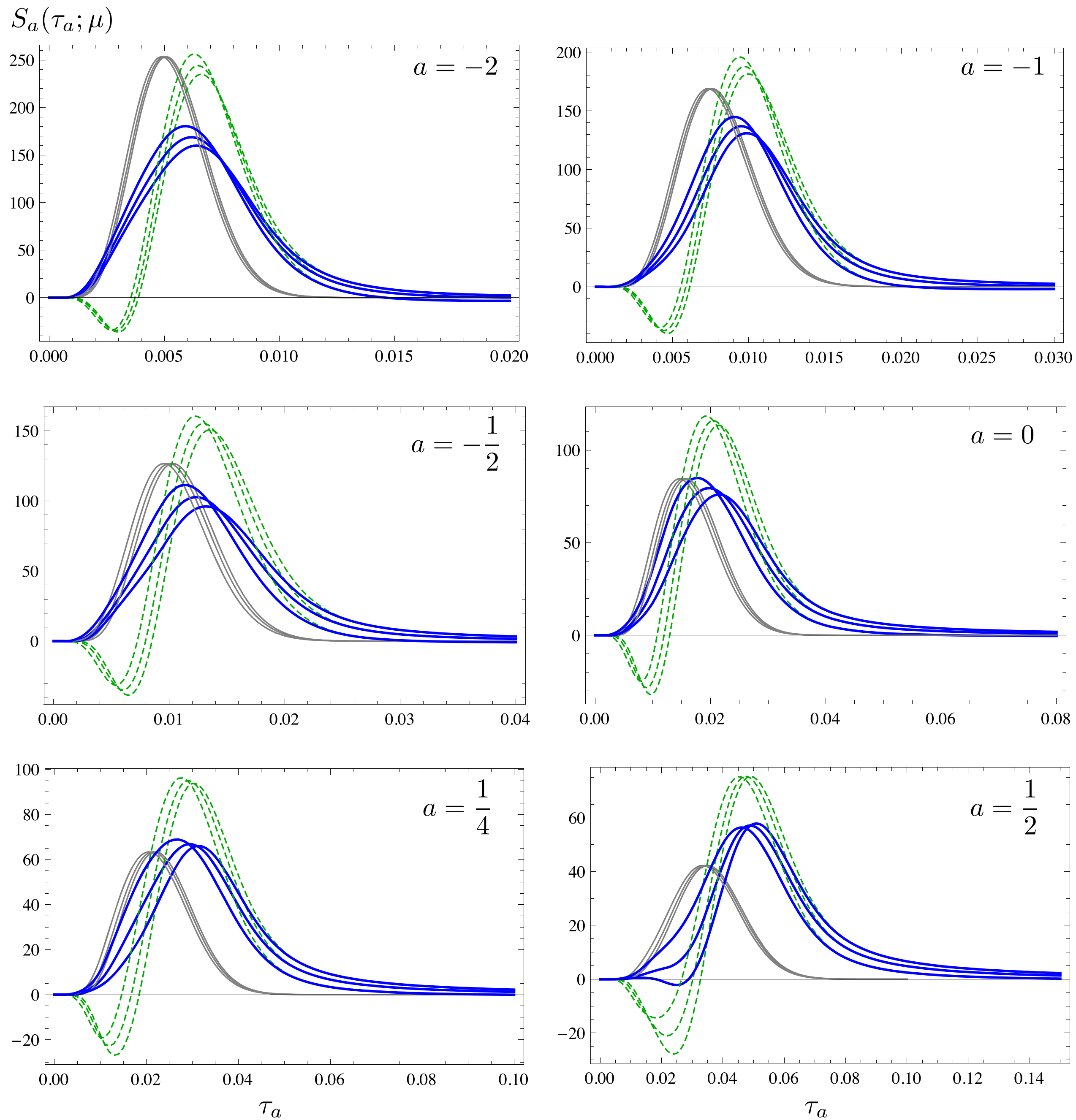
Non-Perturbative Model Function

Numerical Results

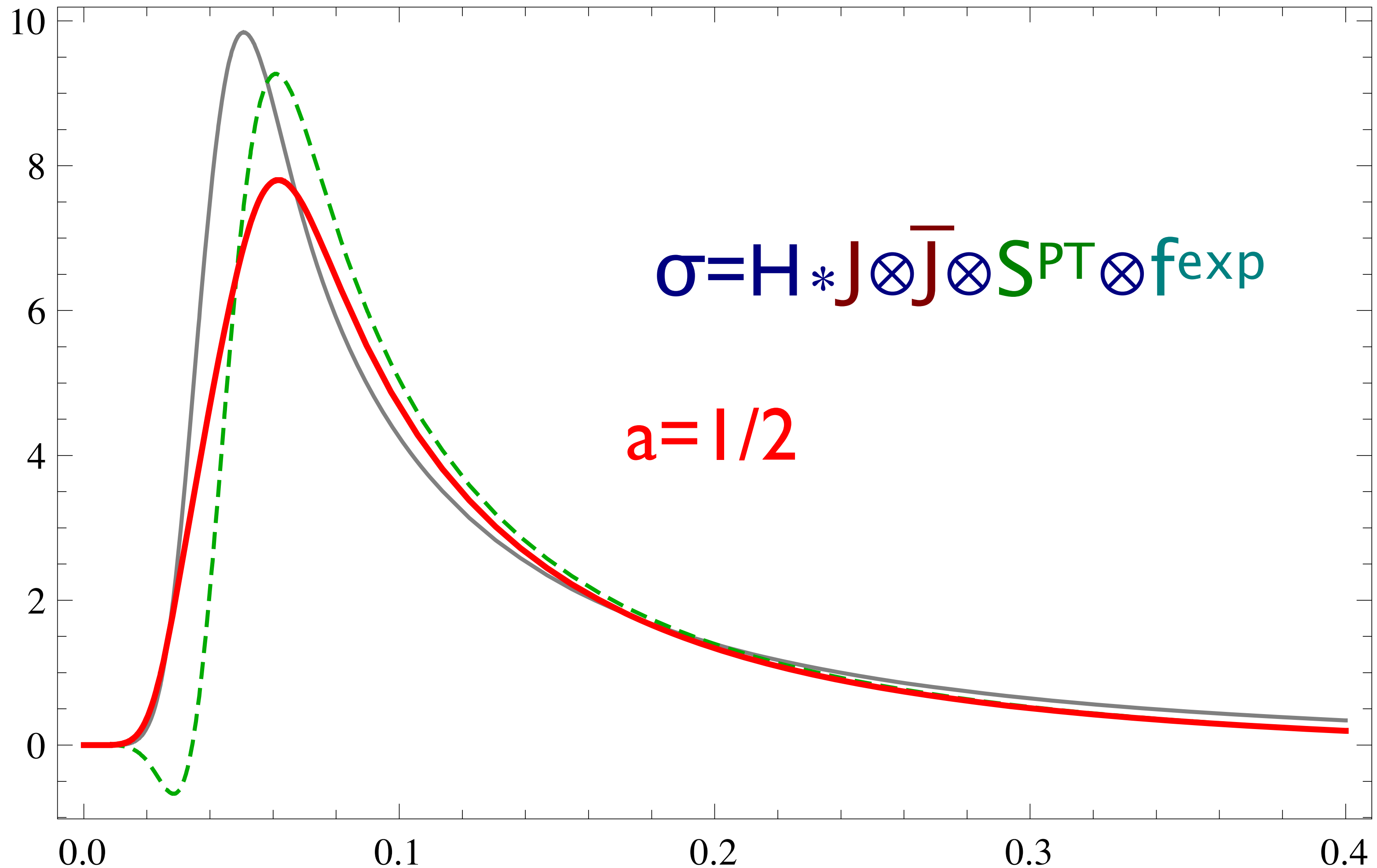
Tree level

1-loop gapped

1-loop gapped with renormalon subtraction



Non-Perturbative Model Function



NLL/LO with gapped soft function

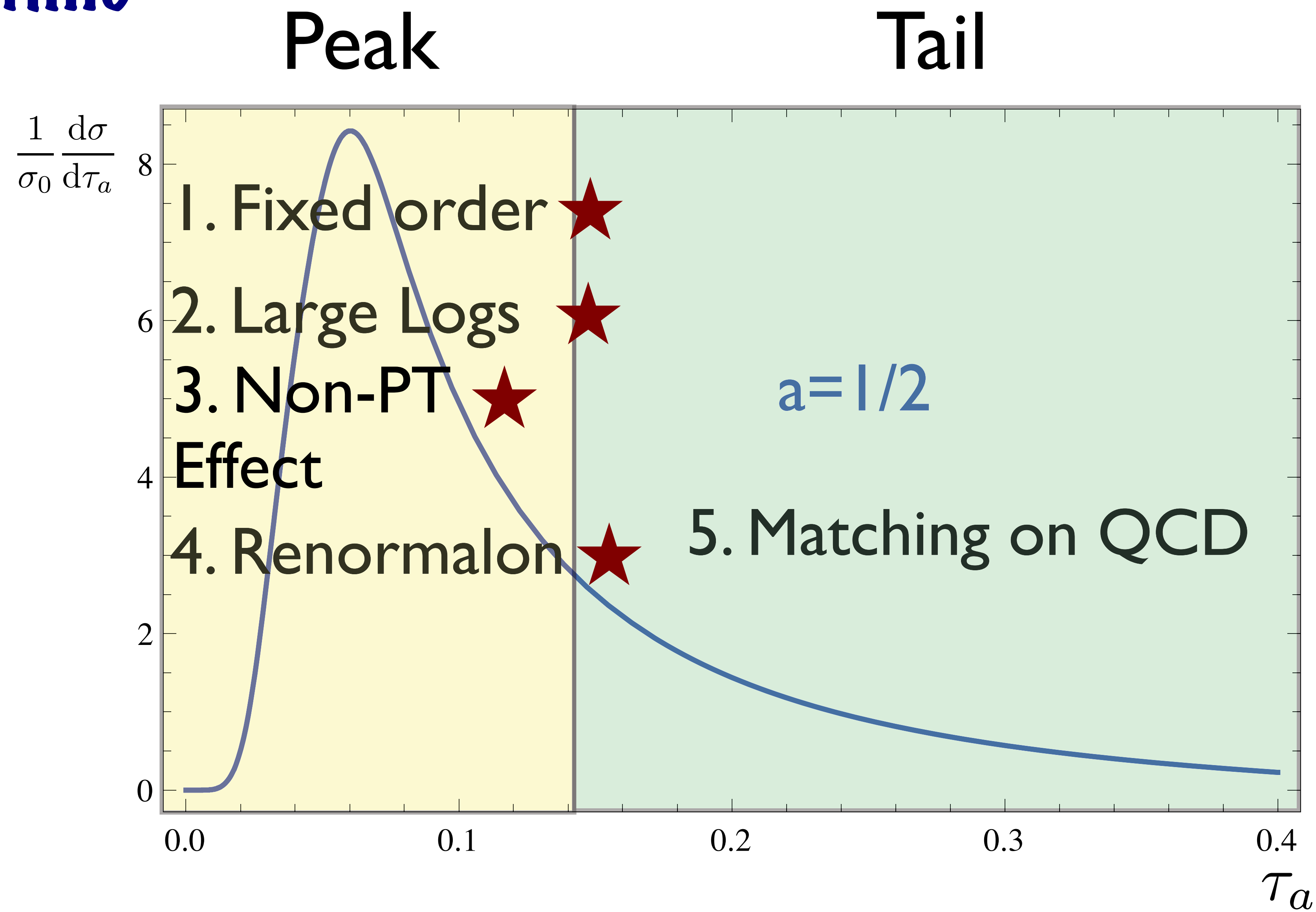
NLL/NLO distribution with gapped soft function

NLL/NLO distribution gapped and renormalon subtracted

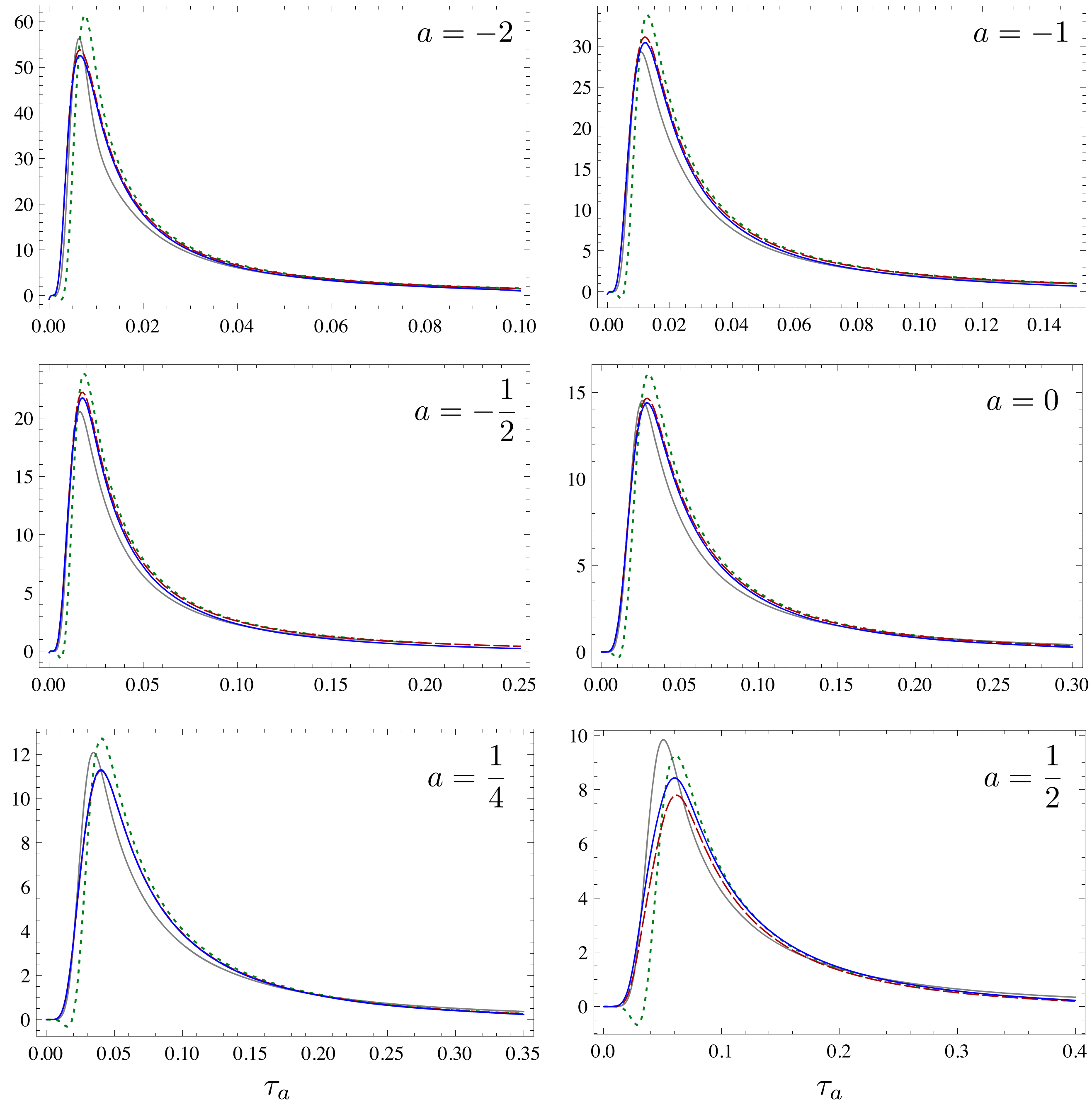
Outline

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

$\mu_s = Q\tau_a$



Numerical Results



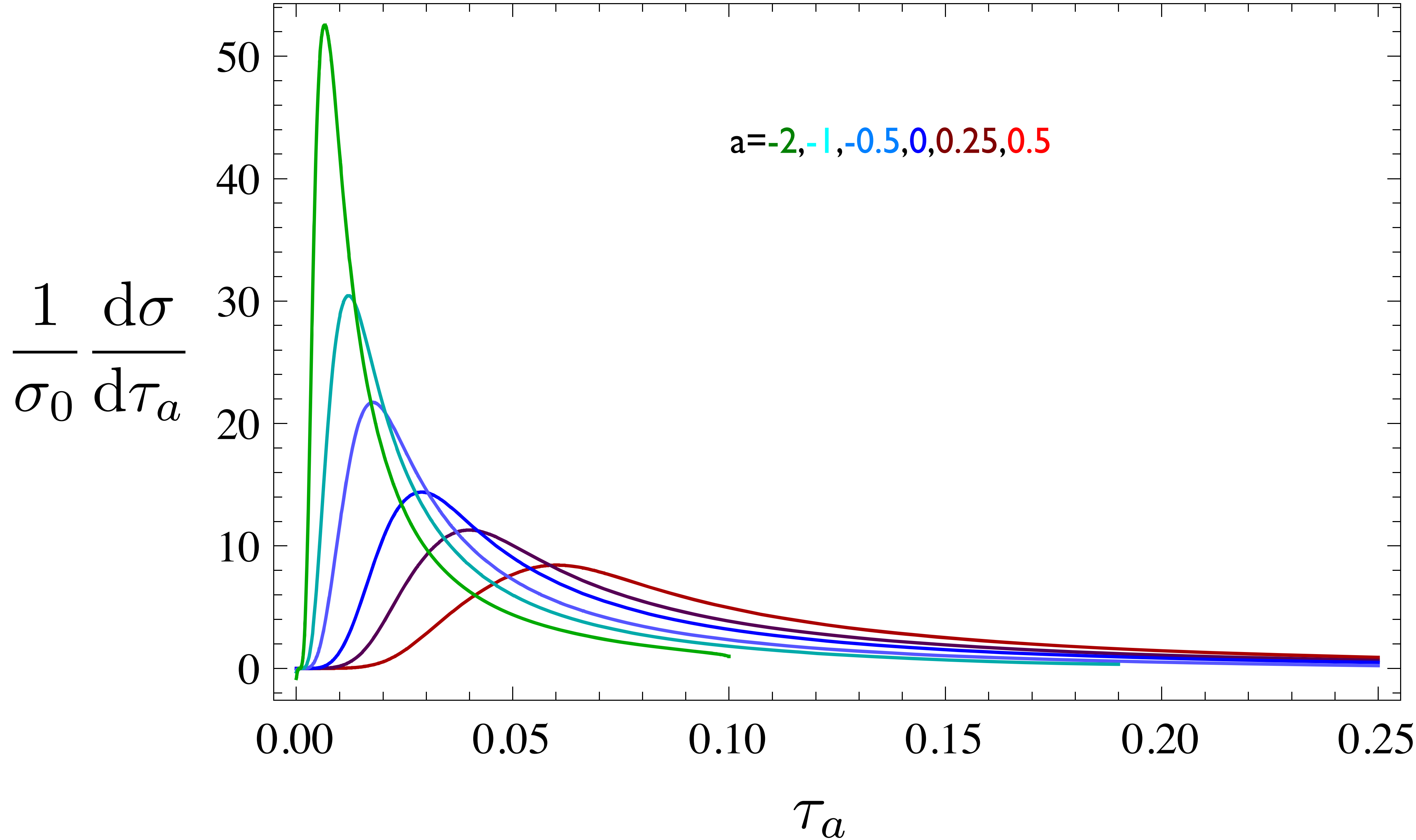
NLL/LO+gapped

NLL/NLO+gapped

NLL/NLO+renormalon

NLL/NLO+renormalon
+matched

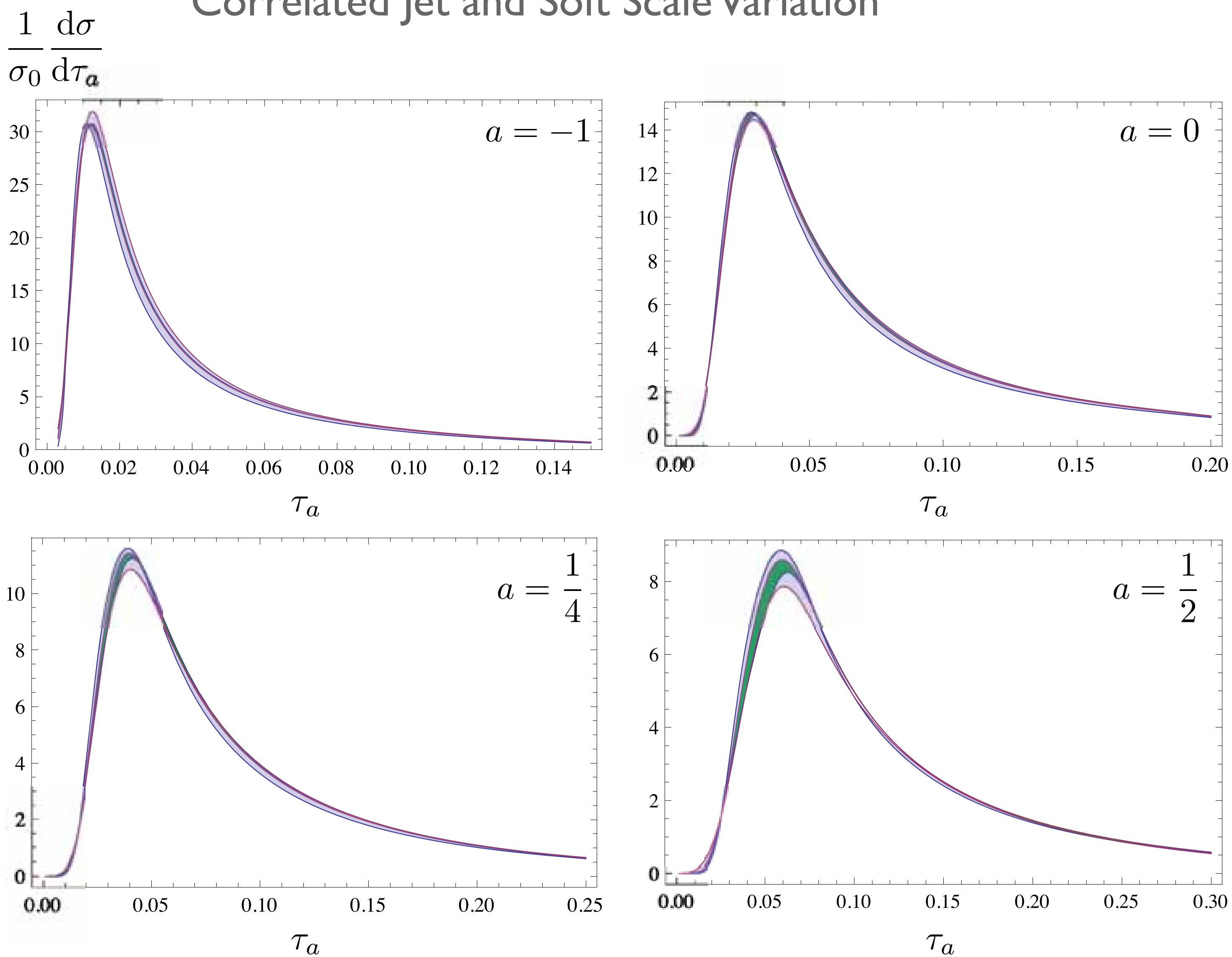
Numerical Results



Numerical Results

Hard Scale Variation

Correlated Jet and Soft Scale Variation



Comparison to Previous Results

Spurious Landau Poles

Fourier Space

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

- In the momentum space the convolution simply becomes a product

$$\sigma(\nu) = H * \bar{J}(\nu) * J(\nu) * S(\nu)$$

$\nu \leftrightarrow 1/\tau$

$S \sim (1 + \alpha_s(\mu_s) \ln(\mu_s \nu / Q) + \ldots)$

- Now, again minimizing logs in the momentum space $(\mu_s = Q/\nu)$ induces a Spurious Landau Pole when transforming back to the position space

- These Spurious Poles behave like power corrections, but are non-physical

Catani, Mangano, Nason, Trentadue, hep-ph/9604351

Comparison to Previous Results

Cross-Section in the Momentum space:

$$\frac{1}{\sigma_{\rm tot}}\,\tilde{\sigma}\left(\nu,Q,a\right)=\exp\left\{2\int_0^1\frac{du}{u}\Bigg[\int_{u^2Q^2}^{uQ^2}\frac{dp_T^2}{p_T^2}A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a}-1\right)\right.$$
$$\left.+\frac{1}{2}B\left(\alpha_s(\sqrt{u}Q)\right)\left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}}-1\right)\Bigg]\right\}$$

NLL/LO Berger, Sterman, hep-ph/0307394

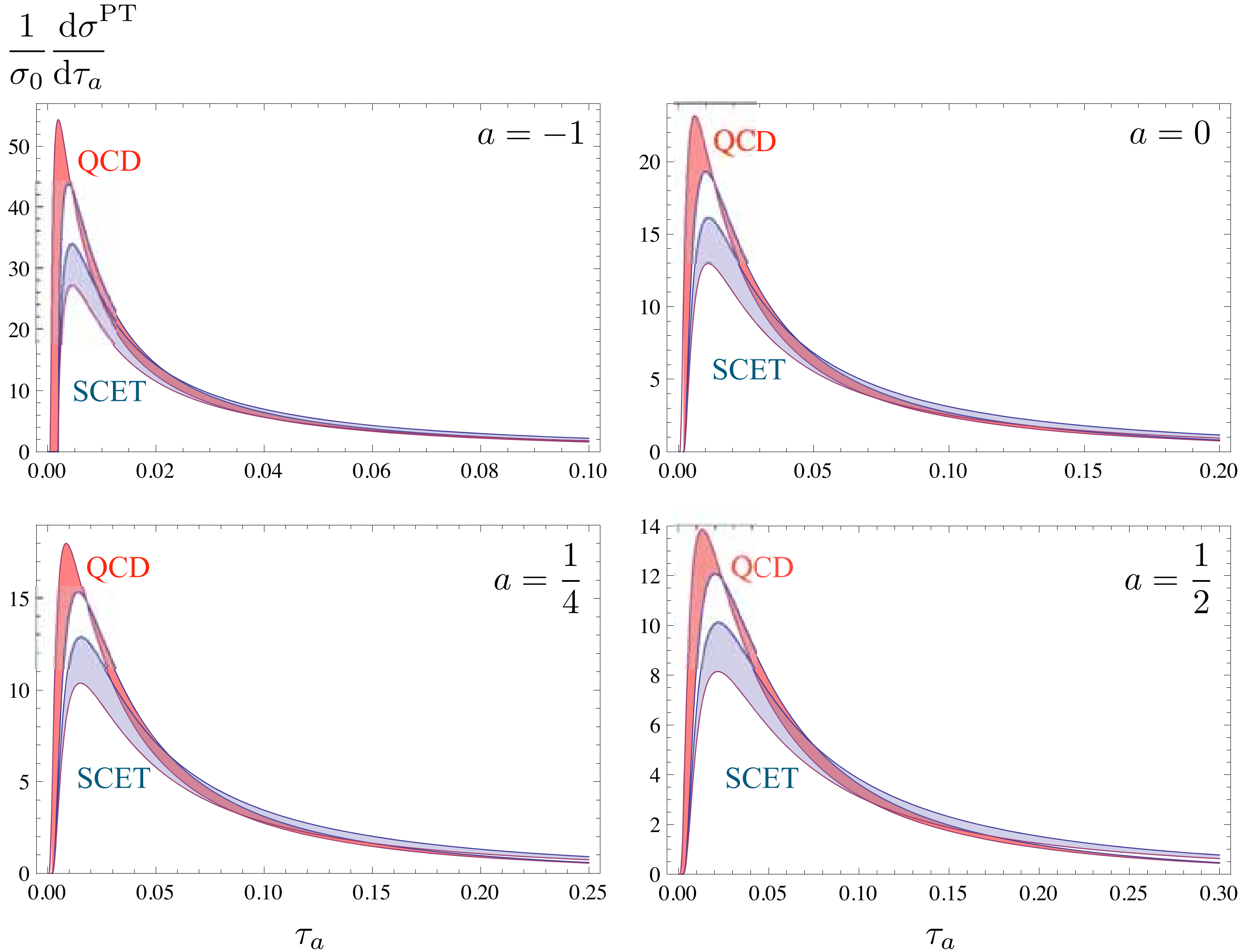
Additional singularities arise from transforming back to the position space, which look like power corrections, but are non-physical

Catani, Mangano, Nason, Trentadue, hep-ph/9604351

Comparison to Previous Results

Angularity Distribution taken from Sterman, Berger[NLL/LO]

Angularity Distribution in SCET [NLL/LO]



Comparison to Previous Results

•In Both Effective Theory and the Traditional QCD approaches there are large power corrections in the peak region

•However the Traditional QCD approach to factorization has additional Spurious Landau Poles due to the lack of flexibility to fix the RGE scales for Jet and Soft functions

Conclusions

- We resummed Angularities for all a<1 to NLL/NLO in SCET
- Effective Theory allows us to avoid spurious Landau poles via flexibility of the choice of the scales

- We adopted a model for non-perturbative corrections, developed for Thrust and generalized it to arbitrary Angularity

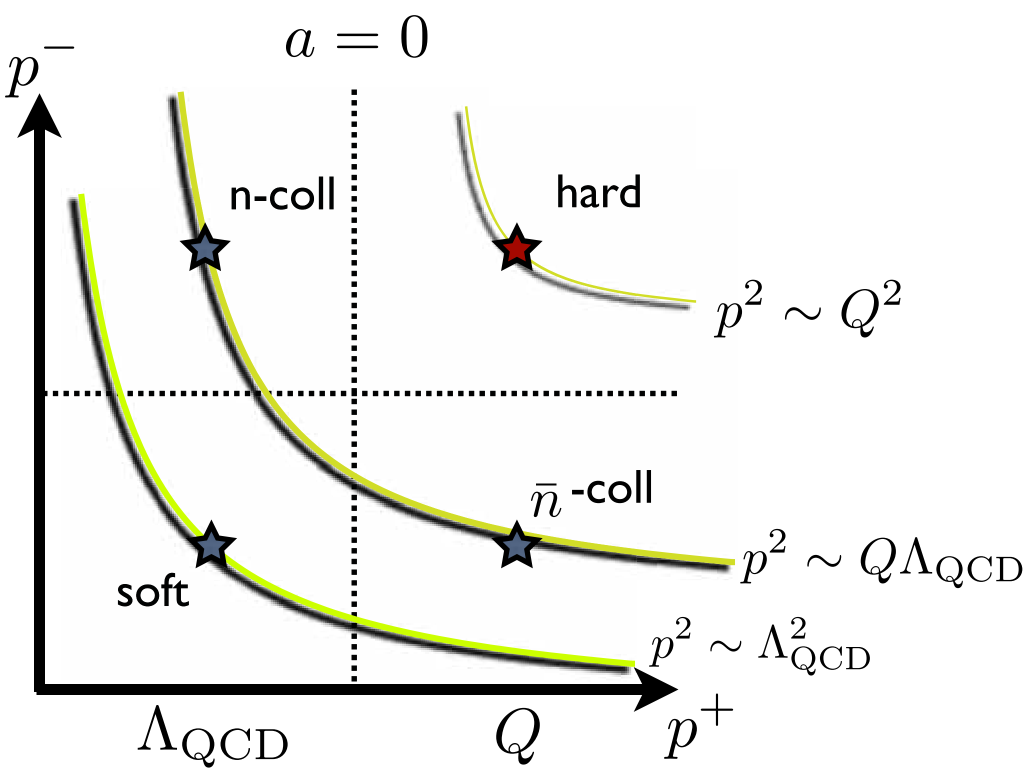
- Effective Theory approach can be straightforwardly generalized to higher orders

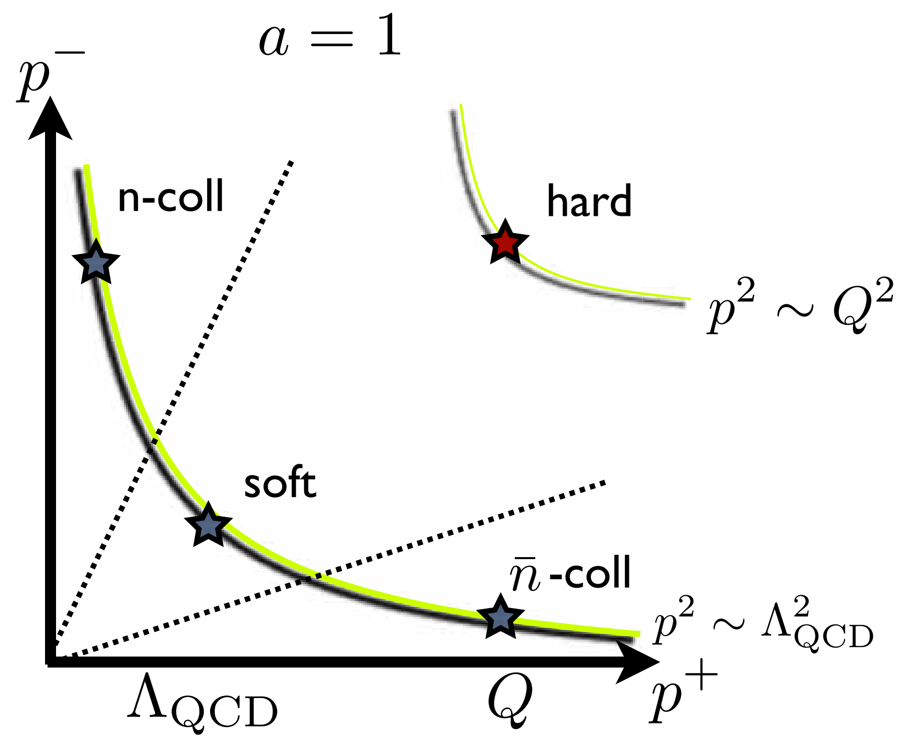
- Angularities can be used to study the substructure of the jets in both leptonic and hadronic collisions

Backup Slides...

Fixed Order Calculations

SCET_{I} vs. SCET_{II}





RGE Evolution

The Consistency Relations

$\sigma(\tau_a) = H * \bar{J} \otimes J \otimes S$

- The Factorization Formula is valid for both Bare and Renormalized Functions to all orders in Perturbation Theory $Z_H * Z_J \otimes Z_{\bar{J}} \otimes Z_s = 1$
- Acting with $\mu d/d\mu$ we get relation between Anomolous Dimensions to All orders in PT $\gamma_H \delta(\tau) + 2\gamma_J + \gamma_S = 0$
- Resulting Consistency Relations Are:

$$\Gamma_S[\alpha_s] = \frac{1}{1-a}\Gamma_H[\alpha_s]$$

$$\Gamma_J[\alpha_s] = -\frac{1-a/2}{1-a}\Gamma_H[\alpha_s]$$

$$2\gamma_J[\alpha_s] + \gamma_S[\alpha_s] = -\gamma_H[\alpha_s]$$

		$F=S$	$F=J$
	j_F	1	$2-a$
Cusp	Γ_F^0	$-8C_F\frac{1}{1-a}$	$8C_F\frac{1-a/2}{1-a}$
Non-Cusp	γ_F^0	0	$6C_F$

Matching in the Tail Region

- Matching in the Tail region is needed since the SCET degrees of freedom capture only collinear and soft interactions, thus valid in the Peak region

- One of the ways to go is to include three jet operators in SCET and match QCD onto such Effective Theory

Bauer, Schwartz, hep-ph/0607296
Marcantonini, Stewart, arXiv:0809.1093

- We choose to simply interpolate between the Peak region, where SCET is valid and the Tail Region, where resummation is not important, so fixed order QCD is valid here

Catani, Trentadue, Turnock, Webber, (1993)
Becher, Schwartz, arXiv:0803.0342

Matching in the Tail Region

- The Interpolation we are looking for is given by:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a}^{\rm PT}\bigg|_{\rm NLL/NLO} = \frac{1}{\sigma_0}\frac{d\sigma_2}{d\tau_a}^{\rm PT}\bigg|_{\rm NLL/NLO} + r_a(\tau_a)$$

- where $r_a(\tau_a)$ is the difference between NLO Fixed Order QCD and SCET cross-sections:

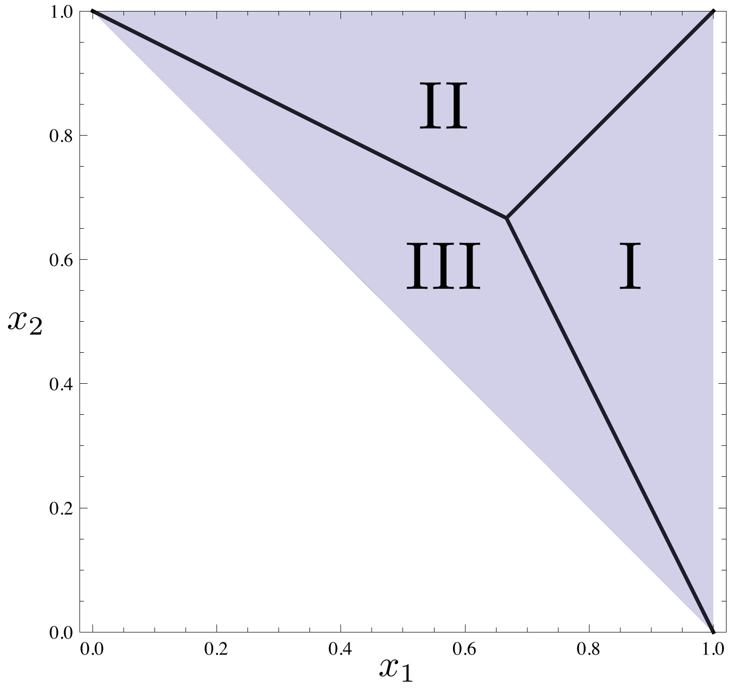
$$r_a(\tau_a) = \frac{1}{\sigma_0}\left(\frac{d\sigma^{q\bar{q}g}}{d\tau_a} - \frac{d\sigma_2}{d\tau_a}\right)_{\rm FO}$$

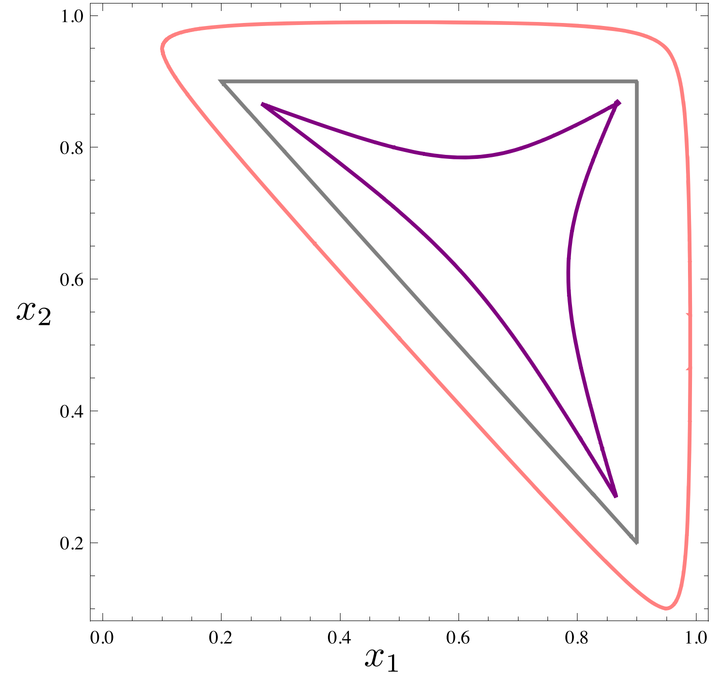
- First term we calculate numerically and for the second one we have analytical formula for all Angularities

Matching in the Tail Region

NLO QCD cross-section

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{q\bar{q}g}}{\mathrm{d}\tau_a} = \frac{\alpha_s C_F}{2\pi}\int \mathrm{d}x_1 \mathrm{d}x_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\delta(\tau_a - \tau_a(x_1, x_2))$$

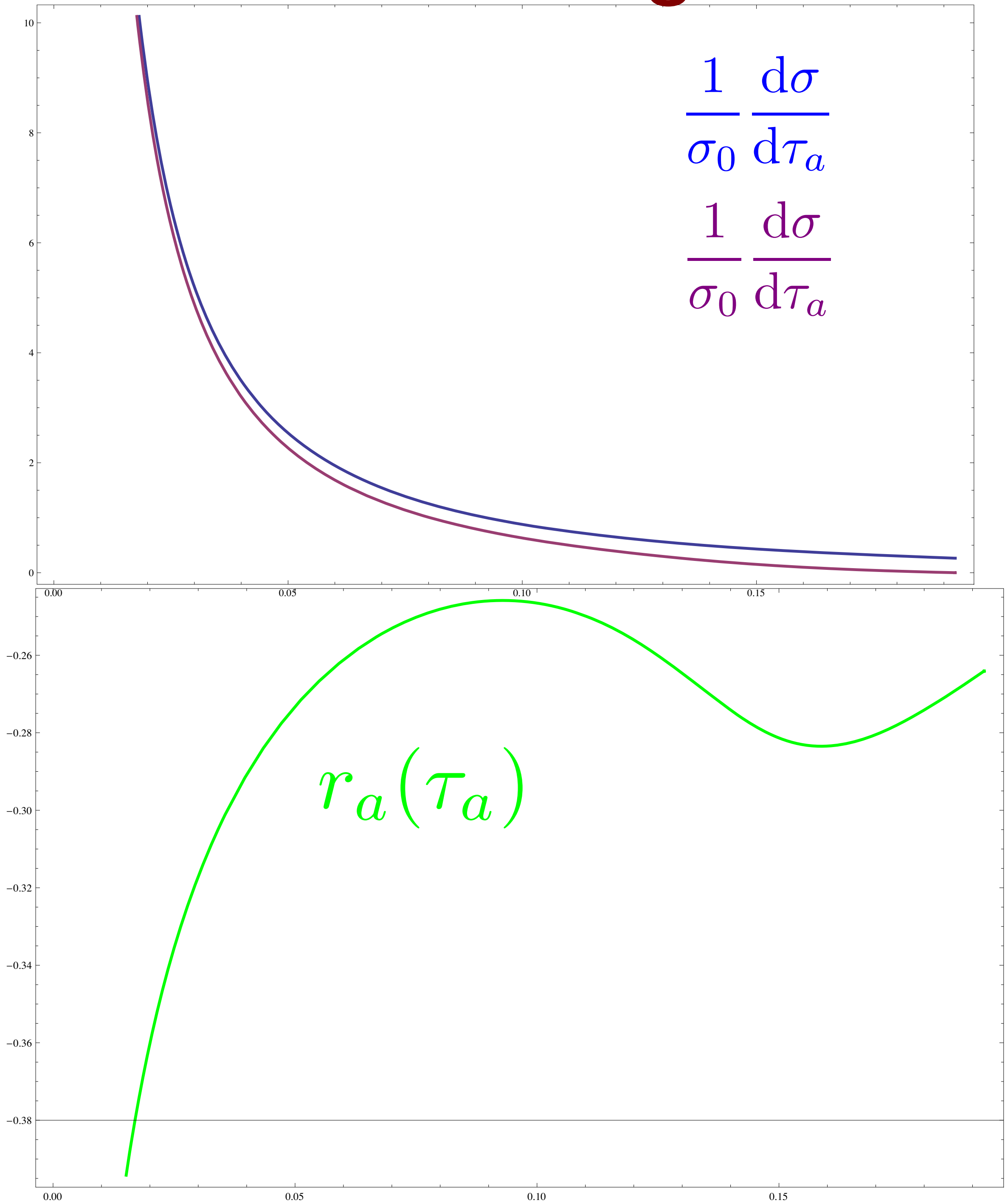




Crosscheck:

$$\int_0^{\tau_a^{\max}} \mathrm{d}\tau_a \left(\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_2^{\mathrm{PT}}}{\mathrm{d}\tau_a} + r_a(\tau_a)\right) = 1 + \frac{\alpha_s}{\pi}$$

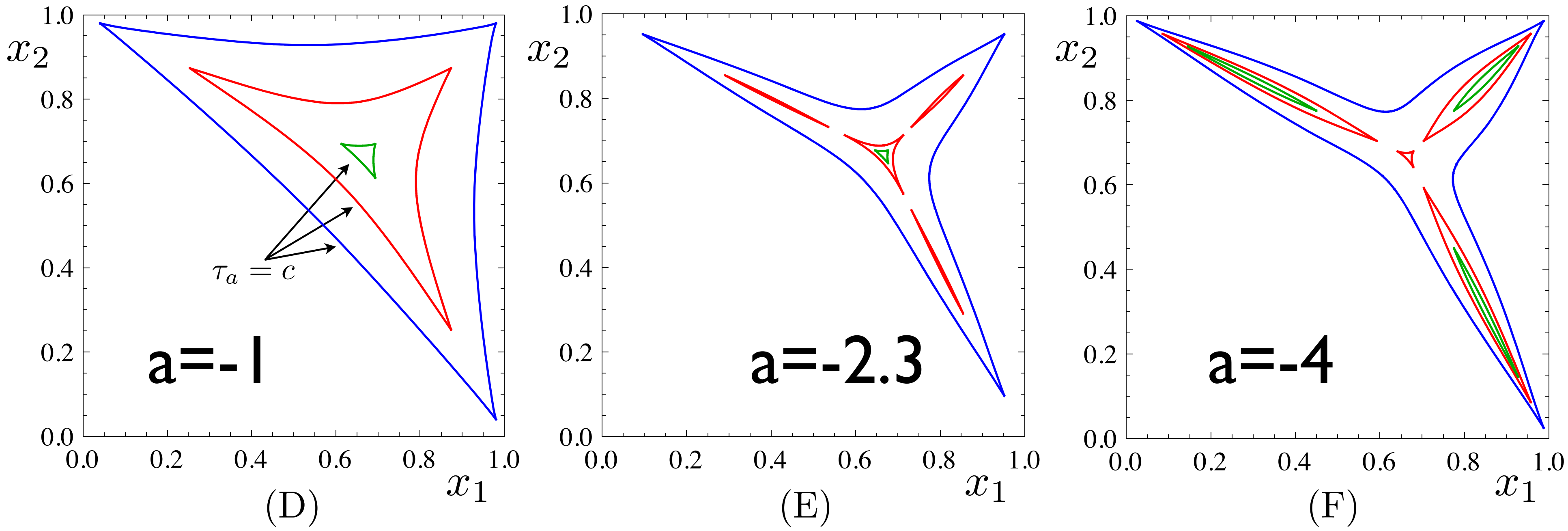
Matching in the Tail Region



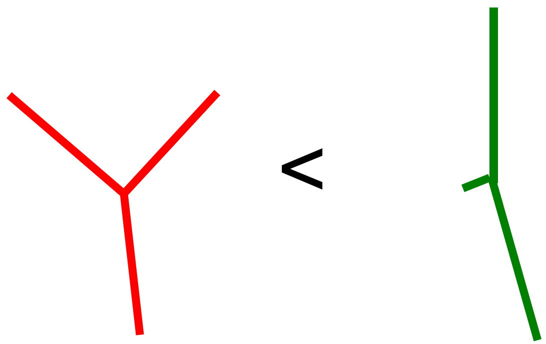
Numerical Example for a=-1

Matching in the Tail Region

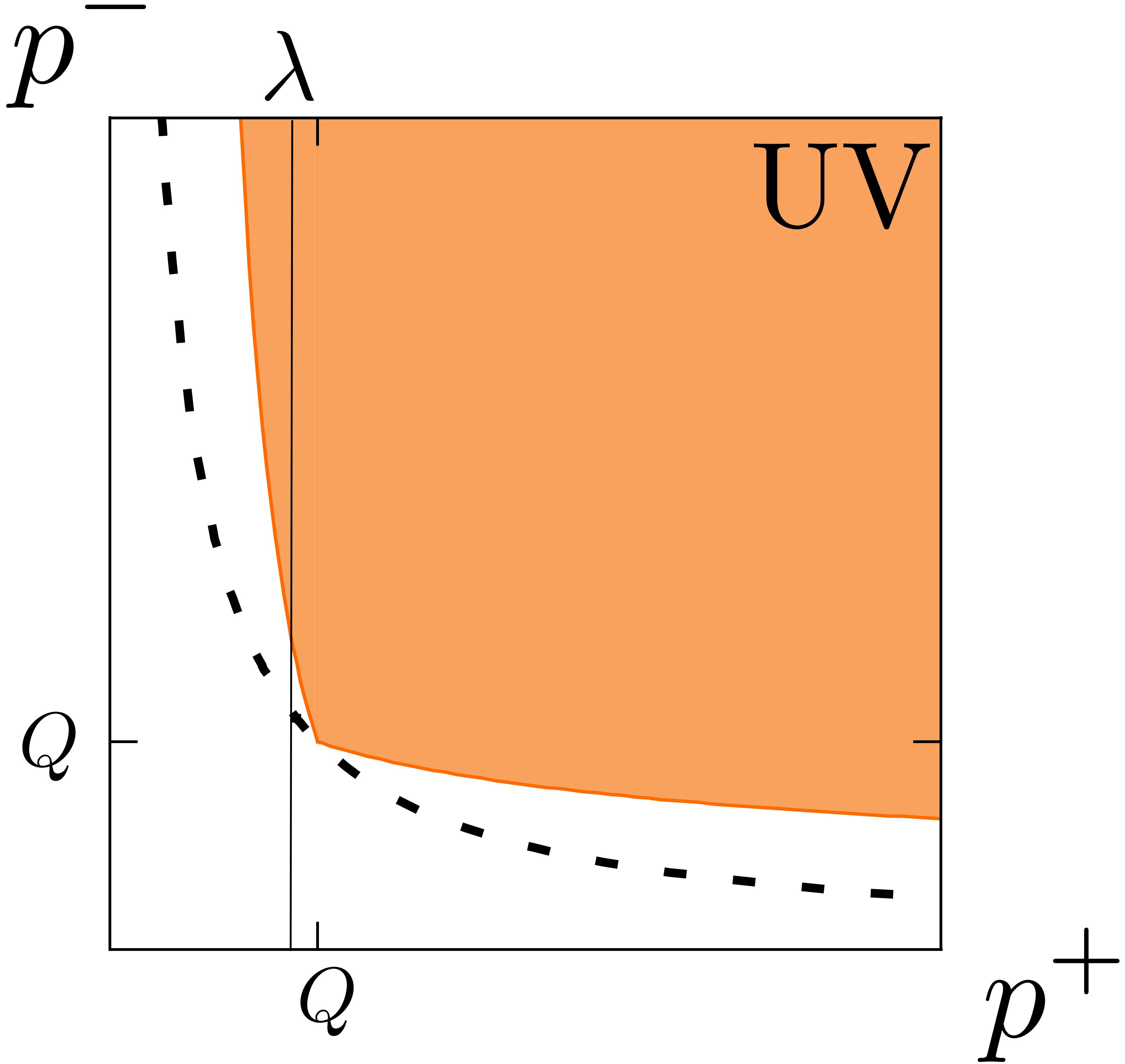
Weird Behavior of Angularities for a<-1.978...



Two jet event should have smaller angularity than 3 jet event



This is only true for a≥-2!!!



p^-
λ
UV
Q
Q
p^+