Poisson Distribution of Radioactive Decay

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In this experiment we observe the distribution of radiation emitted by a ^{137}Cs source. Using a scintillation counter, we count the number of gamma rays emitted by the radiation source at four different mean count rates: 1 sec^{-1}, 4 sec^{-1}, 10 sec^{-1}, and 100 sec^{-1}. From this we can plot the distribution of counts/sec versus the frequency of count rate. We find the distributions for the different mean count rates comparable to Poisson and Gaussian distributions. We also find that the Gaussian distribution can approximate the Poisson distribution very well at high mean rates.

INTRODUCTION

A ^{137}Cs source is an excellent, predictable gamma ray source. It randomly releases radiation at a predictable average rate. Because the radiation releases are independent events, we should be able to model radioactive decay of ^{137}Cs with a Poisson distribution. If we are able to do this, we can make predictions about the spread of radiation over time from such a radioactive source if we can determine the mean rate of emitted radiation.

THEORY

A useful model for predicting the outcome of random, independent events is the Poisson distribution, defined by the equation:

$$P(x;\mu) = \frac{\mu^x e^{-\mu}}{x!} \tag{1}$$

This distribution has its origins in the Binomial distribution, which models the success of an event x with a given probability p over n measurements, and is given by the equation:

$$Pr(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \tag{2}$$

If we take μ, the mean rate of events, to equal pn, we can then evaluate $Pr(x)$ as n goes to infinity:

$$\lim_{x\to\infty} \frac{n!}{x!(n-x)!}\frac{\mu}{n}^x(1-\frac{\mu}{n})^{n-x} \tag{3}$$

$$\lim_{x\to\infty} \underbrace{\frac{n!}{n^x(n-x)!}}_{\approx 1}\frac{\mu^x}{x!}\underbrace{(1-\frac{\mu}{n})^n}_{\approx e^{-\mu}}\underbrace{(1-\frac{\mu}{n})^{-x}}_{\approx 1} \tag{4}$$

$$\approx \frac{\mu^x e^{-\mu}}{x!} \tag{5}$$

In order to apply the Poisson distribution to a certain process, we must first determine whether or not the process is in "steady state with mean rate μ." If we take X to be the number of events occurring over a time T, then

$$\lim_{x\to\infty}\left(\frac{X}{T}\right) = \mu \tag{6}$$

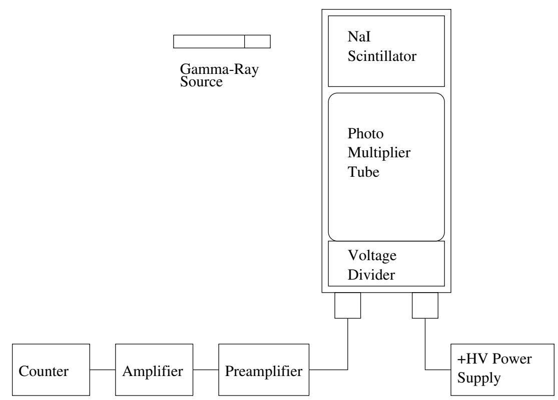

FIG. 1: Diagram of experimental setup showing the source, scintillator, photomultiplier tube, preamplifier, and amplifier. (source: *Poisson Statistics* 8.13 lab guide)

Additionally, if a process follows a Poisson distribution, we will also find that the standard deviation should equal, or come close to, $\sqrt{\mu}$.

EXPERIMENTAL SETUP

We used a scintillation counter (Fig. 1) and exposed it to a ^{137}Cs source to measure its radioactive decay. Each time the ^{137}Cs source gives off a burst of gamma radiation, the radiation excites some of the NaI molecules in the scintillator. During this excitation, a photon is emitted. The number of photons emitted is dependent on the energy of the exciting radiation. The photons then strike the photocathode of the photomultiplier tube, producing electrons. Each electron travels through a series of dynode layers, which multiply the number of electrons, resulting in a slightly amplified signal at the output of the photomultiplier. This output is then fed into a preamplifier, where we inverted the signal, and then to the amplifier, where we made signal gain adjustments that coincided with our voltage threshold on the counter.

We achieved our mean count rates of $\approx$ 1 sec^{-1}, 4 sec^{-1}, 10 sec^{-1}, and 100 sec^{-1} by varying the distance from the source to the scintillator and by adjusting the gain on the amplifier.

PROCEDURE

Using the setup described above, we recorded the number of counts in one second for each mean rate of approximately 1 sec^{-1}, 4 sec^{-1}, 10 sec^{-1}, and 100 sec^{-1} one hundred times. We then recorded the number of counts over a period of 100 seconds for each of those mean count rates.

We achieved each mean count rate by adjusting the distance from the ^{137}Cs source to the scintillator and changing the gain on the amplifier.

ANALYSIS

First, we plotted the cumulative average, $r_c(j)$, for each of our mean count rates. For each count at sequence number j we calculated

$$r_c(j) = \sum_{i=1}^{j} \frac{x_i}{t_i} \qquad (7)$$

where x_i is the number of counts recorded at time t_i and plotted the cumulative average along a counts/sec versus time graph (Fig. 2). We found that after recording the count rate 100 times, the mean count rate μ approached a steady state, which approximately equaled each of our target mean rates.

We then plotted our data along counts/sec versus the frequency of count rate and fitted it to a Poisson distribution (Fig. 3). For each of the four means, the actual standard deviation observed closely matched the standard deviation expected, $\sqrt{\mu}$ for a Poisson distribution. Additionally, both Poisson and Gaussian distributions appeared to fit well to the data, except for the 100 sec^{-1} rate where further binning could be applied to produce a better χ^2.

Alongside the Poisson distribution in Figure 3, we also plotted a fit for the Gaussian distribution. We noted that as the mean count rate increased, the Gaussian distribution closely approximated the Poisson distribution. This is clearly evident when comparing the χ^2 of both distributions for the 100 sec^{-1} count rate. This is expected, since

$$\lim_{\mu\to\infty} \frac{\mu^x e^{-\mu}}{x!} \approx \frac{1}{\sqrt{2\pi\mu}} e^{\frac{-(x-\mu)^2}{2\mu}} \qquad (8)$$

which is, essentially, the Gaussian distribution.

RESULTS

Based on the close fit of the Poisson distribution to our data and the additional fact that the calculated standard deviation closely coincided with the theoretical standard deviation, $\sqrt{\mu}$, we determined that the radioactive decay produced by the ^{137}Cs source followed a Poisson distribution.

Additionally, if we take the mean and expected standard deviation (based on a Poisson fit) for each of the 100 second recordings

Count Rate	Counts in 100 sec	μ	$\sigma = \sqrt{\mu}$
1 sec^{-1}	56	0.56	0.75
4 sec^{-1}	283	2.83	1.68
10 sec^{-1}	1144	11.44	3.38
100 sec^{-1}	10519	105.19	10.23

we find that the values for μ and σ correspond to the values calculated from our previous data of one hundred 1 sec count recordings.

ERRORS

For the graph showing the cumulative mean count rates (Fig. 2), we used

$$err(j) = \sqrt{\frac{r_c(j)}{j}} \qquad (9)$$

$$= \left(\frac{1}{\sqrt{j}}\right)\left(\sum_{i=1}^{j} \frac{x_i}{t_i}\right)^{\frac{1}{2}} \qquad (10)$$

where $r_c(j)$ is the cumulative average at sequence number j, to determine our error bars.

At each point in Figure 3, we took the square root of that count rate's occurrence to determine the error bars. Since we assumed a Poisson distribution, the frequency of occurrence for each point is the mean, μ, for that point.

CONCLUSIONS

We were able to fit a Poisson distribution to the radioactive decay of a ^{137}Cs source emitting gamma rays. Additionally, we further verified the Poisson fit by comparing the experimental standard deviation to the theoretical standard deviation of $\sqrt{\mu}$ and found them closely related.

If we perform additional 100 sequence recordings of this data at the same count rates, we will likely see a better Poisson distribution. The error bars for each frequency bin will likely be reduced by taking the standard deviation of the frequency bins at each count rate from

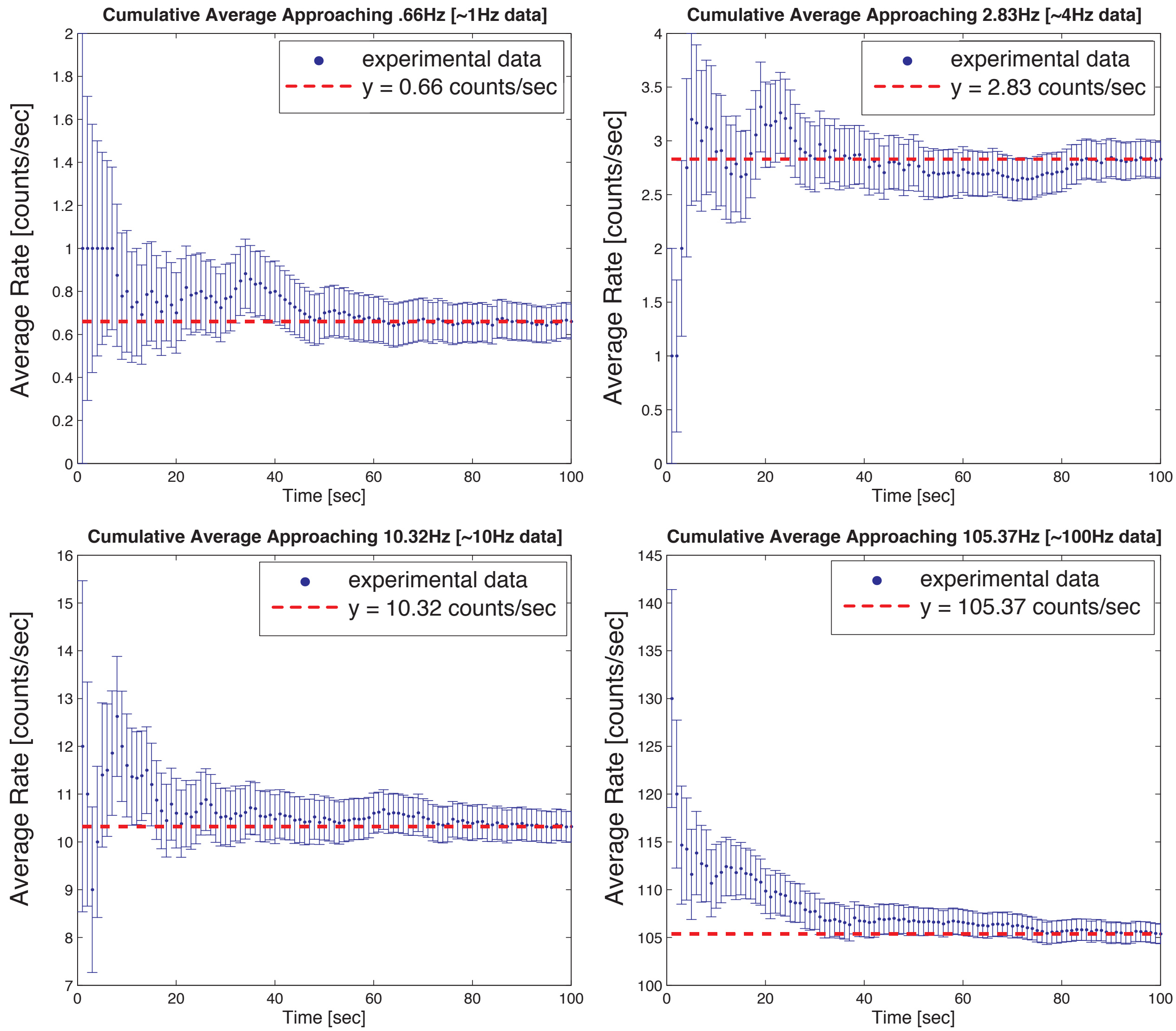

FIG. 2: Graphs for each of the mean rates showing that the mean μ approaches a steady state after a long time: 0.66 sec^{-1} (top left), 2.83 sec^{-1} (top right), 10.32 sec^{-1} (bottom left), and 105.37 sec^{-1} (bottom right).

the repeated 100 sequence recordings. Another way of verifying a better Poisson fit for radioactive decay would be to take a sequence of recordings much greater than than 100.

ACKNOWLEDGEMENTS

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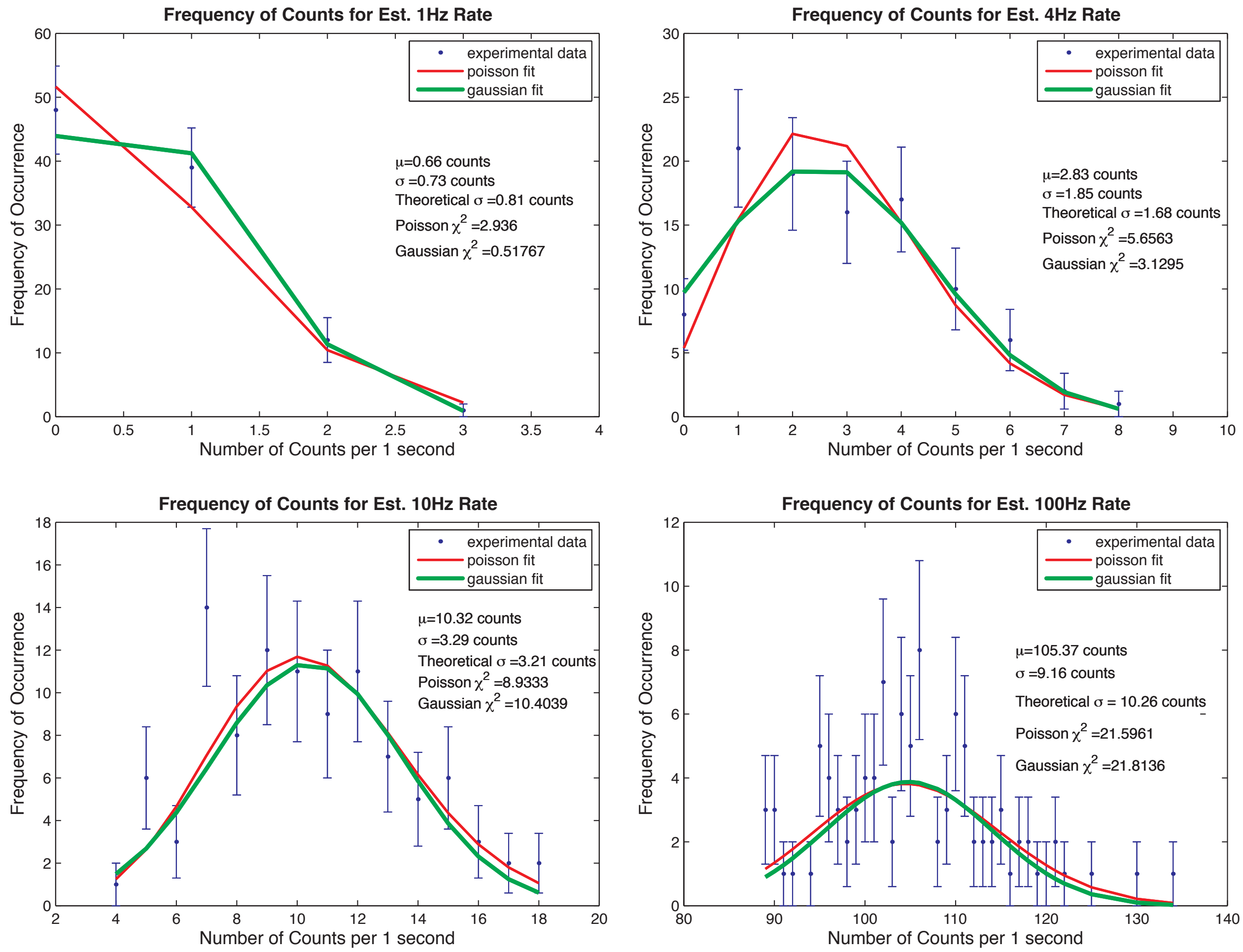

FIG. 3: Graphs for each mean count rate showing a decent fit for both Poisson and Gaussian Distributions. Additionally, the standard deviation for each count rate is $\approx \sqrt{\mu}$ (compare σ and Theoretical σ).