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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 24

With our knowledge of torque...

calm down.

With our knowledge of torque and angular momentum, we can now attack rolling objects which roll down a slope.

For instance, the following...

I have here a cylinder or it could be a sphere, for that matter, and this angle is β .

I prefer not to use α because that's angular acceleration, and there is this friction coefficient with the surface, μ , and this object is going to roll down and you're going to get an acceleration in this direction, a .

And I will evaluate the situation when we have pure roll.

That means the object is not skidding and is not slipping.

What is pure roll?

If here is an object, the cylinder is here with radius R , and I'm going to rotate it like this and roll it in this direction, the center is called point Q .

Once it has made a complete rotation, if then the point Q has moved over a distance $2\pi R$, then we call that pure roll.

When we have pure roll, the velocity of this point Q , and the velocity of the circumference, if you can read that-- I'll just put a c there-- are the same.

In other words, v_Q is then exactly the same as v circumference, and v circumference is always ωR .

This part always holds, but for pure roll, this holds.

You can easily imagine that if there is no friction here, that the object could be standing still, rotating like crazy, but Q would not go anywhere.

So then we have skidding and we have slipping and then we don't have the pure roll situation.

If the object is skidding or slipping, then the friction must always be a maximum here.

If the object is in pure roll, the friction could be substantially less than the maximum friction possible.

Now I would like to calculate with you the acceleration that a cylinder would obtain.

When it pure rolls down that slope, it has mass M , it has length l and it has radius R .

And I would like you to use your intuition and don't be afraid that it's wrong.

I'm going to roll down this incline two cylinders.

They're both solid, they have the same mass, they have the same length but they're very different in radii and I'm going to have a race between these two.

Which one will reach the bottom first? So I repeat the problem.

Two cylinders, both solid, same length, same mass, but one has a larger radius than the other.

There's going to be a race.

We're going to roll them down, pure roll.

Which one will win? Win win? Will win? Who thinks that the one with the largest radius will win? Who thinks the large with the smallest radius will win? Who thinks there will be no winner, no loser? Wow, your intuition is better than mine was.

We'll see how it goes.

Keep in mind what your vote was and you will see it come out very shortly.

Okay, let's put all the forces on this object that we know.

This one is Mg .

And we're going to decompose that into one a longer slope, which is $Mg \sin \beta$, and one perpendicular to the slope.

We have done that a zill... zillion times now.

And this one equals $Mg \cos \beta$.

Then there is right here a normal force, and the magnitude of that normal force is $Mg \cos \beta$, so there is no acceleration in this direction and then we have a frictional force here for which I will write F_f .

There is an angular velocity at any moment in time, ω , which will change with time, no doubt.

And then this center point Q , which is the center of mass, is going to get a velocity v and that v will also change with time.

And the v of the point Q which changes with time is v of the circumference, because that is the condition of pure roll.

That equals ωR .

This is always true, but this is only true when it is pure roll.

I take the time derivative.

The derivative of the velocity of that point Q is, per definition, its acceleration, so I get a equals $\omega \dot{\text{ times }} R$, and that equals αR , α being the angular acceleration.

So this is the condition for pure roll.

Now I'm going to take the torque about point Q.

When I take the torque about point Q, N has no effect because it goes through Q, and g has no effect because it goes through Q, so there's only one force that adds to the torque.

If this radius is R , the magnitude is Rf and the direction is in the blackboard.

But I'm only interested in the magnitude for now, so I get R times the frictional force.

This must be $I \alpha$, I being the moment of inertia for rotation about this axis through point Q, times α , but I can replace α by a/R .

So I get the moment of inertia about Q times a/R .

And this is my first equation, and I have as an unknown the frictional force, and I have as an unknown a , and so I cannot solve for both.

I need another equation.

The next equation that I have is an obvious one, that is Newton's second law: $f = MA$.

For the center of mass, I can consider all the mass right here at Q.

We must have $f = MA$.

And so M times the acceleration of that point Q, which is our goal, by the way, equals this component, equals $Mg \sin \beta$.

That is the component downhill.

And minus F_f , the frictional force, which is the component uphill, and this is my equation number two.

Now I have two equations with two unknowns.

So I can solve now.

I can eliminate F_f and I will substitute for F_f in here this quantity divided by R .

And so I get Ma equals $Mg \sin \beta$ minus moment of inertia about point Q, times a divided by R squared.

And now notice that I've eliminated F of f , and so now I can solve for a .

So I'm going to get...

I bring the a 's to one side.

So I get a times M plus moment of inertia divided by R squared equals $Mg \sin \beta$, and a now we have.

I multiply both sides with R squared.

I get $MR^2 g \sin \beta$ upstairs, and downstairs I get MR^2 plus the moment of inertia about that point Q .

This is my result, and all I have to put in now is the moment of inertia of rotation about that axis.

I want to remind you, though, that it is only true if we have a situation of pure roll.

So we can now substitute in there the values that we have for a solid cylinder.

If we have a solid cylinder, then the moment of inertia about this axis through the center of mass, which I've called Q , equals $\frac{1}{2} MR^2$.

And if I substitute that in here, notice that all my M 's...

MR^2 squares go away.

I get $1 + \frac{1}{2}$, which is $1 \frac{1}{2}$.

Upside down becomes $\frac{2}{3}$.

So $a = \frac{2}{3}$ times g times the sine of β .

There is no M , there is no I and there is no R .

So if I have two cylinders, solid cylinders with totally different mass, totally different radii, totally different length and they have a race, neither one wins.

Very nonintuitive.

Every time that I see it I find it kind of amazing.

Notice that everything disappears.

M , R and I disappear.

So those of you who said that if I take two cylinders with the same mass, different radii, those of you who said that there is no winner, there is no loser, they were correct.

But even more amazing is that even the mass you can change.

You can change anything as long as the two cylinders are solid.

That's what matters.

So if we take a hollow cylinder, then the moment of inertia about this axis through the center of mass, through Q , if this...

if really most of the mass is really at the surface, then it's very close to MR^2 , and then the acceleration--

if I substitute in here MR^2 , I get a 2 there-- equals $\frac{1}{2}$ times g times sine β .

So this acceleration is less than this one.

So the hollow cylinder will lose in any race against a solid cylinder regardless of mass, regardless of radius, regardless of length.

And I want to show that to you.

We have a setup here and I'll try to show that to you also on the screen there, but for those of you who are sitting close, it's probably much better

that you just look at the demonstration right here.

I have here... ooh.

Uh-uh.

I have here to start with a very heavy cylinder made of brass and this one is made of aluminum.

They have very different masses, same radii, same length.

Should make no difference.

There should be no winner, there should be no loser.

I'm going to start them off at the same time.

I hope you can see that there.

This is... this is the starting point.

Can lower it a little.

I will count down three to zero, and then you can see that they reach the bottom almost at the same time.

So very different in mass.

The mass difference is at least a factor of three.

All other dimensions are the same.

Three, two, one, zero.

Completely in unison.

Not intuitive for me.

Now I have one that has a very small radius compared to this one.

This is a sm... small aluminum rod.

Maybe you can see it here, television.

This is way more heavy, almost 30 times heavier.

Should make no difference.

As long as it's solid, should make no difference.

No winner, no loser.

Radii are different, masses are different.

Should make no difference.

Okay? Here we start the race.

Three, two, one, zero.

And they hit the bottom at the same time.

But now here I have a hollow one, and you better believe it, that it's hollow.

So now all the mass is at the circumference, and now it takes more time.

Now the acceleration as you w...

as you will see, is half times $g \sin \beta$; in the other case it was $2/3$.

And you may want to think about it tonight, why this one takes more.

It has to do, of course, with the moment of inertia, but again, it's independent of mass, radius and length.

So it's purely a matter of geometry.

This one is going to be the loser, and this one, regardless of mass or length, is going to be the winner.

So you see them.

One is hollow, one is not.

This is very light; this is very heavy.

I'll put the hollow one on your side.

Three, two, one, zero.

The hollow one lost and even fell on the floor.

Yeah, I find these things always quite amazing, that nature works this way, and I'm impressed that most of you or many of you had the right intuition when they said it would make no difference for the two solid cylinders.

We now come to the most nonintuitive part of all of 8.01 and arguably perhaps the most difficult part in all of physics and that has to do with gyroscopes.

And I really urge you to pay a lot of attention and not even to miss ten seconds, because you're going to see some mind-boggling demonstrations which are so incredibly nonintuitive that unless you have followed the steps that lead up to it, you won't have any idea what you're looking at.

It will be fun, it will be cute, but it won't do anything for you.

Imagine that you and I go in outer space.

No gravity.

We're somewhere in outer space and we have this bicycle wheel there.

And I'm going to put a torque on this bicycle wheel in this direction, so I'm going to put my right hand towards you and my left hand away from you.

And I'll do that for a short amount of time and I'll let it go.

It's obvious what's going to happen.

This wheel is going to spin like this forever and ever and ever.

I've given it a little torque.

That means if there's torque, there's a change of angular momentum.

The angular momentum change must be torque times delta t.

And so I do this and let it go and it will rotate about this axis forever and ever and ever.

Simple, right? Okay.

Now I'm going to torque it in this direction.

So we're in outer space, wheel is standing still and all I do is do this and I let it go.

Then it will rotate forever and ever and ever and ever in this direction.

That's clear.

Now comes the very nonintuitive part.

Now I'm going to give it a spin in your direction and now again I'm going to torque like this.

What will now happen? You will say...

or you might say.

I'm not accusing you of anything.

You might say, well, you give the wheel a spin so this... the wheel will probably continue to spin and you do this, so maybe what you're going to see is that it will rotate like this as it did before and in the same time the wheel will be spinning.

But that cannot be because if the wheel would be spinning like this, then the angular momentum of the spinning wheel is in this direction.

And if I would give it a twist and if it would continue to spin and it would rotate like this, then this spin angular momentum would go around like this and that cannot be because there's no torque on the system, because once I let go, there is no longer any torque.

So it is not possible for the wheel to keep rotating, and as a result of this torque that I give it, that it simply goes around.

That is not possible.

How is nature going to deal with that? I'll show you that on a view graph.

It's very nonintuitive what will happen.

And we will then also... I will also demonstrate it to you.

So here is the situation precisely as I described it to you.

You are, as an observer, in this direction.

This is the direction of 26.100.

So you are viewing the wheels like this.

They're spinning towards you.

That's what I will do shortly and that's what I implied now.

This is my right hand and this is my left hand.

The separation between my height... right and left hand is little "b." So the torque that I apply is bF , is this arm, so to speak, times this force.

And the force is perpendicular to the arm.

I apply a torque for a certain amount of time-- Δt .

When I do that, I apply, I add angular momentum in this direction.

But the wheel was spinning in...

in... in this direction.

You see it.

And so the angular momentum of spin of the wheel is in this direction.

I add angular momentum in this direction and you see that here.

So this was originally the spin angular momentum of the wheel.

I torque for time Δt , and so I add angular momentum like this.

And then I stop.

I only torque for a short amount of time and I stop.

That means after I have stopped, the angular momentum of the system as a whole can no longer change because there's no torque on the system, and the only way that nature now can solve that problem is to tilt this wheel in the way I've indicated here, and to make it spin in this direction and it will stand still.

In other words, I hold it in my hand, the wheel-- I will get it-- I give it a spin.

I hold it in my hand, I spin it towards you...

and I'm going to put my right hand towards you and my left hand away from you.

The spin angular momentum is now in this direction, so I'm going to give it a torque like so.

That means up.

And what will the wheel do? The wheel will do this.

Very nonintuitive.

Watch it.

Isn't that strange? You wouldn't expect that.

I will do it again.

I'm going to torque by pushing my hand towards you, and the wheel does something completely unexpected.

It simply tilts.

If I torque the other way around, then the torque, of course, will make it flip like this.

I'll give it a little bit more spin angular momentum, so now I move my left hand towards me and my... towards you and my right hand towards me, and then I expect that the wheel will do this.

And that's what it does.

Extremely nonintuitive.

These torques applied to a spinning wheel always do something that you don't expect.

However, there is one thing that always helps me in terms of guiding me and that is you can always predict that the spin angular momentum will always move in the direction of the torque, which is this external torque that I applied.

Let's go over that again.

We have here the spin angular momentum that you saw.

It was pointing in this direction.

And I applied the torque in this direction, the vector.

And what does the spin angular momentum do? It goes in the direction of the torque.

And then when I stop with my torque, then of course nothing changes anymore, and so what happens is this wheel tilts.

But notice that L , the spin angular momentum, has moved from here to here.

And I was torqueing in this direction, so it moved towards the torque.

If you have digested this, then you can test yourself now.

Now we have the same wheel.

I'm going to rotate it in exactly the same direction, but now I'm not going to torque like this, in the Z direction, or like this, in the minus-Z direction.

Now I'm going to do this.

Or I'm going to do this.

Try now to really concentrate on what I just taught you.

And try to give an answer to the following question.

The wheel is rotating.

I hold it in my hand and I'm going to torque it like this so that the torque factor is in your direction.

Angular momentum goes like this, torque is like this.

What will the angular momentum vector do? Move in the direction of... of the torque.

What will the angular...

what will the spin angular momentum vector then do if the torque is in this direction? It will do this.

It's going to move in the horizontal plane.

Very nonintuitive, but that's what it will do.

And I will show that to you.

I'm going to spin this wheel.

I'm going to spin it with a high angular momentum, high spin angular momentum.

And then I'm going to sit on this stool and I'm going to torque exactly as you see on the picture on the right.

I'm going to torque like this.

And as long as I torque it like this, that spin angular momentum wants to go around in the horizontal plane.

And when I torque the other way around, it will go back in the horizontal plane.

I'm going to torque exactly as you see on the picture there.

Are you ready? I stop the torque; nothing happens.

I torque backwards; I keep torqueing.

I keep torqueing.

I feel it in my hand.

I really have to push.

I keep torqueing.

And I stop torqueing and it stops.

The angular momentum vector is chasing, so to speak, the torque.

Is that nonintuitive? Very nonintuitive? It's also dangerous sometimes.

We call this motion of the stool, and in this case, the motion of the spinning wheel, we call that precession.

So you apply a torque to a spinning wheel.

Then what you obtain is a precession.

I can show you the precession in another way which is, in fact, very intriguing.

Suppose I have here a string, a rope, like we have there.

And I stick into that rope, I attach to the rope this wheel, just like so.

And I let it go.

Well, we all know what will happen.

Clunk.

It's clear.

All right.

But now I'm going to spin it before I let it go.

So here at the bottom, at this point P, there is a loop.

And here is the... the axis of rotation of the bicycle wheel which is solid brass, it's a solid piece, and I give that a length little "r," not to be confused with capital "R," which is the radius of the bicycle wheel.

So this is capital R.

And it can rotate about this here reasonably freely.

I call that center point Q and let this be the part of the wheel that is on your side.

I'm trying to make you see it a little bit three-dimensionally.

Suppose now I give it a spin in this direction.

Omega s.

"S" stands for "spin." In what direction is now the spin angular momentum? Use your hands, your thumbs.

Spinning in this direction.

Yeah.

Spinning this direction, angular momentum is in this direction.

That's a spin angular momentum.

L spin.

Well, there is a force on this system, Mg , and that force is in this direction.

It has a mass M , the bicycle wheel, and it has a radius, capital "R," and this part is little "r." So relative to point P, there is a torque and the torque is R times Mg .

This is 90 degrees so the cross product is nice.

The sine of the angle is one.

So the torque relative to point P is r times M times g .

In what direction is that torque? R cross F .

In what direction is that torque? Use your hands, thumbs, whatever you want.

You think in this direction? I disagree.

I disagree.

R cross F is...

you must be kidding.

In the blackboard.

It's not out of the blackboard; it's in the blackboard.

R cross F is in the blackboard.

There is a torque in this direction.

Nature, gravity provides that torque.

What will the spin angular momentum do? It's going to move in the direction of the torque.

It's going to chase the torque.

So what will it do if the angular momentum is here? What will it do? It will do this.

And as it moves, the torque will always be perpendicular to the plane through the string and r .

You can just see that for yourself why that is.

At this very moment when angular momentum is like this, the torque is in the blackboard because it's $\mathbf{r} \times \mathbf{F}$.

But when I'm here, this \mathbf{r} has changed position, and always remains perpendicular to the wheel.

So the torque will also change direction and so this angular momentum, spin angular momentum will keep chasing the torque and start to rotate freely.

That is exactly what I was doing when I was sitting on the stool, except that I had to apply that torque in my hands like this.

It's exactly the same direction.

I had to apply it all the time, and when I stopped, the precession stopped.

Here, however, the torque will never stop because this Mg will always be there, and I will show that to you shortly.

You may say, "You must be crazy "because you're violating Newton's second law, $\mathbf{f} = M\mathbf{a}$.

"This object got to fall.

"There's only one force on that object.

" $\mathbf{f} = M\mathbf{a}$.

"How can it not? The center of mass must fall with acceleration g ." Aha.

There is not just one force on that object.

What do you think is here? The tension in this cable, T , will be exactly Mg .

And so the net, the sum of all forces on that object is zero.

There is no net force on that wheel but there is a net torque, and that's why it's going to precess.

If there had been a net force, then indeed it would also go down, if this force were larger than this.

So nature is very clever, the way that it deals with these rather difficult problems.

Before I will show you this demonstration by spinning this wheel and then hanging it there in that rope, um, I want to mention that the angular frequency of the precession, which should never be confused with the angular frequency of spinning, is derived for you... it's only a three- or four-minute job, on page 344 in your book.

Now, I will not derive it here but what comes out of it, that it is the torque which is the one that we have here in this case, divided by the spin angular momentum.

That gives you the frequency of the precession.

In our case, for our bicycle wheel, it is rMg and the spin angular momentum of this wheel, if it is rotating with angular velocity ω of s , would be I times ω .

Remember, I is I times ω of a spinning wheel.

So I have here I rotating about point Q -- this is the axis of rotation-- times ω of the spin.

This is the spin and this is the precession.

And then the period of precession would be 2π divided by ω precession.

Let's take a look at that equation and see whether that sort of intuitively makes sense.

First of all, if you increase the torque, the upstairs, then it says that the precession frequency will increase.

That makes sense to me because the torque is persuading the angular momentum to follow it.

So the torque is persuading the spin angular momentum to change.

Well, if the torque is stronger then it is more powerful, so you expect that the precession frequency will be higher.

However, if the spin angular momentum is very powerful, then the spin angular momentum says, "Sorry, torque, I'm not going to go as fast as you want me to go." So when you increase that spin angular momentum in the wheel, it is also intuitive that the precession frequency will go down.

As the wheel spins, it has spin angular momentum, but as it precesses around like this, there will also be angular momentum in this direction because it's rotating like this.

Therefore there is a total angular momentum which is the vectorial sum of the two.

This equation will only hold as long as the spin angular momentum is really dominating the total angular momentum and you can see that immediately, because suppose you make the spin angular momentum zero, that it is not spinning at all.

Do you really think that the precession frequency will be infinitely high? Of course not.

So this only holds in situations where the spin angular momentum is way, way larger than the angular momentum that you get due to the precession.

So there are restrictions.

When the... when the wheel comes to a halt, when it's no longer rotating, you better believe it, then the thing will go clunk.

There's no longer the precession mode.

For our bicycle wheel, to get a feeling for how long the precession will take, uh, we can substitute the numbers in there, our bicycle wheel, the... the... the rod, the brass rod, little r has a length of 17 centimeters, and the... the radius of the bicycle wheel is about 29 centimeters.

And let me make the assumption that all the mass of the bicycle wheel is at the circumference, which is not very accurate, but it's close to that.

I mean, there are some spokes here, but let's assume that everything is here, so then the moment of inertia is MR^2 .

Well, if now you take a frequency of five hertz, spin frequency, you can calculate now ω of the spin frequency.

ω equals 2π times the spin frequency.

And so I know now I can substitute that in there, so I get an ω precession now equals rMg times the moment of inertia.

I assume that all the mass is at the circumference, an approximation, so we get MR^2 and then we get ω_s , which we have here.

We lose the M , and so we get rg divided by ω_s times R^2 .

That is the angular frequency of the precession, and the period of the precession is 2π divided by ω and you find then for the period of the precession about ten seconds.

So if I gave it a spin frequency of five hertz with these dimensions and with this approximation that all the mass is at the circumference, you would expect that it would precess around very gently in about ten seconds.

But I have very little control over that frequency, so it is possible I gave it seven hertz, it's possible I gave it three hertz.

But I will do what I can.

I'll actually give it the maximum one that I can.

That is always guaranteed success.

Where is the wheel? The wheel is here.

So we'll spin it up and then we'll put it in here.

Notice the way I'm spinning it.

I'm holding it away from me now and going to change it and do it differently next.

And there it goes.

About ten seconds.

Isn't that amazing? And it rotates, seen from below, clockwise.

Now it's going this way and I'm going to redo the experiment, changing the direction of rotation, and then it will go the other way around.

And now the angular momentum is rotating like this, is pointing here.

Spin angular momentum is pointing like this, torque is like this, and so the spin angular momentum is changing that...

chasing that torque.

I am the spin angular momentum.

I am the torque.

This is the torque.

It's chasing it.

All right.

So I have this in my right hand.

That's all right.

And now I will... so when I spin it up, that's right.

So let me now change the direction.

I'm turning it over and I'm going to spin it up again.

Angular momentum is now in this direction.

See, it's turning the other way around.

Angular momentum is in this direction.

Torque is now towards me.

Angular momentum is chasing the torque.

I've changed the direction of the spin angular momentum.

I've not changed the direction of the torque, and now it is rotating, as seen from below, counterclockwise.

Before, it was rotating clockwise.

If I can increase the torque by putting some weight here on the axle, I have this...

this actually extends, in our case, and I can put some weight on here, then I actually add to the torque and then you will see that it's... it goes faster.

The precession frequency goes up.

So I will put some weight on there.

So let it first go around, which was roughly ten seconds, roughly calculated, and now I'm going to put two kilograms here at the end.

And now you will see an instantaneous increase in the precession frequency.

You see it goes much faster now.

I take it off and then it goes back to its roughly ten seconds.

So what I have done is I have increased this torque but not at the expense of M , because the reason why the M cancels is because the moment of inertia has an M in it, but if I just hang this object on it, that doesn't change the moment of inertia of the spinning wheel.

None of this is intuitive.

None of this is intuitive.

You can do all of this with a \$5 toy gyro.

And I want to show this to you.

This is my toy gyro.

I have it in my office.

It's great fun.

And this toy gyro is doing exactly the same thing that this is doing.

Let me show you the toy gyro first.

Toy gyro.

Oh, yeah.

Here is a toy gyro.

Can you see it? Maybe I should make it a little darker here.

Can you see my toy gyro? Yeah? I'm going to spin it and then I'm going to hang it exactly the same way that that was hanging.

I'm going to spin it, for those who are sitting close-- whoosh-- and then putting it horizontally and hanging it in a string, and you'll see exactly the same thing is happening.

And now through friction, of course, all this fun ultimately comes to a halt.

I have something very special for you, or I may have something very special for you.

That depends on my helper who is here behind the scenes.

I hear him.

He's there.

Great.

Did you get full speed?

I'm going to the airport and get a little tired and ask one of my friends to help me.

Would you please help me and just carry this suitcase for me around? Pick it up, walk around a little, make some turns.

Turn the other way around, please.

[laughter]

STUDENT: What the hell? LEWIN: What the hell, yeah.

Exactly.

What are you doing, man? You're behaving so strangely.

Make some more turns, man.

We've got to go the...

we've got to catch the plane.

It doesn't quite do what you think it will be doing, right? So in here, you've guessed it.

STUDENT: Spinning wheel or something.

LEWIN: Spinning wheel.

And when you do this, you put a torque on it and it does exactly what you least expect: it flips up.

Isn't that fun? Yeah.

You may get arrested when you go to Logan Airport with this suitcase.

Thank you very much.

It's great.

Spinning objects have a stabilizing effect.

If you take a bicycle wheel, and we have one, and I put it here and I do nothing, it will fall.

No one is surprised.

However, if I give it a little spin, then it doesn't fall.

Why? Because it has angular momentum.

It has spin angular momentum.

And so it doesn't fall.

And it is not only a bicycle wheel.

Look.

Nicely stable.

Not only with a bicycle wheel.

You take... you take a quarter and you put a quarter like this on your desk.

You bet your life it will fall.

Roll it; it becomes stable.

You give it spin angular momentum, it becomes stable.

Take a top.

You put a top on the table, falls over.

You twist it, you give it spin, and the top is stable.

So spin angular momentum has the property of stabilizing things.

And you will see that addressed in one of your assignments, when I want you to address that quantitatively.

This is the basic idea behind inertial guidance systems.

In inertial guidance systems, you have a spinning wheel, at least in the days that the guidance systems had mechanical wheels.

You have a spinning wheel, but that spinning wheel is mounted in such a way that you cannot put a torque on the axis of rotation of the spinning wheel.

That's the way it's mounted.

We call that three-axle-gimbaled gyros.

So the moment that you put a torque on it, the housings-- in this case, the yellow and the black housing-- will start to rotate, and you never managed to get that torque on the spinning wheel.

You never get it on... on this axis.

And therefore, if now you put it on your boat or you put it in a plane, or a missile for that matter, if you can never put a torque on the spinning wheel and if the angular momentum for spin is in this direction, it will stay there forever and ever, assuming that we have no frictional losses.

And if then the plane turns, the direction of the spin angular momentum will not change, but what will happen of course is that this yellow frame will rotate or this black frame will rotate.

And in these bearings here are shaft encoders, and they sense that the rotation that the outer housing makes in order to keep this thing pointing at the same direction.

And that signal is being fed back to the automatic pilot and that keeps the plane flying in the direction that you want to.

So you use, as a reference all the time, the spin angular momentum of your gyro, which is now mounted in such a way that you cannot put a torque on it, even when the plane changes direction, and I want to show that to you.

Okay, this is the direction of my spin angular momentum and I'm the airplane and I'm going to fly.

Look at that spin angular momentum.

It has no respect for me.

It stays in the same direction no matter how I fly.

And the arrow signals that come from the bearings of the yellow housing and the black housing, those arrow signals are fed back to the automatic pilot and so the plane will stay on course.

Now what I can do for you to come to a final test on your thinking, this wheel is suspended in such a way that there is no gravitational torque on it like there was here.

But I can put a torque on it by simply putting some weights on the axis.

And what do you think will happen now if I put some weight here on the axis? So the wheel is spinning, but now I'm going to put a torque on it here.

It is spinning in this direction.

Angular momentum is pointing straight at me, away from you.

I'm going to put a torque on like this, put a little weight there.

Torque will be in this direction.

What will the spin angular momentum do? Torque is in this direction; spin angular momentum's in this direction.

Spin angular momentum will start to chase the torque.

Watch this.

There it goes.

The spin angular momentum is chasing the torque.

You see exactly the same thing that I've shown you before.

And if I make the torque higher, then the precession frequency will go up.

See, it stops now immediately when I take it off.

Put it back on again.

Continues.

Put more on it.

Goes way faster.

What happens now if I put the weight on this side? So I change the direction of the torque.

If I put it on this side, torque is now in this direction, spin angular momentum is in this direction.

It's going to reverse direction.

There we go.

And you see it does.

Amazingly nonintuitive.

If you have problems with this, you're not alone.

See you Wednesday.