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8.01 Physics I: Classical Mechanics, Fall 1999
Transcript – Lecture 15

We're going to talk about a whole new concept, which is the concept of momentum.

We've all heard the expression, "We have a lot of momentum going." Well, in physics, momentum is a vector, and it is a product between the mass of a particle and its velocity.

And so the units would be kilograms times meters per second.

F = ma.

That equals m dv/dt.

That equals d(mv)/dt and that equals, therefore, dp/dt.

So what you see--

the force is dp/dt, and what that means is if a particle changes its momentum, a force must have acted upon it.

It also means if a force acts on a particle, it will change its momentum.

Let us now envision that we have a whole set of particles which are interacting with each other, and the interaction could be gravitational interaction, could be electrical interaction, but they're interacting with each other.

Zillions of them--

a whole star cluster.

I pick one here, which I call mi, and I pick another here, which I call mj.

And there is an external force on them, because they happen to be exposed to forces from the outside world, and so on this one is a force Fi external, and on this one is Fj external.

But they're interacting with each other, either attracting or repelling, and so in addition to these external force, there is a force that j feels due to the presence of i.

And let's suppose they are attracting each other.

This would be F that j experience in the presence of i.

Actions equals minus reactions, according to Newton's Third Law, so this force, Fij, will be exactly the same as Fji, except in the opposite direction.

We call these internal forces.

That's the interaction between the particles.

If these were the only two forces, then the net force on this object, on this particle i, would be... would be a force in this direction.

This would be F net.

Now, you can do the same here.

This would be F net on particle j, and this would be F net on particle i.

But since there are zillions of objects here, there are many of these interacting forces, and so I can't tell you what the net force will be.

The net force is ultimately the sum of the external force and all the internal forces.

What is the total momentum of these zillions of particles? Well, that's the sum of the individual momenta.

So that is p1 plus p2... pi...

and you have to add them all up.

I take the derivative of this--

dp total/dt.

That is p1/dt.

Well, that's the force on number one.

It's the total force on number one.

So it is F1, but it is the net force on F1.

And F... and dp/dt for this particle equals F2 net force, and the force on particle number i equals Fi net, and so on.

And so when we add up all these forces, obviously that is the total force on the entire system.

Now comes the miracle.

The miracle is that all these internal forces eat each other up.

This ji cancels this one if you look at the system as a whole--

not if you look at the individual particles, but on the system as a whole, they all cancel each other out.

And so the total force on the system is simply the total force external.

And all the internal forces--

you can forget about it.

And this means, then, that we come to a key conclusion--

that dp total/dt...

in fact, I have it written down there.

It's so important that I want you to look at it this whole lecture.

Look at this.

You see that dp/dt is the total force externally--

forget all the internal forces--

and what it means...

that if the external force, the total external force on the whole system is zero, it means that the momentum cannot change.

Momentum is conserved, and that's called the conservation of momentum.

It only holds if the sum of all external forces are zero.

We could have hundreds of stars, like a globular cluster, and they could collide with each other, they could explode, they could break apart--

all those forces are internal, they don't count.

The angular mo... the momentum of the cluster as a whole--

not angular momentum, I misspoke.

The momentum of the cluster as a whole would not change if there are no external forces on the system, if the net sum, the total external force, is zero.

So the individual stars will change their momentum all the time, because the individual particles in the individual stars will experience, of course, the internal forces.

I'm not saying that the individual particles do not continuously experience momentum changes.

It's just the system as a whole for which the momentum is then conserved.

We can do a very simple example so that you get a feeling... numerical idea.

I would have a...

an object here as mass, m1, and here I have one, m2, and let this one have a velocity, v1, and this one has a velocity, v2.

One-dimensional problem, just as a warm-up.

They plow into each other and they stick together.

I put some glue on one, and so they stick together.

That's a given.

You have to accept that.

Let this be the direction of increasing x, and so before the collision I have a certain amount of momentum.

I can do this on a horizontal table, which is frictionless.

I can do it on an ice surface, which is nearly frictionless and I ignore air drag.

So this is the situation before, and so the momentum that I have before equals m1 v1--

this is my plus direction--

I could put vectors over there but that's really not necessary, because it's a one-dimensional problem--

plus m2v2.

If you like that, be my guest.

That's the momentum before.

Now they stick together and so when they stick together, their total mass is m1 + m2.

And then their velocity happens to be v prime.

We often give a prime to the situation after the collision occurs.

And so now I can apply, for the first time, the conservation of momentum.

The momentum before must be the same as the momentum afterwards because there are no external forces on the system.

When they collide, you better believe it that there are internal forces.

You better believe that there is glue and it goes "plunk" and they feel those forces, both of them.

The momentum of each one individually is changing, but not the momentum of the system as a whole, and so this equals (m1 + m2) v prime.

And so, if you put in some numbers--

suppose m1 equals one kilogram, and v1 is five meters per second, let m2 be two kilograms and let v2 be three meters per second.

And you see, since they are both in the same direction momentum here is 5 + 6 is 11, so this is 11.

And this is 1 + 2 is 3, so that is 3 times v1 prime, and so v1 prime is eleven-thirds meters per second.

So the conservation of momentum tells you what the speed is after the collision.

When we deal with collisions, the velocities of the objects before the collision are unprimed.

You see here v1, and you see here v2.

Now, the velocities after the collisions, by convention, are primed.

You see here v prime.

Now if after the collision object one and object two have a different velocity, we will call it v1 prime and v2 prime.

Now in this case, we're dealing with an inelastic collision, so m1 and m2 are stuck together.

They are merged, so it is sufficient to just call it v prime.

You could have called it v1 prime, because v1 prime is the same as v2 prime is the same as v prime.

And during the next four minutes, you will see that a few times do I call it v1 prime.

I wish I really hadn't.

It would be better if I had just used v prime.

That is unique and sufficient.

Now, there was a certain amount of kinetic energy before the collision, and of course, we can all calculate that--

kinetic energy before the collision.

That is one-half m1 v1 squared plus one-half m2 v2 squared--

one-half m1 v1 squared, one-half m2 v2 squared.

How much is it? You can do that as well as I can.

If you add that up, you will find 21.5 joules.

Trivial to stick in the numbers.

Now I want to hear from you.

What do you think the situation is with the kinetic energy after the collision? Don't look at the numbers.

Just use your intuition.

It may be wrong.

That's okay, my intuition is often wrong.

So there was a certain amount of kinetic energy.

We had this collision, and I want you to tell me whether you think that the kinetic energy has perhaps increased or decreased or maybe the kinetic energy hasn't changed.

Who thinks the kinetic energy has not changed? Good for you.

I see some parents even.

Who thinks the kinetic energy has decreased? Even better for you.

Hey, there's a professor of physics there.

Can't go wrong there.

And who thinks the kinetic energy has increased? No one thinks that.

Hey, it's amazing.

Let's take a look.

Kinetic energy after the collision.

That would be one-half m1 plus m2 times v prime squared.

Would we agree? Let me write this at a different location.

So this was the 21Ω joules.

Well, you can do this as well as I can.

We know that v1 prime is--

that is eleven-thirds.

And you will find that this number equals 20.2 joules.

So the kinetic energy went down.

Now, you may say, "Well, big deal.

"Little bit of kinetic energy--

1.3 joules.

Who cares about 1.3 joules?" Well, it is a real big deal in physics, let me tell you.

And I can appre...

I can make you appreciate that it is a real big deal by doing the following.

I don't change the masses, but I just am going to change the direction of the impact.

Here is m1 and here is m2.

The speeds remain the same--

v1 and this is v2.

No change in the numbers except that they go now

[whooshes]

head-on.

What is the momentum of particle number one? Well, that is mv.

1 x 5 is plus 5.

Remember, this is the increasing direction of X.

So the momentum is plus 5.

What is the other one? That is 2 x 3, that is 6, backwards.

This is minus 6.

So what is the total...

the total momentum...

let's give it a magnitude equals minus 1? The magnitude is one, of course and the momentum, if I leave this off we would call that minus one because it's a one-dimensional problem.

The momentum before is in this direction.

It's minus one.

So what now is v prime? They stick together.

Here they are.

And afterwards, I'm not even sure whether they go this way or this way.

Yes, I am sure, because the momentum is in this direction, so I predict that v1 prime will be in this direction.

And so this now equals m1 plus m2 times the new v1 prime.

Well, m1 plus m2 is 1 + 2, is 3 kilograms and so you see that v1 prime equals minus one-third meters per second.

So the whole system now goes off with one-third meters per second in this direction.

The kinetic energy before has not changed--

those 21Ω joules, right? That is independent of how I collide them with each other.

What now is the kinetic energy afterwards? This is a great tragedy, because now you get one-half the sum of the masses, which is three times this small number squared.

Goes with V squared, right? And now there is only 0.17 joules left.

Almost all kinet... kinetic energy has been destroyed.

So what you see here in front of your eyes--

that kinetic energy can be destroyed, but momentum cannot be destroyed in the absence of external forces.

Kinetic energy and momentum is a totally different thing.

The momentum of the individual particles gets changed, but the net momentum did not change but the kinetic energy was destroyed.

I can destroy the kinetic energy completely if I want that.

I can arrange a collision so that all kinetic energy has been removed.

Suppose this particle has a mass five and the velocity is one.

This is my shorthand notation.

And this particle has a mass one but it has a velocity five.

The momentum of this system is zero--

plus five in this direction, minus five in this direction.

But you bet your life there is kinetic energy.

After they collision...

they hit each other, we agreed that they would stick, there was glue on it.

Momentum afterwards must therefore still be zero.

Internal forces don't matter no matter what happens.

Kinetic energy is zero.

So the whole system collides--

[bing]--

And afterwards you have here the sum of the total masses and it just stands still, because I told you that they are going to stick together.

I used glue.

If you have a car collision, and two cars hit each other, and I compare the situation just before the collision with just after the collision--

here are the two cars and one has a speed in this direction, the other has a speed in this direction, and they hit each other.

And they stick together, it's a given.

You go

[whooshes]--

one big clump.

So this is before.

And I can give them some velocities.

This is the speed of one, v1, and this is the speed of the other, v2.

Afterwards you see something like this: v prime.

Just a wreck!

The impact time is so short that the change in momentum due to friction with the road--

that would be an external force, friction with the road.

But that can be ignored, is negligibly small.

The cars hit.

There is a huge internal force going on.

One slams on the other and the other slams on one.

There is even fireworks--

metal scrapes over metal.

Friction, but that's internal friction, not friction with the road.

So momentum is approximately conserved if we can ignore the friction from the road during the impact because the impact time is so short.

So let here be car with mass m1 and here is the other car with mass m2.

And we'll make it a two-dimensional problem, because we have seen only one-dimensional problems now.

Let's make it a two-dimensional problem.

So this is the direction for this car, and let's say this is the direction in which the other car is going, velocity v2.

And tragedy has it that this is the place where they're going to collide.

In what direction and what will be the speed after the collision? I only compare just the moment before they hit with just the moment after they hit.

What comes later is a different story.

When you have formed the wreck, clearly it's going to slide on the road, and then there is an external force which is friction, which will slow it down.

It's just during the impact I claim that to a reasonable approximation, momentum will have to be conserved.

And so, what is the momentum of this one? Well, this may be the momentum of this one.

It may have a very small mass, small speed.

And what is the momentum of this one? Well, this may be the momentum of number one and this could be the momentum of number two.

The net momentum is the vectorial sum of these two, which is this.

That is p total of the system.

That is never going to change.

That's before and after the collision exactly the same.

And therefore if you knew this angle theta and you know p1 and you know p2, then you can calculate in what direction the objects are going to slide, but of course you can also calculate then the velocity after the impact, because this total momentum must be the sum of the total of the two cars--

the mass of the two cars--

times v prime.

And so you can calculate everything.

And that's what the police is doing when they find wrecks on the road.

They actually use the... the track, the... the skidding tracks of the wreck to calculate v prime, and then they can try to reconstruct the situation as it was before the collision.

Now, in all these cases, the objects stuck together.

We've only considered cases where they stick together, in which case we call those in physics "completely inelastic collisions." Now, next lecture I will also deal with situations whereby during the collision, the particles bounce off each other.

In completely inelastic collisions, you always lose kinetic energy--

sometimes a little, as you see there; sometimes a lot, as you see there; sometimes everything, as you see there.

Always in inelastic collisions do you lose kinetic energy.

Can we have, in a collision, an increase of kinetic energy? Well, depends on how you define the word "collision." The answer is yes.

And in fact, I will show you an example and even do a demonstration.

In the most simple case that I can think of, I have here a block which has a certain mass, m.

But there is an explosive inside and all of a sudden, it goes "Boom!" It explodes.

Well, before, the speed is zero and so the momentum is zero.

And there comes the bang--

[whooshes]

and one piece flies in this direction with a certain velocity, v2 prime.

This is m2.

And another piece flies off in this direction with mass m1, with velocity v1 prime.

Clearly, momentum must be conserved.

This explosion is only internal forces, and so you can write down...

if you call this the increasing direction of x, so this momentum is positive and this momentum is negative, the total momentum will never change.

It's the same before as it is afterwards, so this must be m2 times v2 prime minus m1 times v1 prime.

What happened with the kinetic energy? Well, kinetic energy has clearly increased.

There was zero kinetic energy to start with.

This one now has kinetic energy and this one has kinetic energy.

Where did that come from? Well, it was the chemical reaction of the explosion.

Momentum, however, was conserved.

See, momentum really doesn't care about these explosions.

That's all internal forces.

So be very careful.

Never confuse momentum with energy.

Energy can change or cannot change, can increase or decrease or remain the same kinetic energy, but if there are no external forces on the system, momentum is always conserved.

And this is what I can show you.

I have a demonstration set up on this air track, and we do it on an air track so there is a minimum of friction.

And we're going to push two cars together and we hold them together with a spring.

The spring is like the explosion, like the dynamite that can push these apart.

So here is one car on the air track, then there is a spring and there is another car.

Let this mass be m1 and this mass be m2, and this surface is nearly frictionless.

And I hold them together with a string so that I can pull the spring in.

So this spring is compressed, and the whole system is at rest, v equals zero.

There is potential energy in that spring.

I take a burner

[whooshes], and I burn away this thread.

This is the situation before.

Momentum before is zero.

Total momentum afterwards would also be zero.

It's identical to what I did there.

And so this object will fly in this direction, and this object will fly in that direction.

And you already can tell what the ratio of the velocities will be, because you will see that v2 prime divided by v1 prime--

you see it right there--

equals m1 divided by m2.

And so I will make some predictions about the speed of these two objects.

The largest mass, by the way, would get the smallest velocity, and the smallest mass would get the largest velocity, the largest speed.

What you see in here are really speeds.

We don't have the information on the direction anymore.

How do we measure the speed? Well, we actually measure the time for the cars to move ten centimeters.

Each car has a little metal plate which is ten centimeters long, and it blanks out a light-emitting diode.

And the moment that the light- emitting diode is occluded, the timer starts, and the moment that the light- emitting diode emerges again, the timer stops.

So that's the way we will measure the time for each car to move by ten centimeters.

The first experiment that I will be doing, m1 is 244 plus or minus one gram and m2 is also 244 plus or minus one gram.

In other words, the uncertainty in the masses is something like 0.4%.

That's just part of life.

I can't do much better.

The time that it takes for object number one to go ten centimeters is obviously ten divided by v1, which is the velocity that we want to compare them with.

Now, the ten centimeters is really only known to an accuracy of about one millimeters.

So that is an uncertainty of 1% in my timing in my velocity, so this is a 1% uncertainty.

So if we measure the times that object number one and object number two will go through this timing device--

a measurement of ten centimeters--

you would expect the times to be the same to within roughly 1 + .5...

say 1Ω%.

There is, however, an additional problem that is very difficult for me to evaluate and that's the following.

If here is the light-emitting diode, and here is that metal plate that I mentioned to you, which comes over this diode, occludes the diode, and says "start the timer," and then the other end, of course, moves off and then the timer stops--

that criterion for on and off for one diode may not be exactly the same as for the other.

Here is one system.

One car will fly in this direction.

And here is another system, and the car will fly in that direction.

And that's not so easy to evaluate unless you do a lot of experiments.

So I would roughly guess--

I'm always on the conservative side--

that this may add an uncertainty in the criterion of the diode of another millimeter.

And so effectively, I really don't know any better than two millimeters what the ten centimeters is.

So I would say we really have to allow here for two percent plus .4%, so I would predict that the times of these two cars will be the same to within roughly 2Ω%.

That would be my prediction.

So we're going to write down here the times, and then we're going to measure them.

So time one and time two, and we will see whether these two numbers are the same within the uncertainty of our measurements.

Here is the air track.

Have to turn on the airflow.

Oh, let me tell, for those of you...

For the parents in my audience who haven't seen this one, this is a marvelous device.

It is a track which is well-constructed, so that these cars fit beautifully on this V-shaped track, and then we blow air out of this track, which lifts these cars up so that they float, and so we can move them in horizontal direction with extremely little friction.

There is no metal touching metal.

The only friction you have is the air drag.

That you can never avoid.

When you go through the air with a car, there is wind--

the same wind that you feel when you drive your bicycle.

But there's very little, and that's why these experiments are done on these air tracks.

These are not cheap, by the way, these air tracks--

they're very expensive.

Well, you... you pay $25,000 tuition, so you might as well get something for it, right?
So I hope that these timers are on.

They are.

And now we have here one car, and notice how frictionless it goes when I touch it.

Very nice, almost frictionless.

And here we have another one, and they have equal mass to within one gram.

And there is a spring between them.

Some of you may be able to see it.

And I'm going to attach now that string to hold them together.

And so I put potential energy in the spring by pushing them together, and they are lined up now here.

They have no momentum.

The timer's not on? Has to be reset.

Thank you.

They're now both zero.

Yeah, you're sure? Okay.

So now I'm going to burn off that wire.

The momentum of each one of those cars will change.

One will go in this direction, the other one in this direction.

Kinetic energy will increase.

That's the potential energy from the spring.

But the momentum of the system as a whole after I burn the wire will remain zero.

Ready for this?
Make me happy.

What do you see?
You're going to make me unhappy? I better walk around.

I can't wait.

220... oh, fantastic, absolutely fantastic! 223, 219 milliseconds.

That is well within the uncertainties.

What was the number, 219?

219 milliseconds and 223 milliseconds.

If I repeat the experiment and I make the string shorter, I will get different primes.

I can never predict the times, because if I make the spring more squeezed, then obviously I will have more potential energy.

And so the speeds of the cars will be higher, but the times will be the same within the uncertainty of the measurements.

So I can never predict the time.

Now we're going to do an experiment whereby I'm going to make m2 twice the mass.

m2 is 488 plus or minus one gram.

Now, that's going to be interesting, because now you really begin to test the conservation of momentum.

The momentum before is zero.

I burn the wire, momentum afterwards is also zero.

But the velocities...

oh, I erased this--

v2 prime divided by v1 prime equals m1 divided by m2.

And so twice the mass will get half the speed.

And so now you're going to see that one car will go twice as slow as the other.

That's the only way that nature can conserve momentum.

Nature has no choice.

Nature can deal with kinetic energy one way or another, but it cannot finagle momentum.

So it must give the more massive car half the speed than it gives the other car.

And so we're going to redo this experiment now with a car here which is twice the mass.

And this one equals...

Where is my other car? Oh, my car is here.

And this one is one mass, so this is the 244, and the one on the...

on your right is the 488.

So I have to bring them together again with a string.

So I... I do the work now.

I always do the work here in 26.100.

I do the work.

I squeeze these springs.

I get paid for that, by the way.

And that's potential energy that goes into the spring.

There it is.

Momentum is zero.

Kinetic energy is zero.

I have to zero the timers.

Oh, and I would like to make a prediction.

I would like to see...

we'll measure T1 and then we'll measure T2, and then we get a number and we get a number, and if we multiply this number by 2.00, then I would like to get this number within the uncertainties of the measurements.

There is always an uncertainty in any measurement that you do.

All right? Make sure that...

are the timers zero?
Okay, there we go.

What do you see? Ah! 406, 193--

whoo! On the button! 406 and 193.

0.406--

is that what you see? Yeah? And 0.193.

Now, I don't even need my calculator to multiply this by two.

I can even do that by heart.

9-3--

.396, .406.

Oops, how careless of me.

I should have used a calculator.

0.193 times two is 0.386 (not 0.396).

Notice that the difference between 0.406 and 0.386 is 5%.

The two are equal within the uncertainty of each measurement which is about 2.5%

Close enough.

The two are in excellent agreement within the 2Ω% uncertainty in each measurement.

So, you see, that's exactly how you can demonstrate the conservation of momentum.

They picked up kinetic energy, but after the wire was burned, the momentum remained zero.

Now I change the topic.

It... it appears that I change the topic, but you will see in a few weeks that it's really related--

not even a few weeks--

you will see that it's really related to the conservation of momentum.

I'm going to explain to you now what we physicists mean by center of mass of a system.

The center of mass of a system is defined as follows: I have here some kind of an object--

not just a point mass, but it has a finite size.

It could be a hammer.

It could be something like this--

a squash racquet.

And let the center of mass be, for instance, here.

This is the center of mass.

I pick any origin I want to.

You're totally free to choose this origin--

doesn't matter where you take it.

And this, now, is the position vector of the center of mass.

And now I carve out a zillion little mass elements, m of i, covering the entire object, and then this is the position vector, r of i.

And the center of mass is defined as follows: the total mass of this object times the position vector of the center of mass is the sum, summed over i--

over all these little elements, i--

times the position vectors of the individual little particles that make up this object.

v center of mass--

I find that by taking the derivative of this equation--

equals one over the mass total.

I bring this on this side, I take the derivative here, and so here I get the sum of m of i times the dri/dt.

And so this is the sum of i... of mi vi, simply taking the first derivative, which changes positions into velocities.

Now, this is also 1 over m total times the total momentum, because I've all these momenta of these individual particles.

They have a...

together, a total momentum.

And so now you come to another very important statement in physics, and that is that the total momentum of an object--

which could be a hammer, which could be a squash racquet--

is the total mass of that object times the velocity of the center of mass.

And if I take the derivative of this, then dp/dt of the total momentum--

of which we learned that the total momentum, dp/dt, is the total external force, we already learned that earlier--

that now is the total mass of the system times the acceleration of the center of mass, because I take the derivative of this equation.

That gives me dP/dt here, and the velocity changes to acceleration.

And look at this! This is really an amazing statement.

This says F = ma.

But if I have here this squash racquet and here is the center of mass, then if I throw this object up in 26.400, as I will do later, then the center of mass behaves as if all the mass of the entire squash racquet was right at that center of mass.

So the behavior of the center of mass is extremely predictable, whereas the behavior of the squash racquet is not.

It may start tumbling.

If the external force on the squash racquet were zero, then it would continue to go always with the same velocity, the center of mass.

If I had here a hammer--

this is a hammer--

and there were no external forces--

I would be somewhere in outer space--

and it would have a certain velocity.

Then the center of mass, but only the center of mass, would have a velocity that never changes, because if there's no external force then there is only a constant velocity.

There's no acceleration.

But the hammer itself may be tumbling.

A little later in time, the hammer may have this position.

A little earlier in time, the hammer may have had this position.

But the center of mass is just one smooth motion.

That is very mysterious that there is one and only one point in any one of us--

in you, in me, in any object--

that the center of mass behaves as if all the matter were together in one point.

And so this is another quite important statement: For the center of mass, the total momentum is the total

[no audio]

...the velocity of the center of mass.
And you take the derivative of that equation and you get F = ma.

And that means if the external force is zero--

you can go back to the upper line again--

if the external total force is zero, then the momentum of the system is conserved, and so the center of mass will then keep its velocity unchanged.

Let's do one calculation to give you some experience on how you derive the center of mass.

I'll take a simple case.

I won't take a hammer.

It's a little bit complicated.

I will take three masses which are held together by very unphysical...

by massless rods, say.

Then I have three point masses and that makes my life a little easier.

Let this be the Y axis and this the X axis, and here at zero I have a mass m.

At a distance l, I have here a mass 2m.

This is also l and this is also l, so this is equilateral triangle.

These are massless rods, and here I have a mass m.

And I'm asking you, where is the center of mass? So you have three point masses, and they are connected with massless rods.

Well, for those of you who have a good feeling for symmetry, they would say certainly it has to lie somewhere on this line.

And it's probably slanted in the direction of the 2m, so it will probably be somewhere here.

And so this will be the position vector r center of mass, and the individual position vectors from the origin here will be this, and this will be a position vector to this object, and the position vector to this object is zero.

And so now we can calculate the position of the center of mass as follows.

We know that the total mass, which is 4m times the position vector, r center of mass--

I go all the way up there on the blackboard; there's my definition for center of mass--

equals the sum of the individual masses times their position vectors.

So it is the sum of i m of i times r i.

And this i goes from one to three.

Now, this is a vectorial equation, and whenever we have a vectorial equation, it sometimes pays off to split it into two components--

a y and to an x component.

And so in the x direction...

of course, the same equation holds for the x component of these vectors.

So now I have that 4m times the x component of the center of mass equals this mass times the x component of its position vector, which is zero, plus this mass, which is 2m times the x position, which is l--

so plus 2m times l--

plus this mass times the x component of this mass, which is one-half l.

So that gives me plus one-half m times l.

My m goes and so I get that x center of mass equals 2Ω divided by four.

That is five-eighths l.

So we were not too far off where we put it.

Now, in the y direction, you can do exactly the same.

You can split it up into the position vector of this object, which is one-half l square root three.

This one has no y component and this one has no y component, so this is very easy.

So you're going to get that 4m times y of the center of mass equals this mass m times the y component of that position vector, and that is one-half l square root of three.

And so you lose your m, and so you see that y center of mass then becomes the square root of three divided by eight times l.

And I think that's about 0.22L, very roughly.

Yes.

And so you see that we didn't put it in so badly.

It's about one-fifth of this distance.

It's about one-fifth higher than this distance.

And so you can calculate the center of mass.

That's really not too hard, if you have discrete points.

If you have a car or if you have an object like this, then it is, of course, much harder to calculate the center of mass.

I will teach you in a few weeks a very easy way to determine experimentally where the center of mass is located--

experimentally, which is different from calculating it analytically, as we just did.

So I mentioned to you that the motion of the center of mass is very uniform in the absence of external forces, and that I can demonstrate for you again with the air track.

We have a system here of two cars which I connected by a spring.

I will turn on the air shortly because the air makes a lot of noise.

This is... these are two cars connected by a spring, and I will give these cars a certain motion and they will go in this direction.

And they will oscillate in a weird way because they are connected with a spring and I keep them connected.

And it will be nearly impossible for us to evaluate the motion of these two cars individually.

But if I give the whole system a certain velocity and then they go like this, and they keep going like this and making crazy things, momentum of the center of mass will not change--

only of the center of mass.

Not of this car, not of that car.

That's the uniqueness of the center of mass.

And so the center of mass will just laugh at us and ignore all these motions and will travel at a constant speed very nicely.

So if you concentrate on that little object, you may be able to see that.

You may need a lot of imagination to see that, because you're going to be distracted by the weird motion of the other two objects.

This is the center of mass.

It's right in the middle.

The objects have the same mass.

There we go.

You see how complicated this motion is of the individual cars? Can you see that the center of mass is moving very uniformly, or can you not see that? Oh, you think you can see that? You have a lot of imagination.

But I will help you.

I will turn on...

I will turn off all the lights and only turn on ultraviolet light.

And ultraviolet light will interact with that little ball, the center of mass, and when I then make it dark, you will only see the motion of the center of mass.

And then you can really see that the center of mass moves in a very civilized way.

I'll bring it out here again.

Okay.

So now I'm going to help you to concentrate.

Are the timers off? Yes, timers are off?
Okay, dark.

Let your eyes get used to the darkness.

Okay, I'm going to do the same thing, and now look at that center of mass.

And we know that these cars are doing crazy things.

Hard to predict but the center of mass behaves decently.

Beautifully! Constant velocity.

I can let it go backwards, so you can enjoy this once more.

Center of mass motion, in the absence of external forces has a constant velocity.

When I throw up a hammer, then the hammer will do very weird things.

The hammer will start to tumble and rotate, but the center of mass will behave in a civilized way.

If I throw up a hammer, then the center of mass and only the center of mass will just go along a perfect parabola as if it were just a tennis ball.

Now at one point...

I will do it with a squash racquet--

at one point the squash racquet may be like this, and at another point, the squash racquet may be like this, and a little later, it may be like this, but the center of mass of the squash racquet will perfectly go along a parabola.

And so we have here a squash racquet and we have here the center of mass.

I have also here a regular...

well, it's not quite a tennis ball, but close enough.

This one, you would expect it to behave perfectly like a parabola.

From this one, you would not expect it to behave like a parabola.

Let me throw this one up in light, and you will see that it has very strange motion.

If I show the whole thing in UV, then you will see the same kind of beautiful parabola as you would see with this ball--

something like this.

Forget the fact that it lights up.

You remember it was the last lecture that I wanted to remind you of.

So now we're going to turn off the lights and I want to show you the motion of the center of mass.

You see the center of mass here? Can you all see it? Okay, there we go.

You ready? Concentrate only on the center of mass.

Nice parabola or not? I'll do it again.

You see the center of mass? Can you see it? You can still see it, right?
Wonderful parabola for me! All right.

Enjoy the presence of your parents.

Have a good weekend.

See you Monday.