

Ideas from the classification of cycle structure

First, note that if $x \leq c$, we have

$$\sigma_{abc}(x) = x + a + b.$$

For ease of notation, let $d = c - a$. If $c < x \leq b + c$, we have

$$\sigma_{abc}(x) = x - d,$$

and if $x > b + c$, we have

$$\sigma_{abc}(x) = x - a - b.$$