

Idea of the construction of $\mathbf{p}_k(n)$

Let A_k be the event that $k|(a+b)$ and $(b+c)$, and let B_k be the event that $(a+b)/k$ and $(b+c)/k$ are relatively prime integers.

Then

$$\mathbf{p}_k(n) = Pr(A_k) \cdot Pr(B_k \text{ given } A_k)$$