

Definition of $\mathbf{p}_k(n)$

Since

$$\mathbf{p}(n) = 1 - \sum_{s=2}^n \mu(s)f(n, s),$$

we have

Theorem

$$\mathbf{p}_k(n) = \begin{cases} f(n, k)(1 - \sum_{s=2}^{n/k} \mu(s)f(n/k, s)) & \text{if } n \mid k \\ f(n, k)(1 - \sum_{s=2}^{n/k} \mu(s)\Gamma(n/k, s)) & \text{if } n \nmid k \end{cases}$$

where $\Gamma(n, k)$ is a similar function to $f(n, k)$.