A Global Fit for Thrust at NNNLL: Precision determination for $\alpha_s(M_Z)$

Vicent Mateu

Max-Plank-Institute for Physics
Munich

SCET workshop 09 (Boston MIT)

24 - 03 - 2009

Taskforce : A. Hoang & V. Mateu - MPI
R. Abbate & I. Stewart - MIT
M. Fickinger - University of Arizona
[AFHMS]

MAX-PLANCK-GESELLSCHAFT

FlaviA net

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[AFHMS] A Fundamental Honest Measure of Strong coupling

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Motivations & Outline

- **Motivations**
  - Fit lots of data so far unused.
  - Treat non-perturbative effects from solid grounds.
  - Perform global fits to ALL thrust data.
  - Get a precise value of $\alpha_s(M_Z)$.

- **Experimental data**
  - Definition of event shape variables: thrust.
  - Sample of data.

- **A theory for all regions**
  - SCET: fixed order results & resummation of logs & non-perturbative effects.
  - Subtraction of renormalons.
  - Non-singular terms.
  - Power Corrections.

- **Preliminary results**
  - Tail fits: a two-paramter fit.
  - Final (preliminary) value for $\alpha_s(M_Z)$.
  - Comparison with other analysis.

Determinations of $\alpha_s(M_Z)$

$\alpha_s(M_Z)$ is a key parameter for the analysis of all collider experiments.

DIS
τ decays
$\Gamma(Z \to \text{hadrons})$
$e^+e^- \to$ jets
quarkonia
…

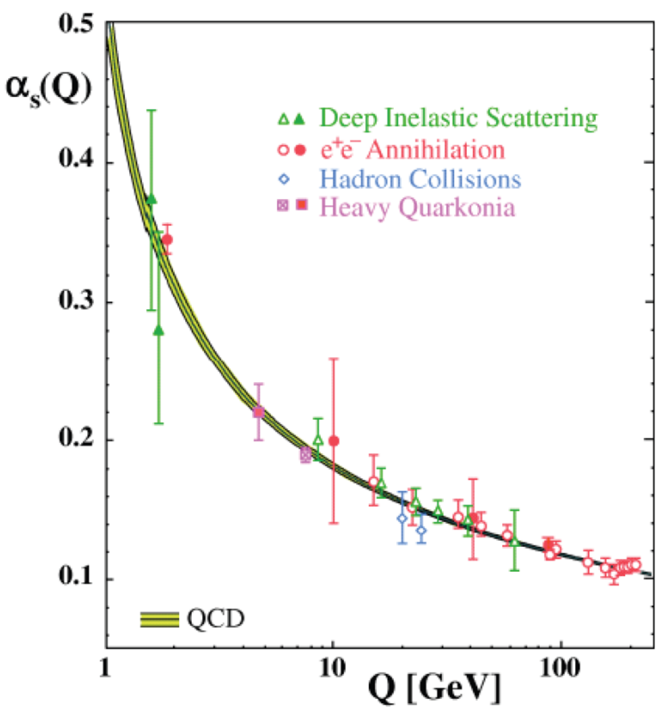
$$\boxed{\alpha_s(m_Z) = 0.1176(20)}$$

PDG'05 [Hinchliffe]

$$\boxed{\alpha_s(m_Z) = 0.1170 \pm 0.0012}$$

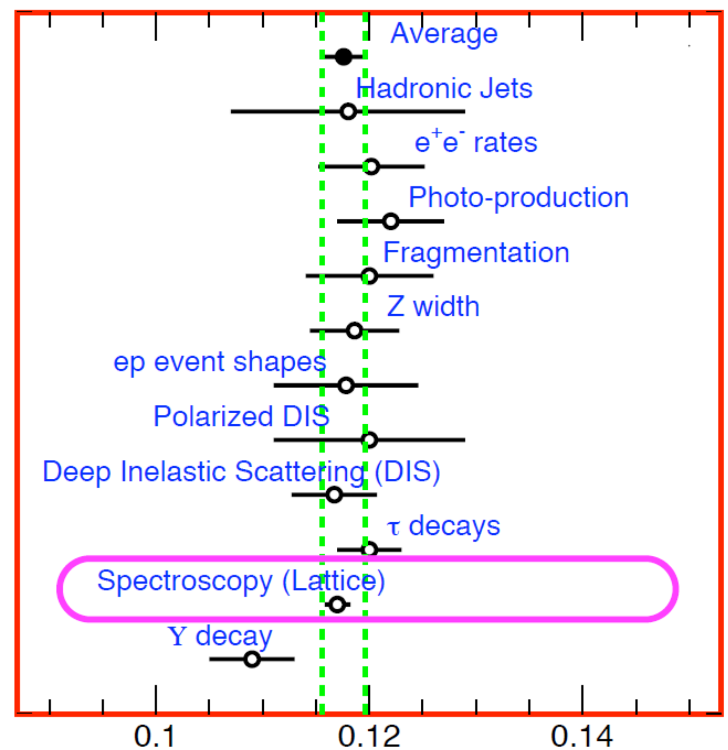
[Mason et al '05]

Staggered quarks



Determination over a wide range of energies

Must always face hadronization effects !

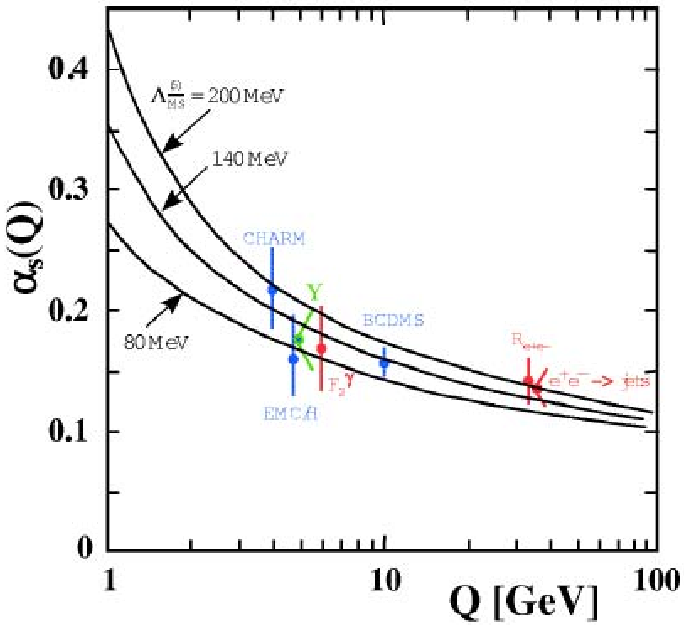


Another World Average

S. Bethke's Review

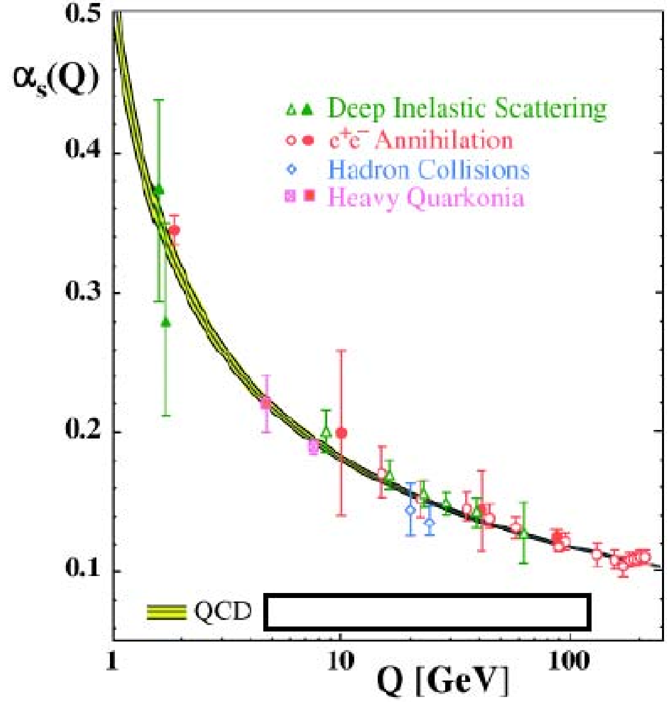
hep-ex/0606035

1989:



$$\alpha_s(m_Z) = 0.11 \pm 0.01$$

2006:



$$\alpha_s(m_Z) = 0.1189 \pm 0.0010$$

not all measurements
are used for this average

Summary of Results

theory errors dominate

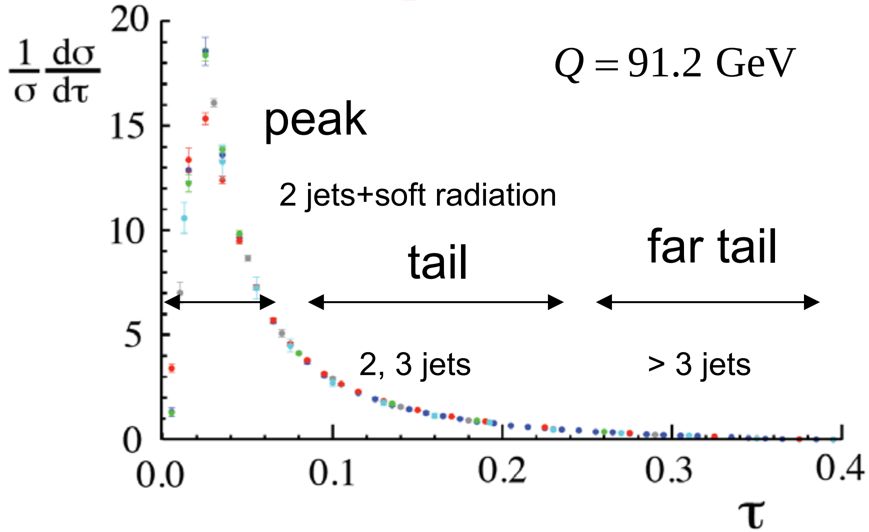
measurements used in avg.

e^+e^- event shapes

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
DIS [pol. SF]	0.7 - 8		$0.113\,^{+0.010}_{-0.008}$	±0.004	$^{+0.009}_{-0.006}$	NLO
DIS [Bj-SR]	1.58	$0.375\,^{+0.062}_{-0.081}$	$0.121\,^{+0.005}_{-0.009}$	–	–	NNLO
DIS [GLS-SR]	1.73	$0.280\,^{+0.070}_{-0.068}$	$0.112\,^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO
DIS [ν; xF_3]	2.8 - 11		$0.119\,^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO
DIS [e-p $\rightarrow$ jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT
e^+e^- [F_2^γ]	1.4 - 28		$0.1198\,^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO
e^+e^- [σ_{had}]	10.52	$0.20\ \pm0.06$	$0.130\,^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO
e^+e^- [jets & shps]	14.0	$0.170\,^{+0.021}_{-0.017}$	$0.120\,^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum
e^+e^- [jets & shps]	22.0	$0.151\,^{+0.015}_{-0.013}$	$0.118\,^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum
e^+e^- [jets & shps]	35.0	$0.145\,^{+0.012}_{-0.007}$	$0.123\,^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO
e^+e^- [jets & shps]	44.0	$0.139\,^{+0.011}_{-0.008}$	$0.123\,^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
$p\bar{p}\rightarrow b\bar{b}X$	20.0	$0.145\,^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO
$p\bar{p}$, $pp\rightarrow\gamma X$	24.3	$0.135\,^{+0.012}_{-0.008}$	$0.110\,^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO
$\sigma(p\bar{p}\rightarrow$ jets$)$	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO
e^+e^- $\Gamma(Z\rightarrow$ had$)$	91.2	$0.1226\,^{+0.0058}_{-0.0038}$	$0.1226\,^{+0.0058}_{-0.0038}$	±0.0038	$^{+0.0043}_{-0.0005}$	NNLO
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum
e^+e^- [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum
e^+e^- [jets & shps]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum

Experimental data

$e^+e^- \xrightarrow{Q} \text{jets}$

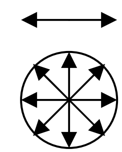


$$\boxed{1-\tau = \max_{\hat{n}} \frac{\sum_i |\hat{n}\cdot\vec{p}_i|}{Q}}$$

τ is an event shape variable

$\tau \to 0 \quad \Rightarrow$ dijet

$\tau \to 0.5 \Rightarrow$ spherical



At each Q there is a distribution in τ

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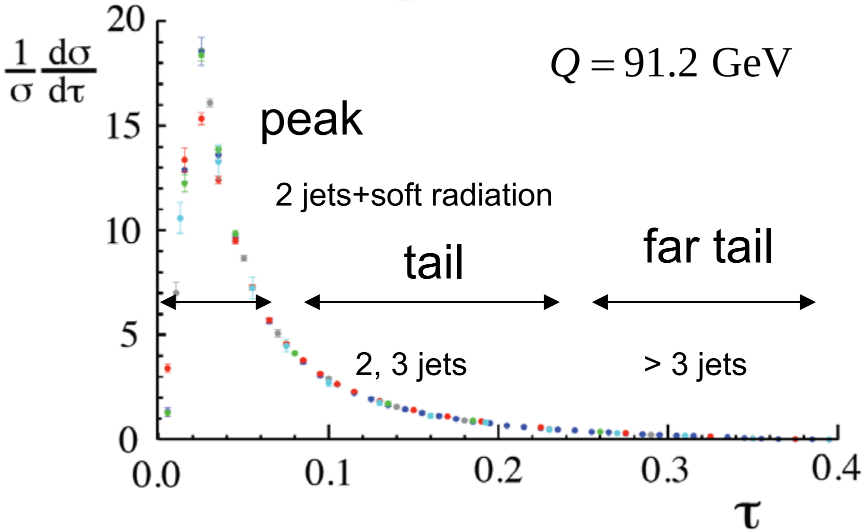

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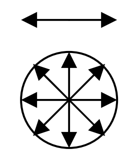

At each Q there is a distribution in τ

	Experiment	Values of Q
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
	OPAL	{91.0, 133.0, 177.0, 197.0}
	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{14.0, 22.0, 35.0, 44.0}
	JADE	{35.0, 44.0}
KEK	AMY	{55.2}

Improvements on earlier work

In SCET 08 we had two Interesting talks

T. Gehrmann $\longrightarrow$ NNNLO fixed order result

M. Schwartz $\longrightarrow$ NNNLL' perturbative analysis

In our work we have...

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1. Full treatment of non-perturbative effects with field theory

   (systematic treatment of errors from power corrections)

2. Interface between power and perturbative corrections (renormalon subtraction).
3. A method to simultaneously describe all three regions.
4. Account for factorization theorem for subleading orders, derived in SCET.
5. b-mass effects incorporated, treated within SCET – QCD.
6. QED effects incorporated, treated within SCET – QCD – QED.

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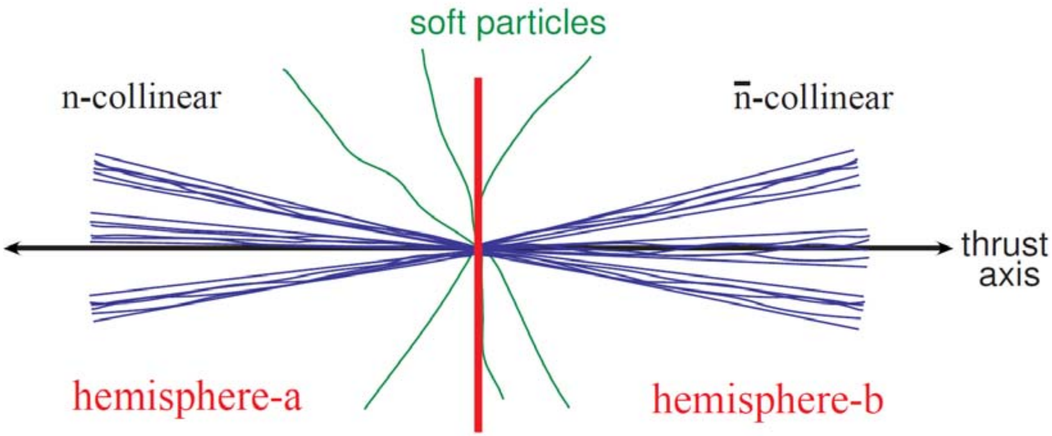
Global fit for all values of τ and Q

Fit for $\alpha_s(m_Z)$ and non-perturbative effects simultaneously

SCET basics

$$e^+e^- \xrightarrow{\gamma,Z} 2\text{ jets} + X_{soft}$$



Particles in each hemisfere have

$$p_+ = E + p_n \sim O(Q)$$

$$p_- = E - p_n \sim O(\Delta_{QCD})$$

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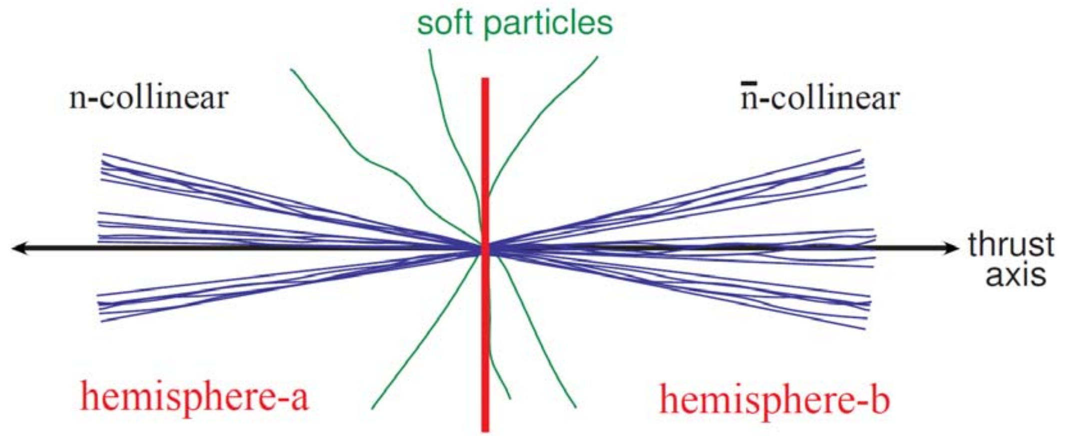

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3 energy scales ⟶ EFT

large logs !

$$Q^2 = \left(E_{e^+} + E_{e^-}\right)^2 = \left(\sum_{hem\,a+b} p_i^\mu\right)^2 \approx 2\sum_{hem\,a} E_i^2 \sim (100 \text{ GeV})^2$$

$$\mu_{Jet}^2 = \left(\sum_{hema} p_i^\mu\right)^2 \approx \sum_{hem\,a} (E_i^2 - \vec{p}_i^{\,2}) \sim \sum_{hem\,a} p_i^+ p_i^- \sim Q\,\Delta_{QCD} \sim \left(20 \text{ GeV}\right)^2$$

$$\mu_{soft}^2 = p_{hadron}^2 = \Delta_{QCD}^2 \sim \left(2 \text{ GeV}\right)^2$$

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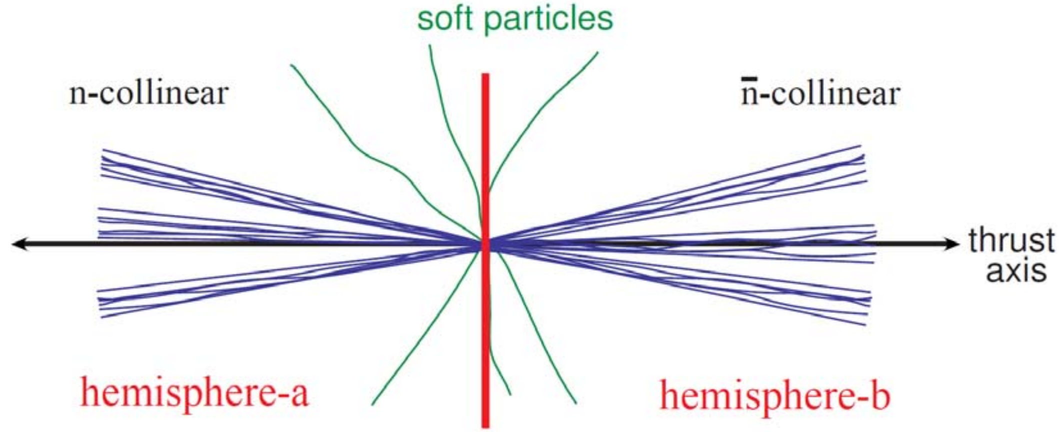

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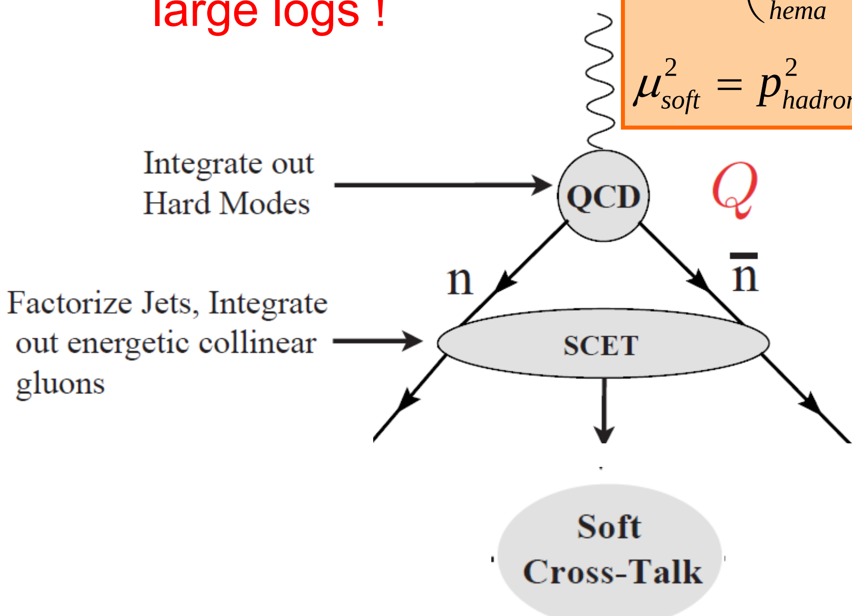

Matching

QCD SCET representation

$$\bar{q}\,\Gamma^\mu q \quad \to \quad \bar{\xi}_n\, W_n\, Y_n^\dagger\, \Gamma^\mu\, Y_{\bar{n}}\, W_{\bar{n}}\, \xi_{\bar{n}}$$

Collinear Wilson lines Soft Wilson lines

Factorization theorem

$$\frac{d\sigma}{d\tau} = \sigma_0\, H(Q,\mu) \int d\ell\, d\ell'\, J_T(Q\tau - \ell, \mu)\, S_p(\ell - \ell' - 2\Delta, \mu)\, S_{\text{model}}(\ell', \lambda)$$

Has large logs

Electroweak factor

(family dependent)

Hard matching coefficient

(function)

Jet function

(distribution)

Partonic Soft function
(distribution + renormalon)

Model Soft function

(non-perturbative)

Valid at leading order in power counting. Efficient for the dijet region ! (SCET)

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Complete basis!

$$S_{\mathrm{model}}(\ell) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n\, f_n\!\left(\frac{\ell}{\lambda}\right)\right]^2$$

[See talk by Zoltan Ligeti]

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Hoang & Stewart

$$\Delta = \delta(R,\mu) + \bar{\Delta}(R,\mu)$$

Shifts τ

$$\delta(R,\mu) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n \delta_n(R,\mu)$$

Infrared scheme parameter

Removes the u=1/2 renormalon in S_p order by order in perturbation theory

We must sum up the large infrared logs in $\bar{\Delta}(R,\mu)$ using infrared renormalization group equations

Resummation of Logs

Resummation of large logs!

No large logs any more

$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell\, d\ell'\, U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell, \mu_s)$$

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In Fourier space all convolutions turn into product of functions

Removes renormalon

$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int dx\, e^{ix(Q\tau - \bar{\Delta})} \tilde{U}_J(x,\mu_Q,\mu_s) \tilde{J}_T\left(\frac{x}{Q},\mu_j\right) e^{-2ix\delta} \tilde{S}_p(x,\mu_s) \tilde{S}_{\mathrm{model}}(x,\lambda)$$

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Shifts τ

$$S_T(\ell,\mu) = \int d\ell' \left[S^{(0)}_{\text{part}}(\ell-\ell',\mu) + S^{(1)}_{\text{part}}(\ell-\ell',\mu) + S^{(2)}_{\text{part}}(\ell-\ell',\mu) + \ldots \right]$$
$$\times \left[1 - 2\delta_1 \frac{d}{d\ell'} - 2\delta_2 \frac{d}{d\ell'} + 2\delta_1^2 \frac{d^2}{d\ell'^2} + \ldots \right] F(\ell' - 2\bar{\Delta})$$

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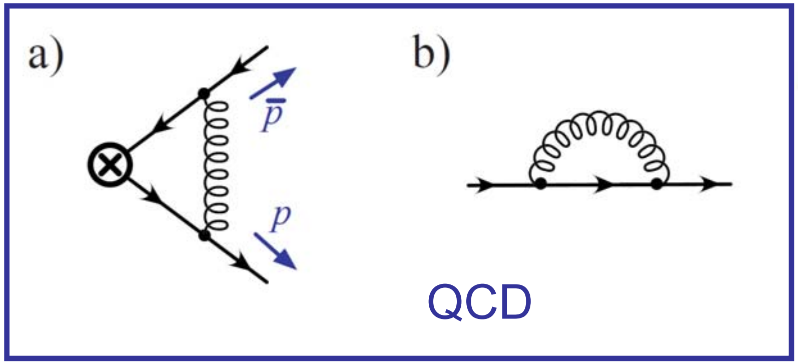
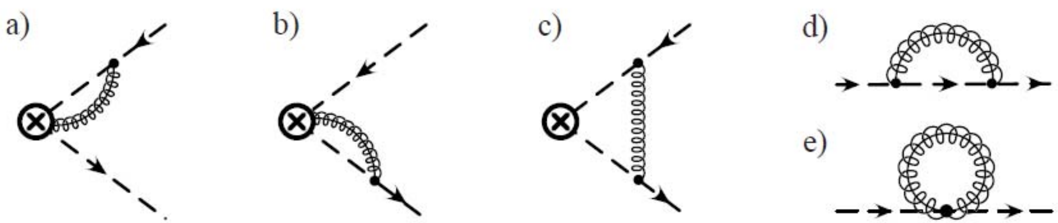
The renormalon arises from separating perturbative and non-perturbative effects in dim-reg. Subtracting the renormalon gives stability to perturbation theory, and here achieves a positive cross section at very small tau.

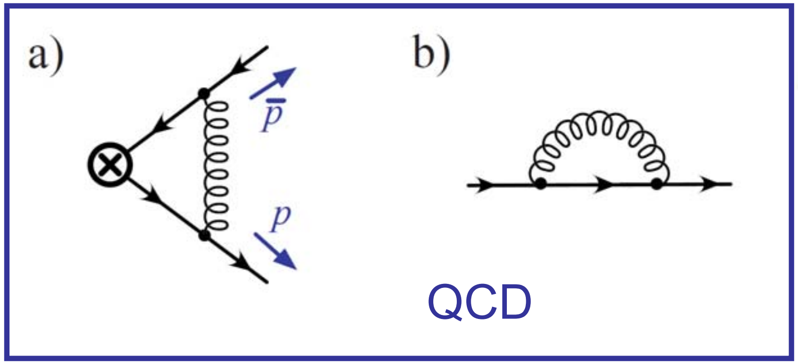
Hard, jet and soft pieces

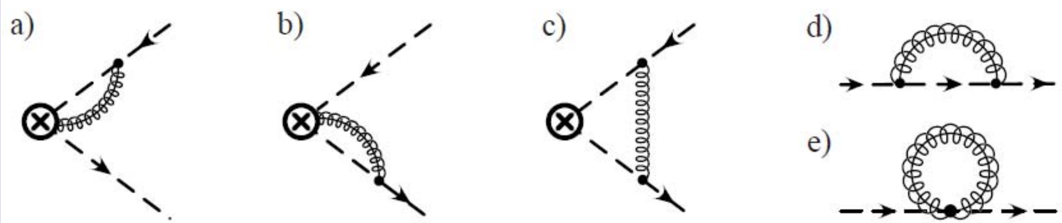




Hard coefficient: from matching

Hard, jet and soft pieces





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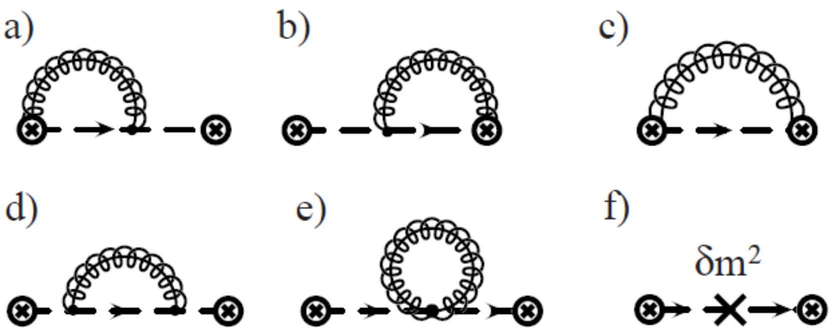
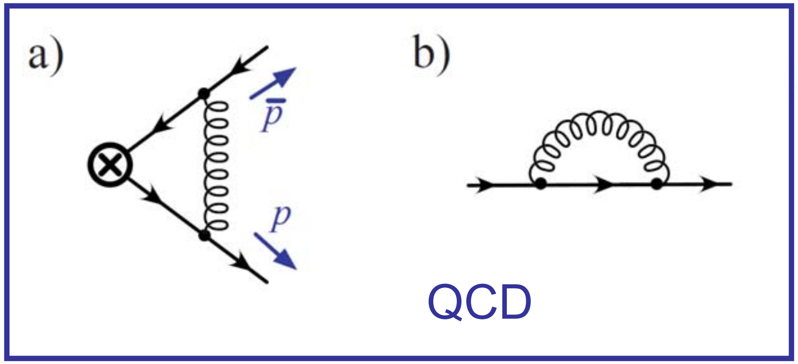
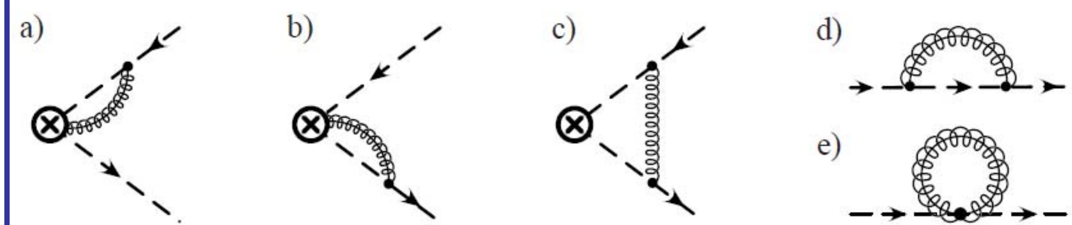


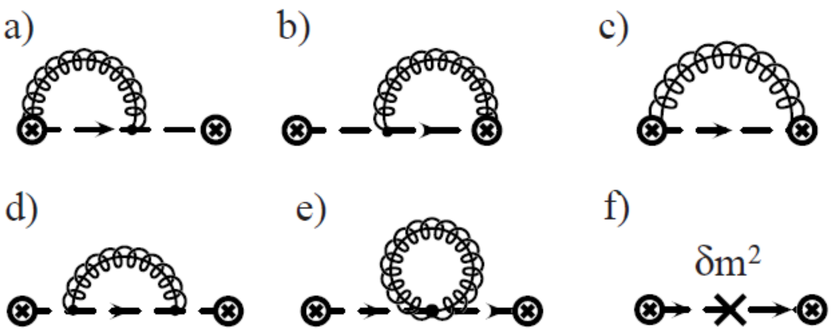
Jet function

Hard, jet and soft pieces

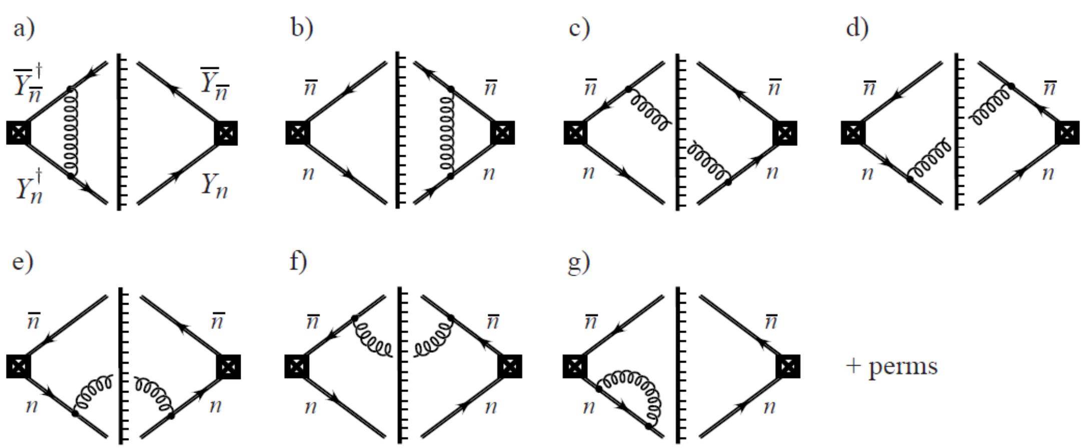




Hard coefficient: from matching



Jet function



Soft function:

It is non-perturbative

Perturbative for large tau

Ingredients for the calculation

Order or the analysis

Matrix elements

IR and UV running of gap parameter

Renormalon subtraction terms

	cusp	non-cusp	matching	alphas	γ_Δ^μ	γ_Δ^R	$\delta(\Delta)_i$	non-singular
LL	1	–	tree	1	tree	tree	–	–
NLL	2	1	tree	2	1	1	–	–
NNLL	3	2	1	3	2	2	1	1
N^3LL	4^P	3	2	4	3	3	2	2
N^3LL'	4^P	3	$\boxed{3^*}$	4	3	3	3	3

From a Padé approximant

log information known, and sum of non-log terms known.

From fixed order full QCD calculations

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log information known, and sum of non-log terms known.

From fixed order full QCD calculations

The renormalon subtractions $\delta_i(R,\mu)$, and the UV and IR running of the gap $\Delta(R,\mu)$ are determined from the soft function

The ‘primed’ analysis enhances the matching order by one. Relevant !

Ingredients for the calculation

Hard coefficient

NNLL

Known up to $O(\alpha_s^2)$ NNLO

Anomalous dimension known up to $O(\alpha_s^3)$

Cusp anomalous dimension known up to $O(\alpha_s^3)$

Moch, Vermaresen & Vogt

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Soft function

- Analytically known up to $O(\alpha_s)$ — Schwartz; Fleming et al
- Numerically known up to $O(\alpha_s^2)$ NNLO — Becher & Schwartz; Hoang & Kluth

Adding the $O(\alpha_s^4)$ cusp with a Pade approximation $\longrightarrow$ NNNLL' analysis

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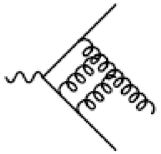
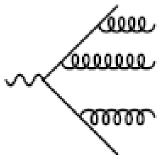
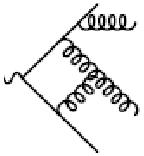
Fixed order		
1 - loop	Known analytically	Ellis et al.
2 -, 3 - loops	Known only numerically	Glover et al, Gehrmann et al.







Power corrections

$$\frac{\Delta_{QCD}}{\mu_s} \approx \frac{\Delta_{QCD}}{Q\tau}$$

$$H\ J_T \otimes S_T^{pert} \otimes S_T^{\text{model}}$$

$$\frac{\mu_s^2}{\mu_J^2} \approx \tau$$

$$\frac{d\sigma_{\text{nonsingular}}}{d\tau} \otimes S_T^{\text{model}}$$

$$\frac{\Delta_{QCD}}{\mu_H} \approx \frac{\Delta_{QCD}}{Q}$$

Requires SCET subleading calculation

Same effect for all tau

Numerically irrelevant

The relative importance is region dependent.

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- Tail and far tail: expansion causes shift

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Theoretical uncertainty $\quad \frac{\delta\alpha_s}{\alpha_s} \sim \frac{\Delta_{QCD}}{Q} \sim 0.5\%$

Concerning renormalons and gap subtraction, we treat the non-singular in the same way as the singular, to ensure complete cancellation in the far tail and it is consistent with the subleading factorization theorem. We convolute it with the same Soft function model.

Subleading perturbative contributions

$$\boxed{\frac{d\sigma}{d\tau} = \underbrace{\sigma_0\, H(Q,\mu) \int d\ell\; J_T(Q\tau-\ell,\mu)\, S_T(\ell,\mu)}_{\text{Peak and tail}} + \underbrace{\left.\frac{d\sigma^{partonic}}{d\tau}\right|_{\text{non-singular}} \otimes\; S_{\text{model}}}_{\text{Tail \& far tail}}}$$

In the far tail region, fixed order perturbation theory is the optimal description.

We no longer have three scales, resummation messes up cancellation.

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Peak and tail

Tail & far tail

In the far tail region, fixed order perturbation theory is the optimal description.

We no longer have three scales, resummation messes up cancellation.

Use of factorization theorems at subleading order derived with SCET.

We define the non-singular terms as the full QCD fix order result with the expanded SCET subtracted.

How do we determine non-singular?

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{fixed order}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{SCET(no resummation)}} + O(\tau^{0,1\ldots})$$

Subleading terms

(do not known how to resum logs

but do know factorization theorem)

Full QCD

SCET expanded out

$$\boxed{\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{non-singular}} \equiv \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{fixed order}} - \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\right|_{\text{SCET(no resummation)}}}$$

known analytically to three-loop accuracy*

known numerically at 2 -, 3 - loops

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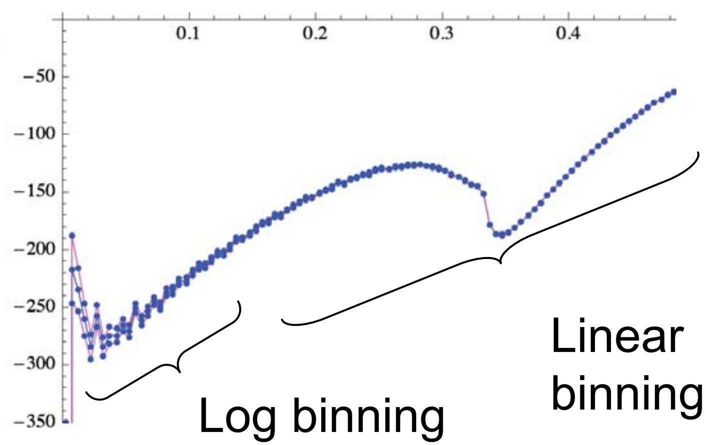
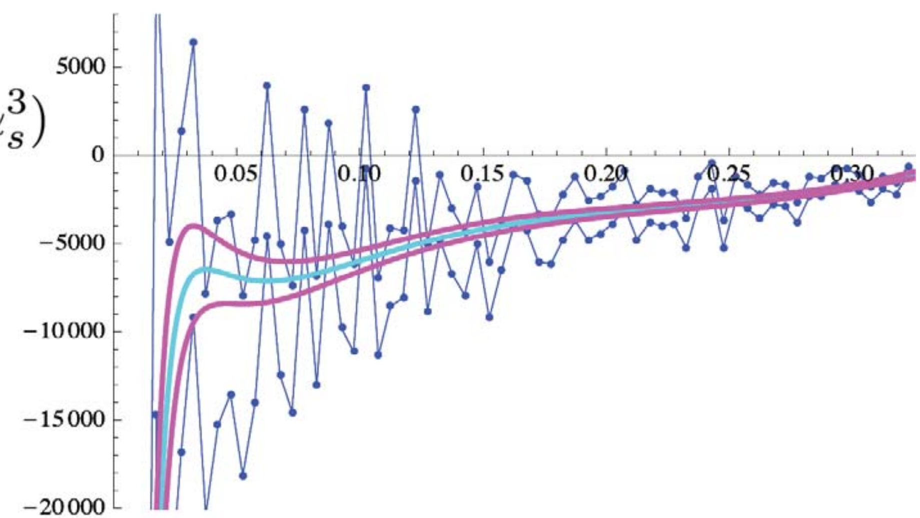
Gets very noisy at small τ !

We use a fit function at small τ and an interpolating function for the rest

$\mathcal{O}(\alpha_s^2)$



$\mathcal{O}(\alpha_s^3)$



Non-perturbative contributions

Hoang & Stewart

Contains an infrared renormalon

Removes renormalon

"Infrared scale" dependence

$$\boxed{S(\ell,\mu)=\int \mathrm{d}\ell'\, \underbrace{S_{\text{part}}(\ell-\ell'-2\delta,\mu)}_{\text{Gives the right anomalous dimension}}\, \underbrace{S_{\text{model}}(\ell'-2\bar{\Delta})}_{\text{Parametrizes the non-perturbative effects}}}$$

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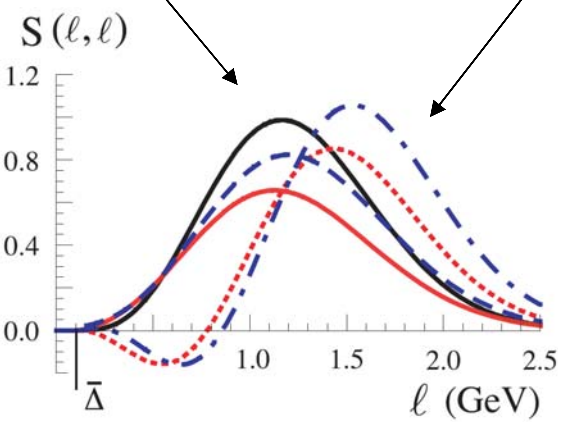
Gives the right anomalous dimension

Parametrizes the non-perturbative effects

Renormalon subtracted

No subtraction

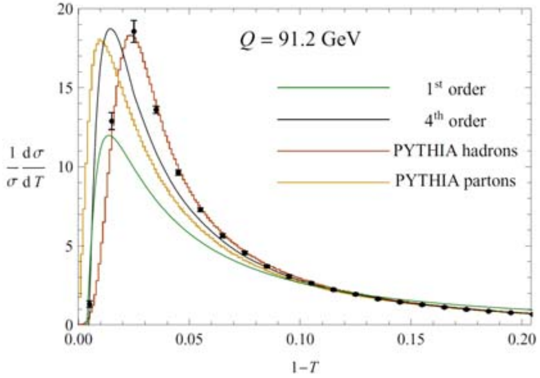


$$S(\ell,\mu)=\int d\ell'\left[S^{(0)}_{\text{part}}(\ell-\ell',\mu)+S^{(1)}_{\text{part}}(\ell-\ell',\mu)+S^{(2)}_{\text{part}}(\ell-\ell',\mu)+\ldots\right]$$
$$\times\left[1-2\delta_1\frac{d}{d\ell'}-2\delta_2\frac{d}{d\ell'}+2\delta_1^2\frac{d^2}{d\ell'^2}+\ldots\right]F(\ell'-2\bar{\Delta})$$

$\bar{\Delta}(R,\mu)$ must be evolved to avoid large logs.
We evolve from $\bar{\Delta}(R_0,\mu_0)\equiv\Delta_0$ to $\bar{\Delta}(R,\mu)$ to sum them up.

R is a scheme parameter
Δ_0 is a model parameter

Treatment of non-perturbative effects



Most LEP analyses used montecarlo generators to estimate NP corrections

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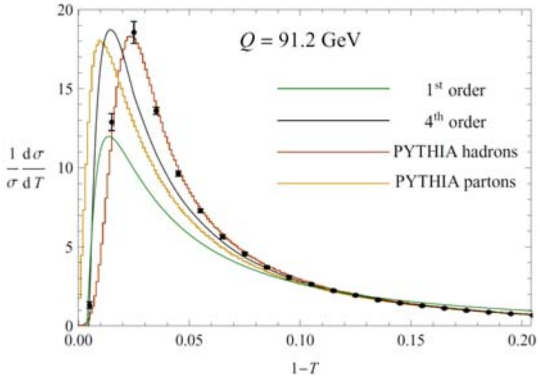

Most LEP analyses used montecarlo generators to estimate NP corrections

Davison & Webber's Model: freezes α below some scale μ_I

$$\left.\frac{d\sigma}{d\tau}\right|_{\tau} = \left.\left(\frac{d\sigma}{d\tau}\right)^{\text{perturbation theory}}\right|_{\tau+\Delta\tau}$$

$$\Delta\tau \sim \alpha_{\text{eff}}\frac{\mu_I}{Q} \sim \frac{\Lambda_{QCD}}{Q}$$

Definite pattern in Q

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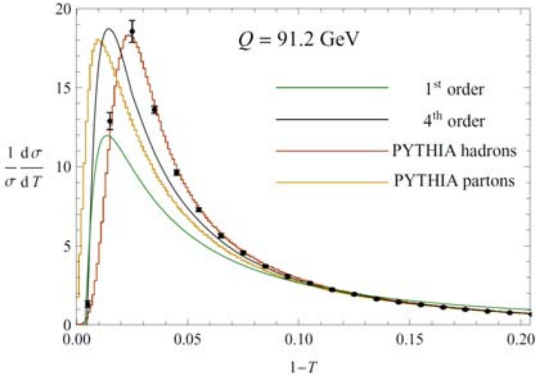

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Definite pattern in Q

Predictible from QCD/SCET factorization !

In the tail region $l_{\text{soft}} \sim Q\,\tau \gg \Lambda_{QCD}$

and we can expand the soft function

$$S(\tau) = S_{\text{pert}}(\tau) - S'_{\text{pert}}(\tau)\frac{2\Omega_1}{Q} \approx S_{\text{pert}}\left(\tau - \frac{2\Omega_1}{Q}\right)$$

$\Omega_1 \sim \Lambda_{QCD}$ Is a non-perturbative parameter

Ω_1 is defined in field theory

Lee & Sterman

Shifts distributions to the right !

Treating all regions together

The renormalon subtraction introduces a scheme parameter R.

The gap function $\bar{\Delta}$ has both ultraviolet (μ) and infrared (R) running.

We must turn off the resummation in the multijet region

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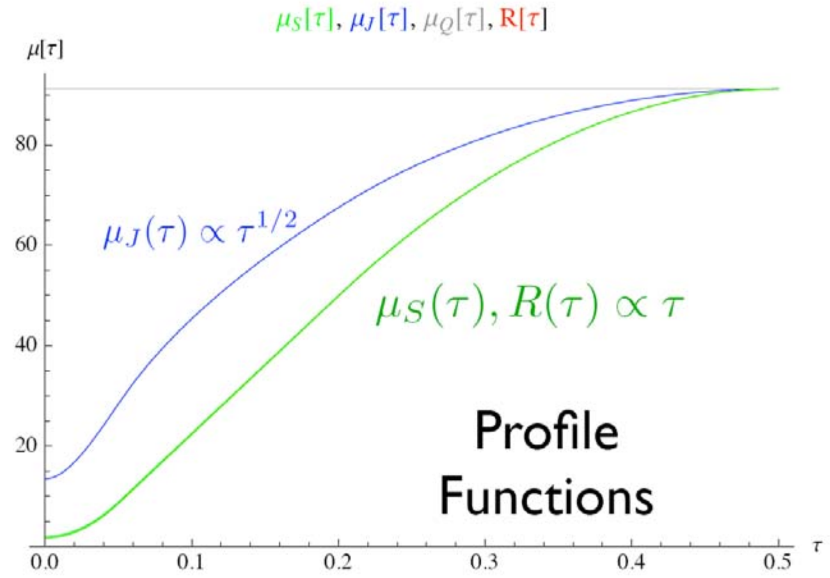
Let us μ_J, μ_s and R depend on τ



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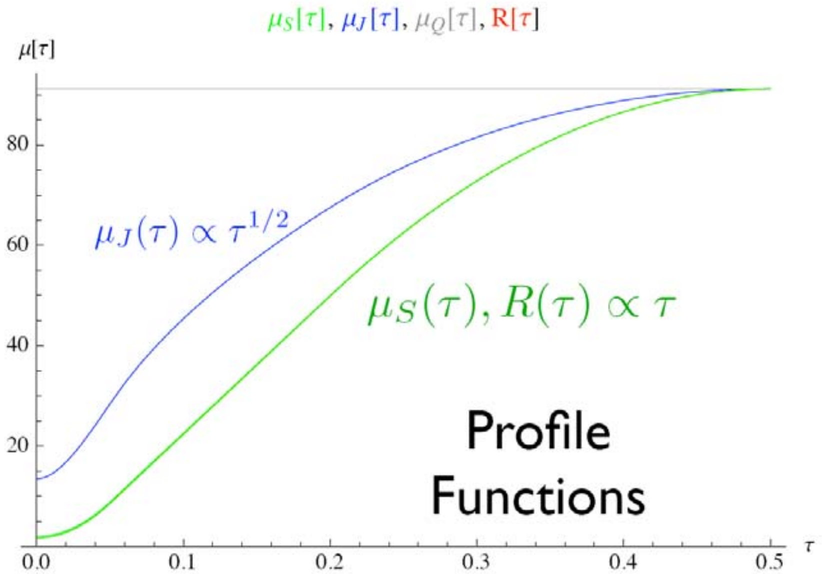

So all scales merge to the hard scale for the multijet region, and the transition occurs smoothly.

Still there is room for estimating the perturbative uncertainties by varying these profile function within the constraints.

Mass corrections

Fleming , Hoang, Mantry & Stewart

Now the electroweak effects tell apart the up- and down-type quarks

Our theoretical uncertainty for a massless production is at the 1% level

Hard, soft and running kernels unchanged.

Use $\overline{\text{MS}}$ running mass at the jet scale.

Shifts thresold to $\dfrac{2m^2}{Q^2} \longrightarrow 1-\sqrt{1-\dfrac{4m^2}{Q^2}}$

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$$J_T(p,\mu) \to J_T(p,m,\mu) = \begin{cases} J_{\text{distribution}}(p,m,\mu) \to \text{ Fourier transform} \\ J_{\text{function}}(p,m,\mu) \to \text{ Treat in momentum space} \end{cases}$$

Non-singular piece analytically known [AFHMS]

Important for bottom

Small for charm

The one-loop mass corrections are about a 2% effect

$$\text{Theoretical distribution} = \frac{\sum\limits_{q=u,d,s,c,b} \left.\dfrac{d\sigma}{d\tau}\right|_q}{\sum\limits_{q=u,d,s,c,b} (\sigma_0)_q}$$

One-loop level suffices

QED corrections

[AFHMS]

Affect all matrix elements: Hard, Jet and Soft

$\alpha_s(\mu)$ and $\alpha(\mu)$ have coupled evolution equations.

One can solve them perturbatively.

The non-singular term is trivially obtained

All running kernels are affected. There are interesting QED – QCD mixing effects !

Different for up and down quarks! Need to take into account electroweak factors.

One can consider $\alpha \leq \mathrm{O}(\alpha_s^2)$

No renormalon subtractions for QED

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Stick to NLO in Matrix elements and NNLL for the running

They have a small effect on $\alpha_s(m_Z)$

They shift Ω_1

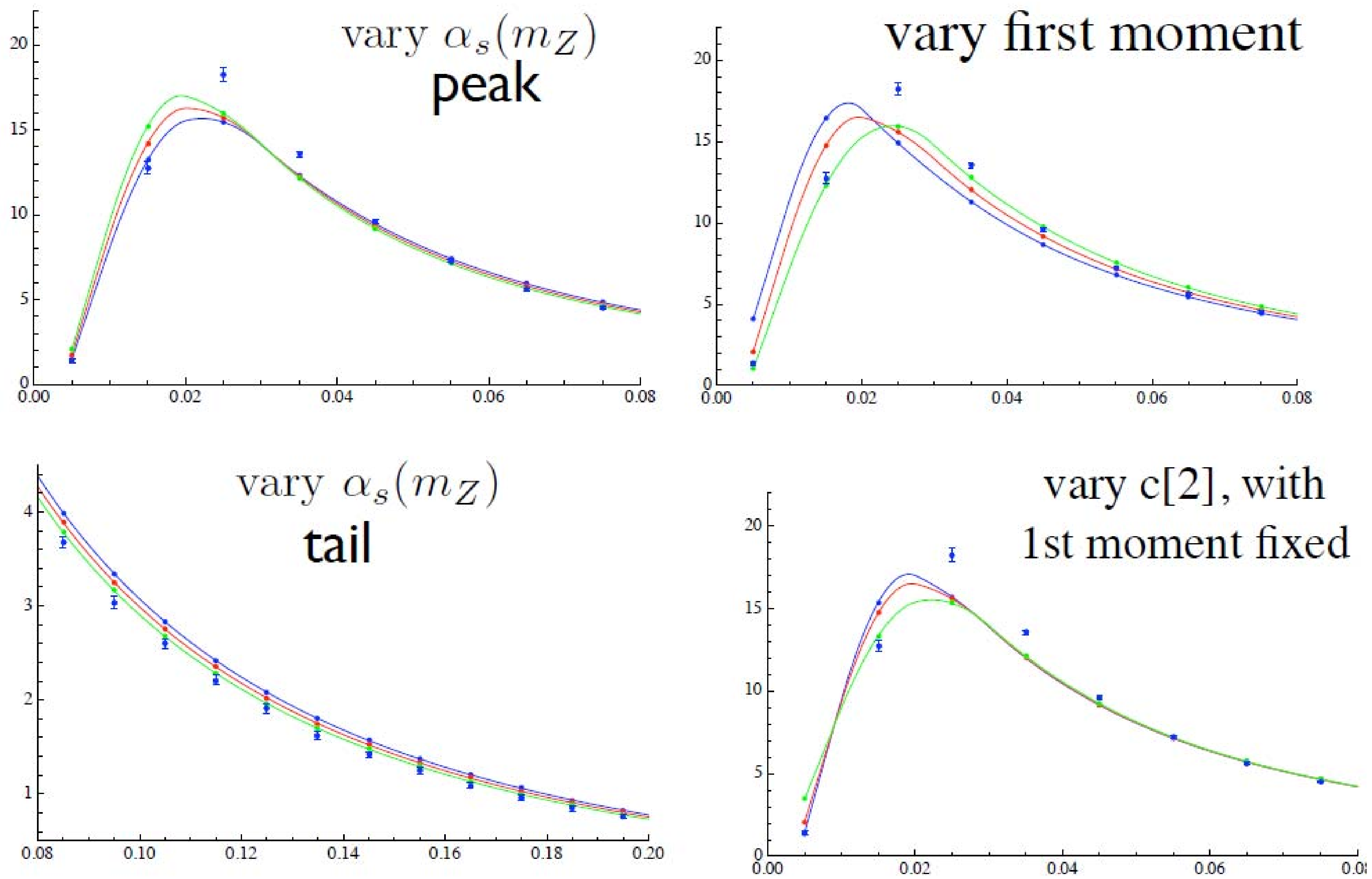
Different for up and down quarks! Need to take into account electroweak factors.

One can consider $\alpha \leq \mathrm{O}(\alpha_s^2)$

No renormalon subtractions for QED

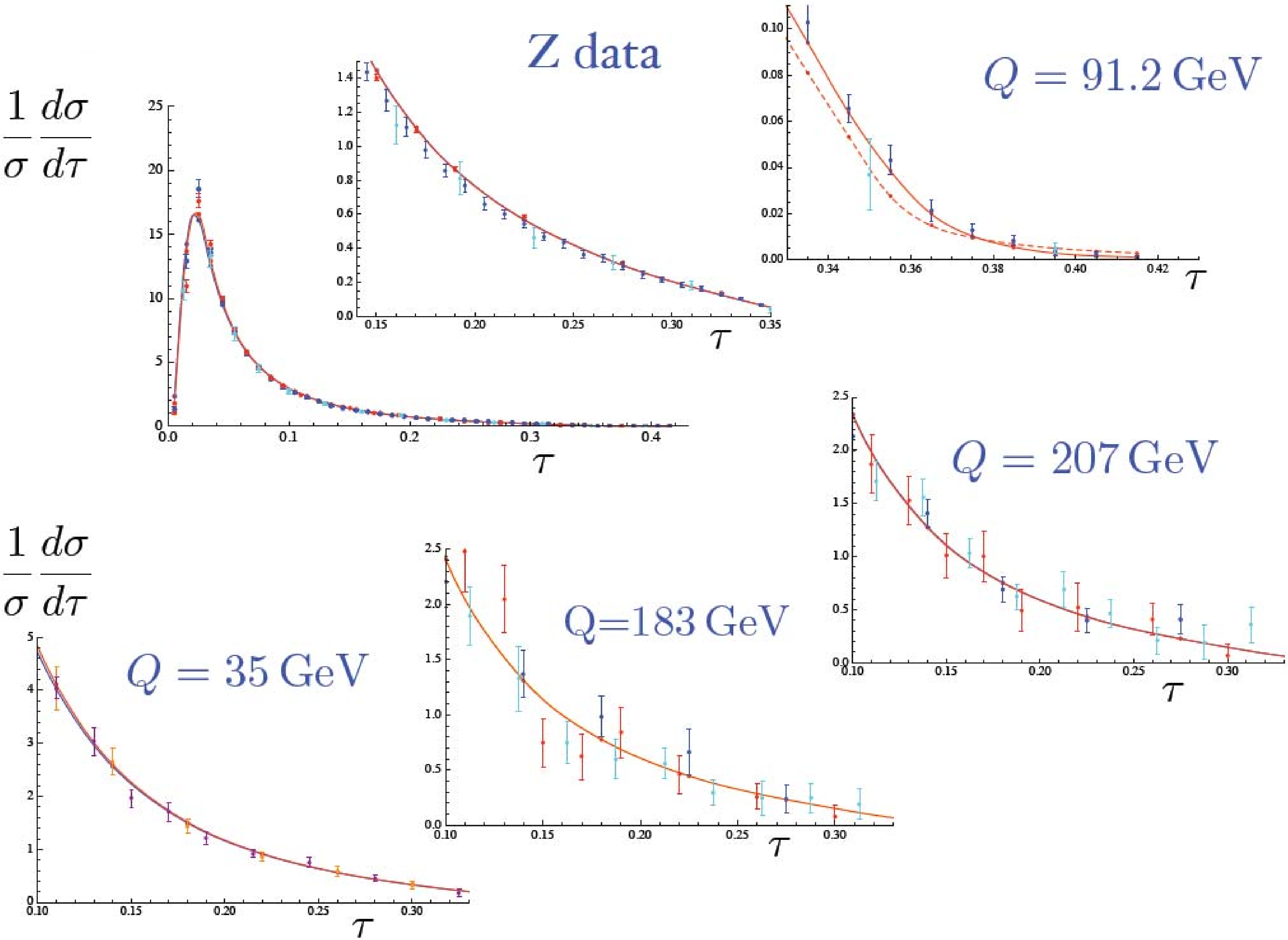
Do not need to worry about initial state radiation, since these effects are minimized in the normalization procedure

Effects of various pieces



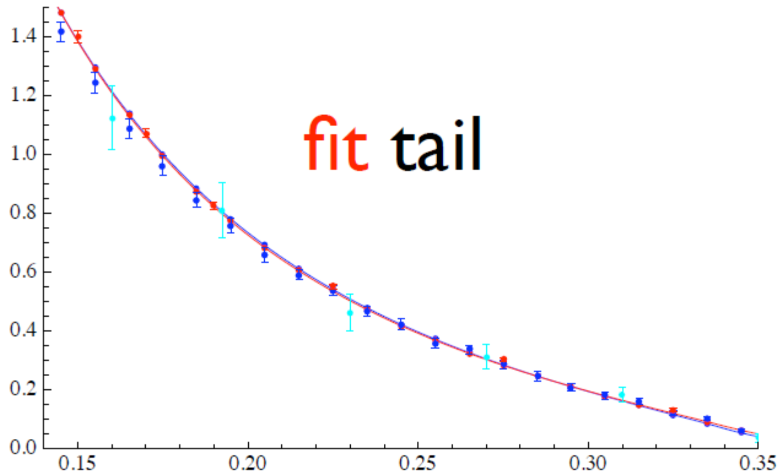
Fit for Ω_1 $\alpha(M_Z)$ and c_2

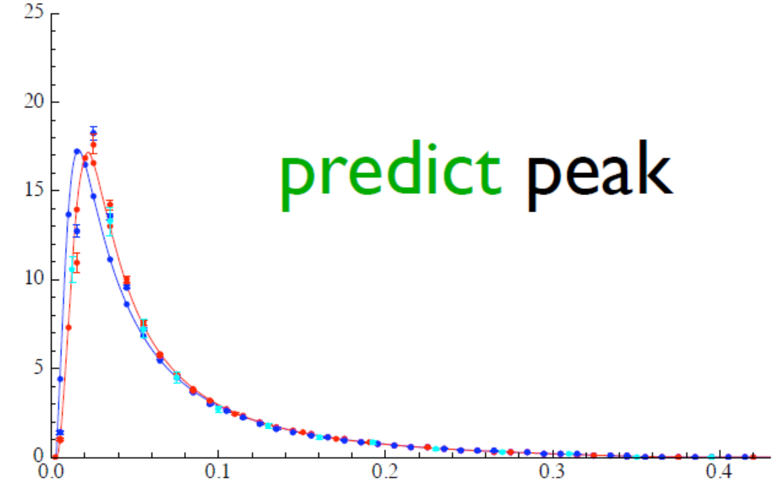
Fit in all regions (peak, tail & far tail)

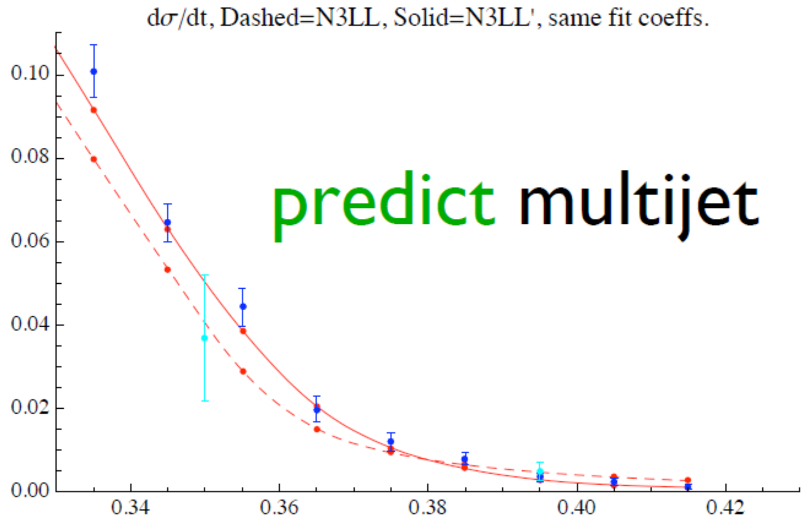


Two parameter fit in tail region

In the tail $[\tau, 0.14, 0.33]$ only the shift matters. So we fit for Ω_1 & $\alpha(M_Z)$ simultaneously







Central values for tail fits

For the full dataset

(Preliminary)

	$\alpha_s(m_Z)$	$2\Omega_1$	χ^2/dof
$\mathrm{N^3LL}$	0.1165	1.100	0.823
$\mathrm{N^3LL'}$	0.1137	0.988	0.772
$\mathrm{N^3LL'}$ (μ up, fix Ω_1)	0.1139	(0.988)	0.761
$\mathrm{N^3LL'}$ (μ down, fix Ω_1)	0.1131	(0.988)	0.785
$\mathrm{N^3LL'}$ (μ up)	0.1146	0.922	0.760
$\mathrm{N^3LL'}$ (μ down)	0.1130	1.010	0.784
$\mathrm{N^3LL'}$ (normalized)	0.1134	0.975	0.787
$\mathrm{N^3LL'}$ (add QED)	0.1138	0.980	0.773
$\mathrm{N^3LL'}$ (add QED & mass)	0.1142	0.921	0.776

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We use LEP working groups correlation model for systematic errors

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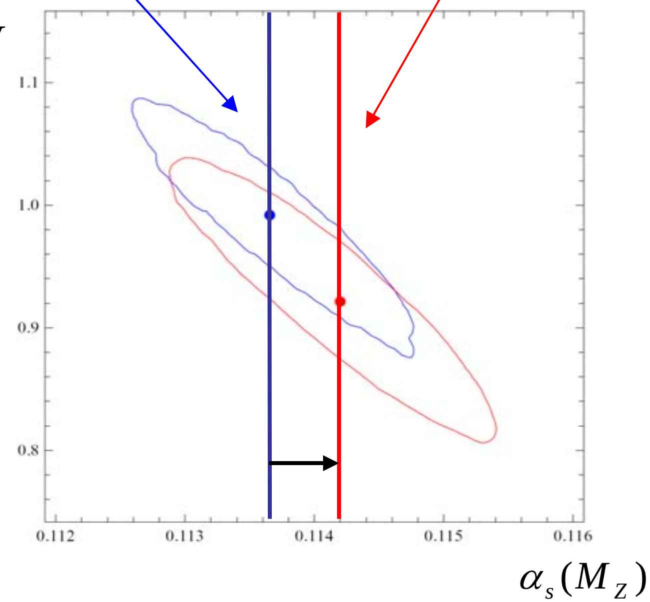
Massless no QED

Mass & QED

$$\boxed{\alpha_s(M_Z) = 0.1137 \pm 0.0008 \pm 0.0011}$$

$$\boxed{\alpha_s(M_Z) = 0.1142 \pm \underbrace{0.0008}_{} \pm \underbrace{0.0011}_{}}$$



Perturbative error

Syst.,Stat., & hadronic error

reduced by a factor of 2 - 3 !

$$\frac{\chi^2}{\mathrm{d.o.f.}} = 0.77$$

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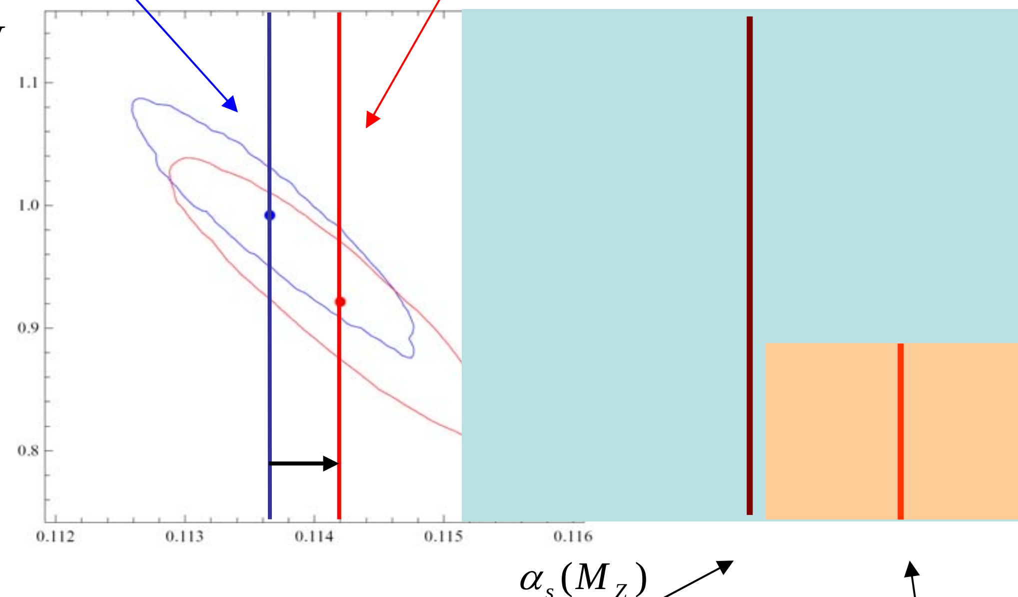

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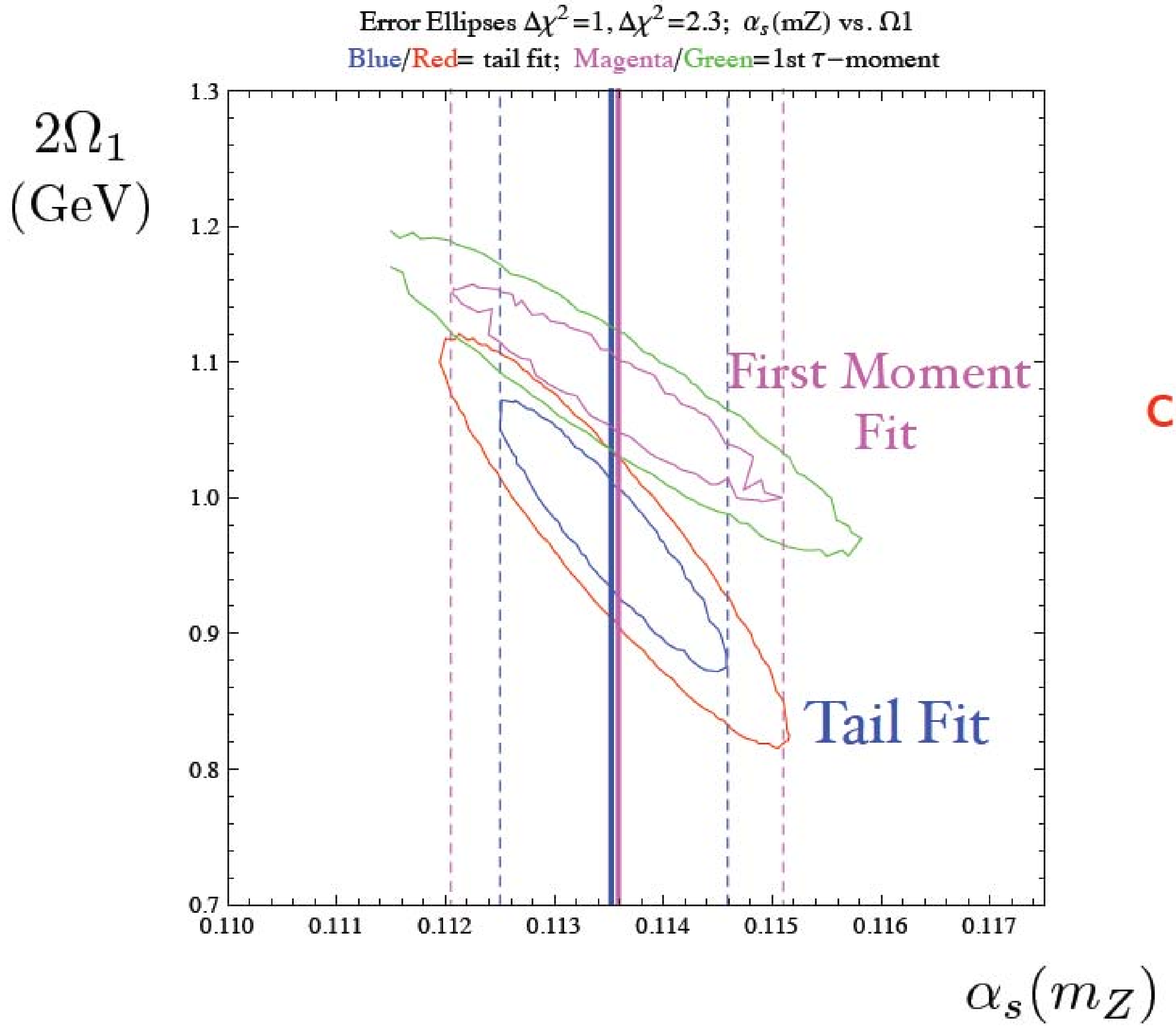
PDG world average

Bethke average

Our value is lower than the World average $\longrightarrow$ montecarlo hadronization gives larger values

Tail Fit & first Moment Fit Results



consistent

Comparison with other α^3 analyses

Becher & Schwartz

- Data: Aleph & Opal
- Resummation: In the amplitude (EFT RG Equations)
- No non-perturbative effects in central value
- Result $\alpha(M_Z) = 0.1172 \pm 0.0021$

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Davison & Webber

- Data: Many Q´s
- Resummation: In the distribution [NLL+O(**α³**)]
- Non-perturbative effects in a model (large logs in subtraction)
- Result $\alpha(M_Z) = 0.1164 \pm 0.0040$

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- Non-perturbative effects in a model
- Result $\alpha(M_Z) = 0.1164 \pm 0.0040$

$$\boxed{\alpha_s(M_Z) = 0.1142 \pm 0.0008 \pm 0.0011}$$

Dissertori, Gehrmann-De Ridder Gehrmann, Glover & Heinrich

- Data: Aleph
- Resummation: fixed order
- Non-perturbative effects treatment: Montecarlo generator
- Result $\alpha(M_Z) = 0.1240 \pm 0.0029$

Conclusions

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets.
- SCET has finally provided theorists with a mean to catch up to the experimental precision of LEP. $\boxed{\alpha_s(M_Z) = 0.1142 \pm 0.0008 \pm 0.0011}$
- Global fit of all data with all Q's and all τ's .
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- Perform a fit to all regions (including peak and far tail).
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The future for high precision determinations of the strong coupling constant looks good!