Review of Factorization Theorems for Quarkonia using SCET and Open Questions

Adam Leibovich
University of Pittsburgh

SCET Workshop 2009
March 26, 2009

Outline

- Early calculations
- NRQCD
  - The pieces
  - Factorization in NRQCD
  - Some failures of NRQCD
- SCET and Quarkonia
  - What has been done
  - Example of factorization
- Some open questions

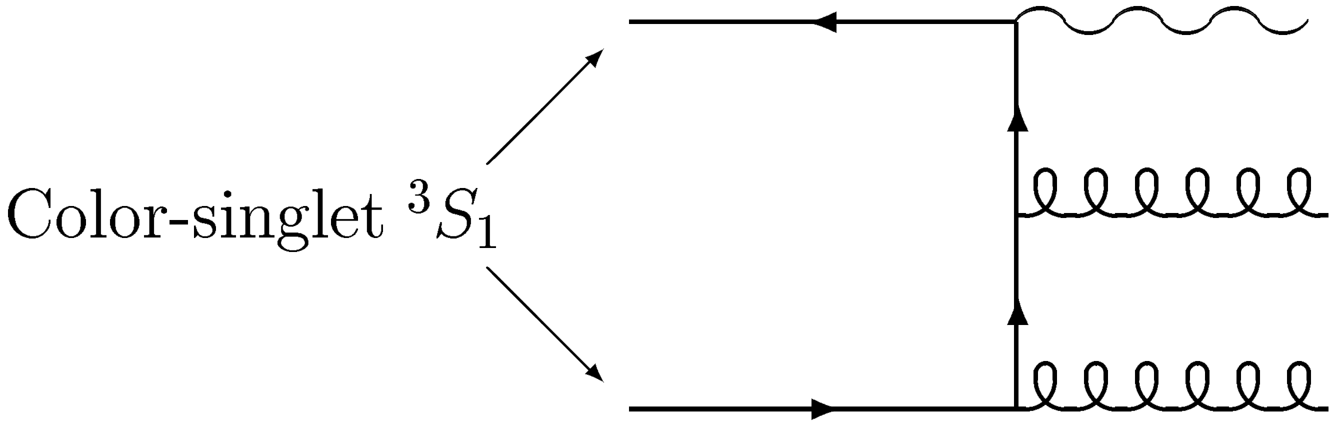
Ancient History

Quarkonia always found to be interesting

Large mass ➟ hope of perturbative calc

$$m_Q \gg \Lambda_{\rm QCD}$$

Early calculations used Color-Singlet Model



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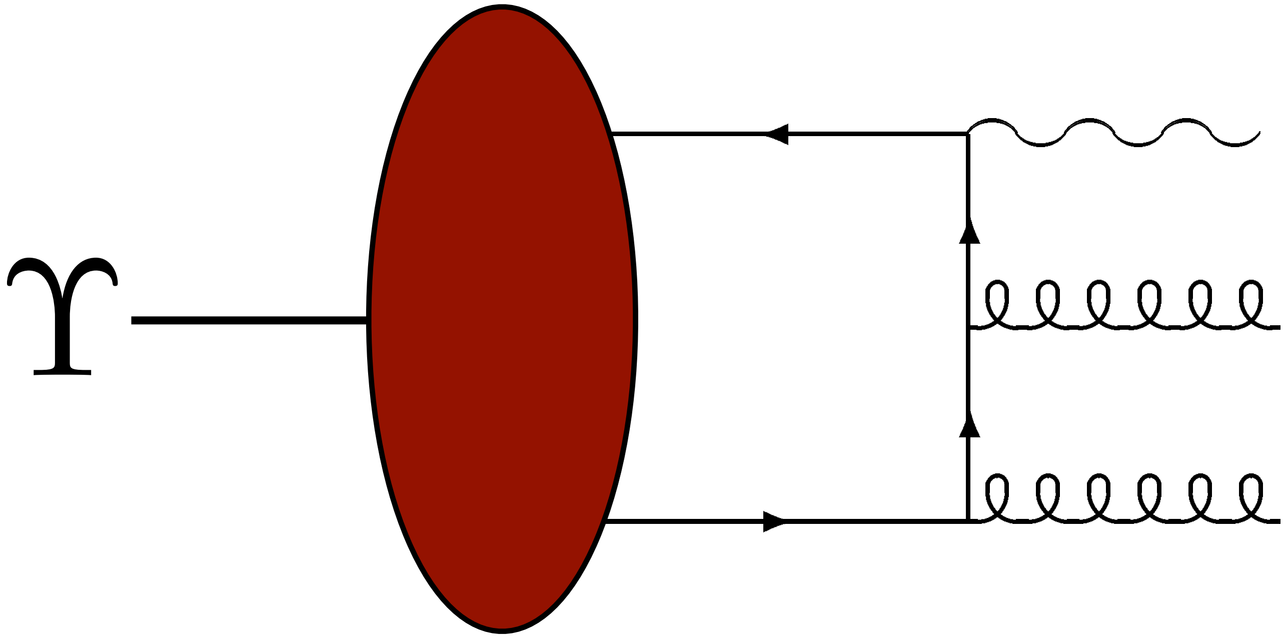

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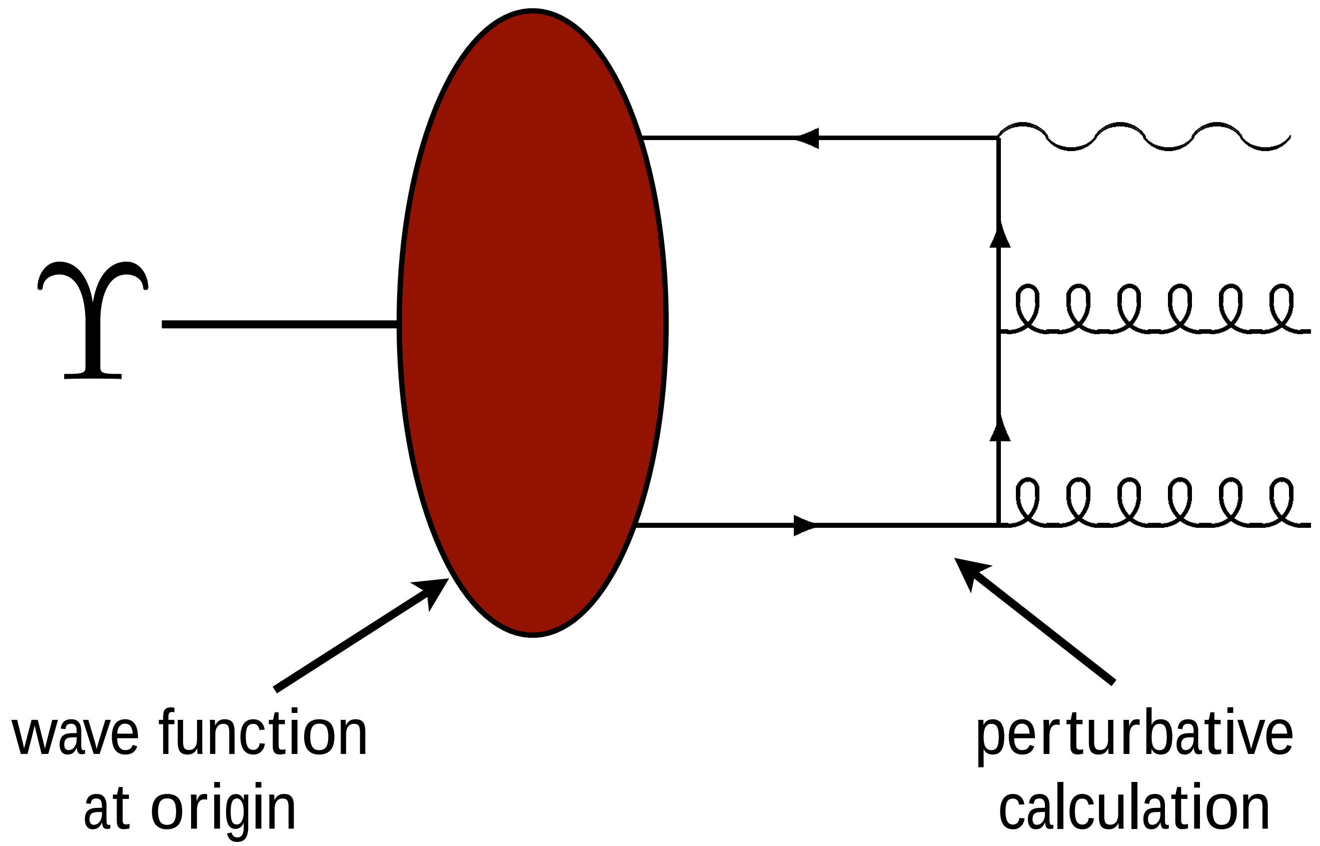

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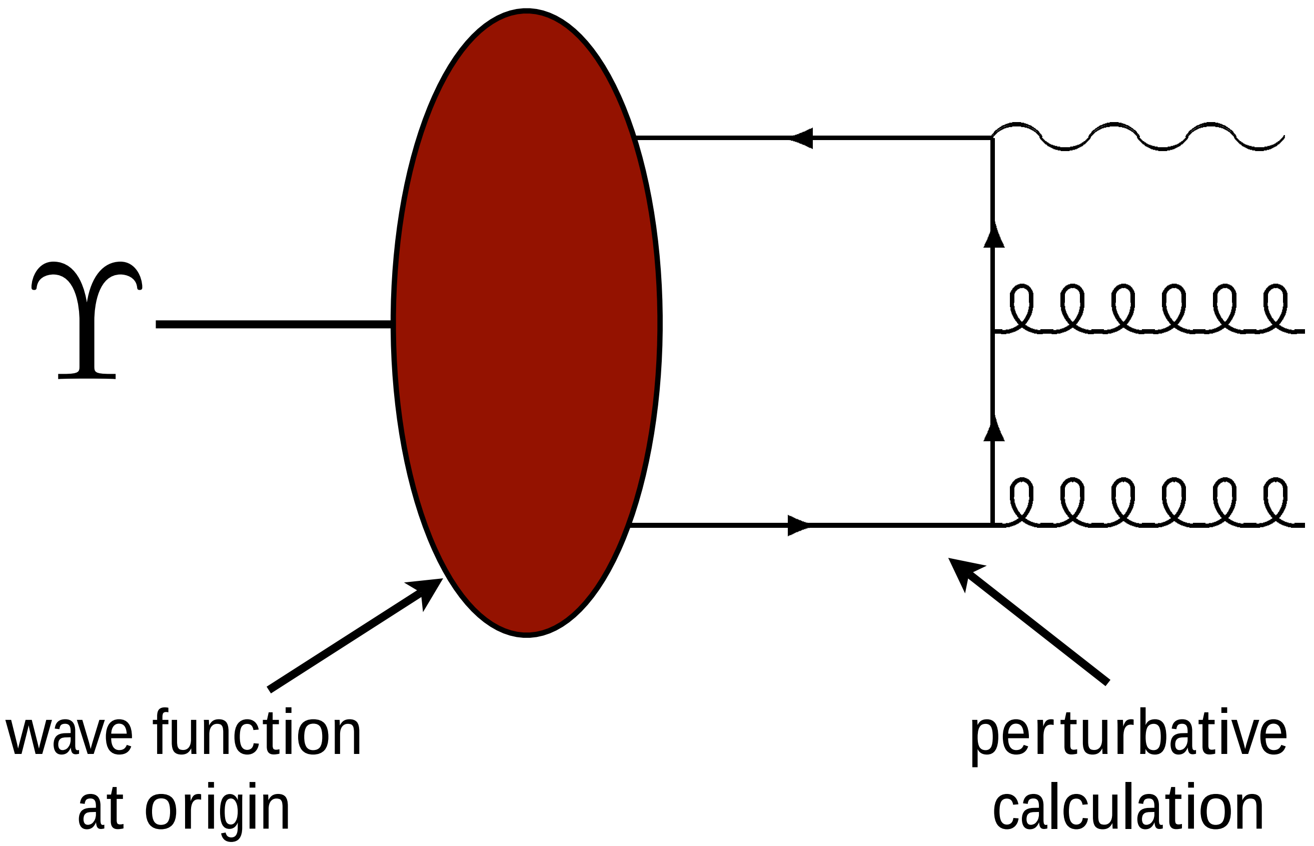

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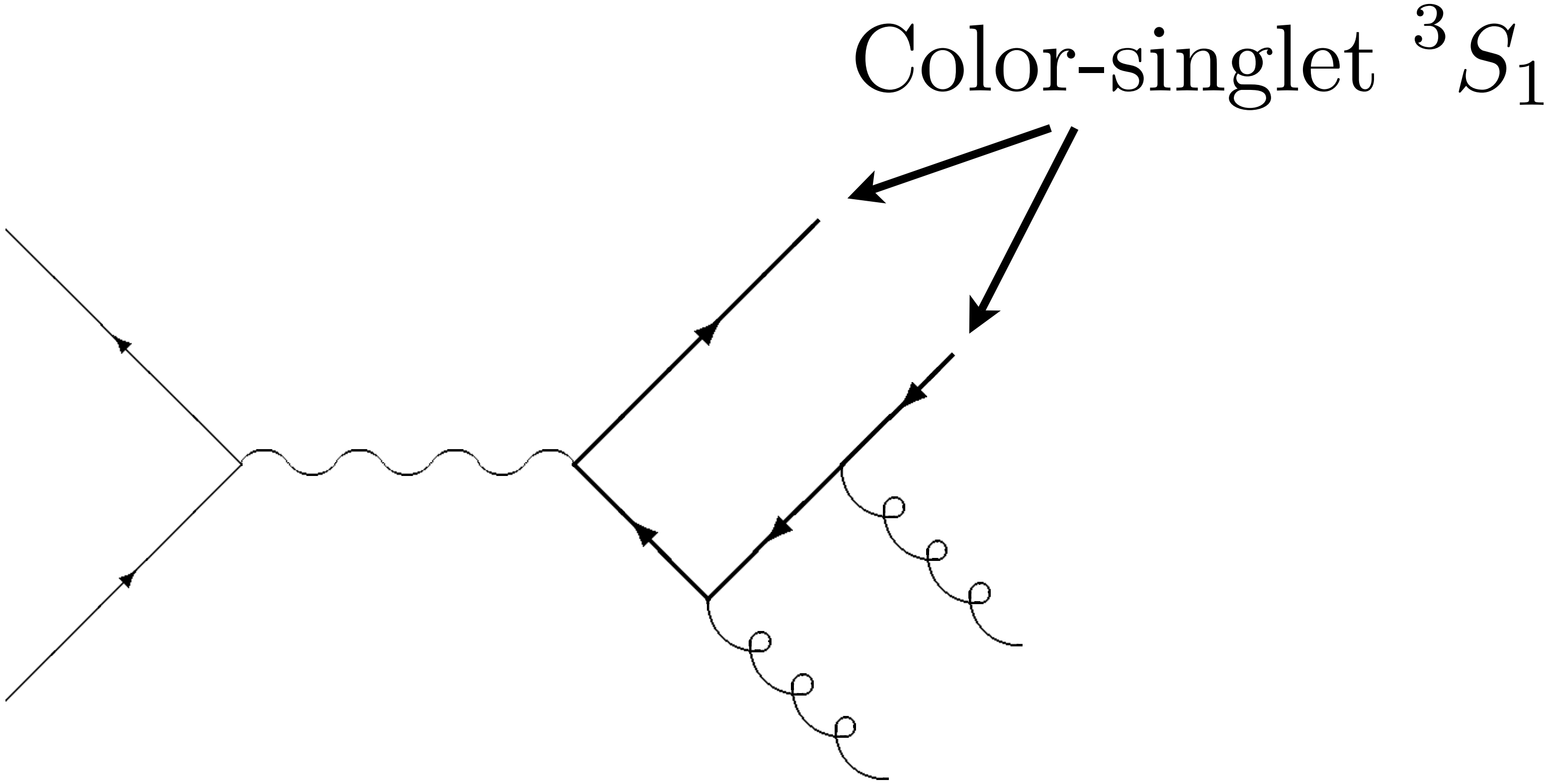

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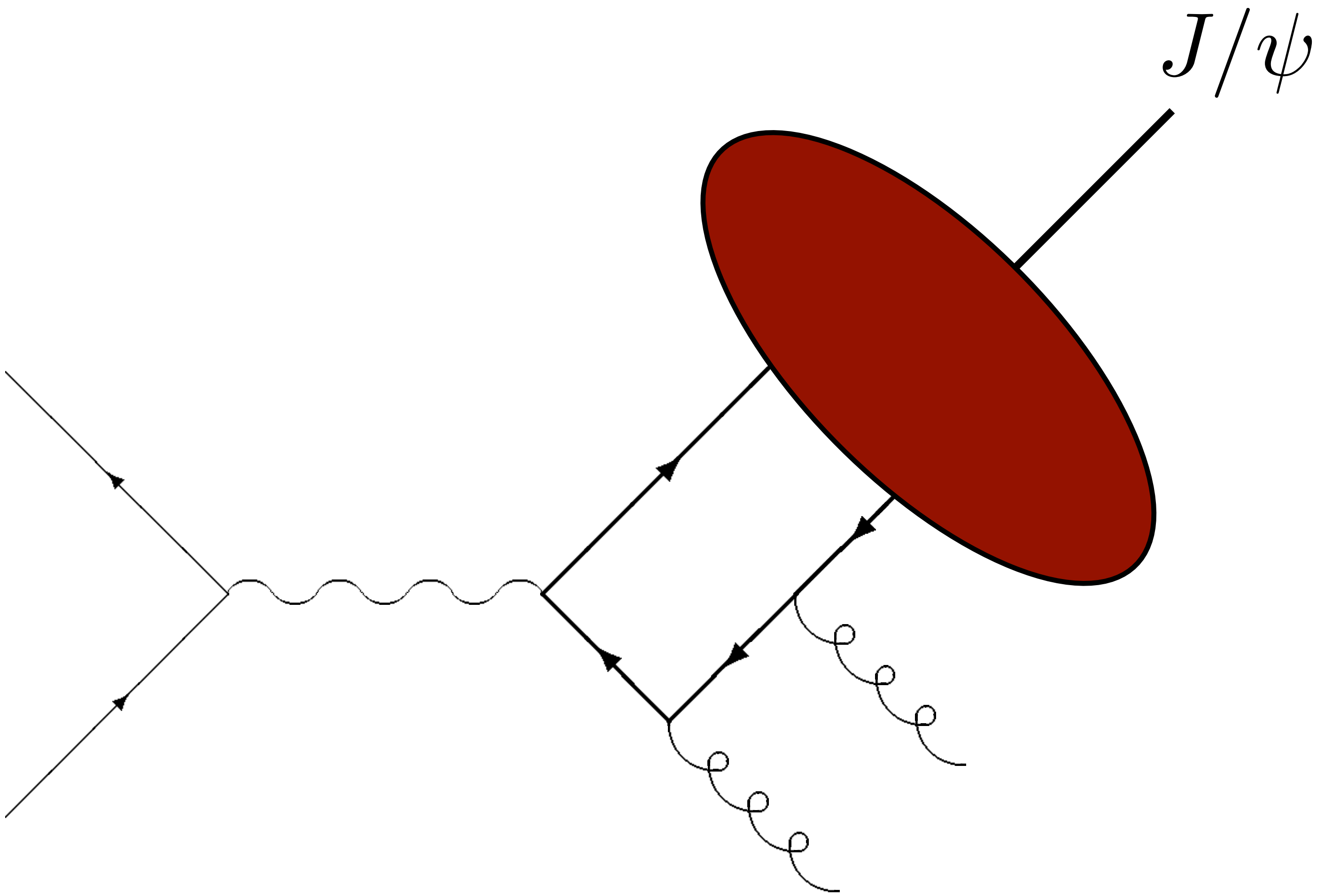

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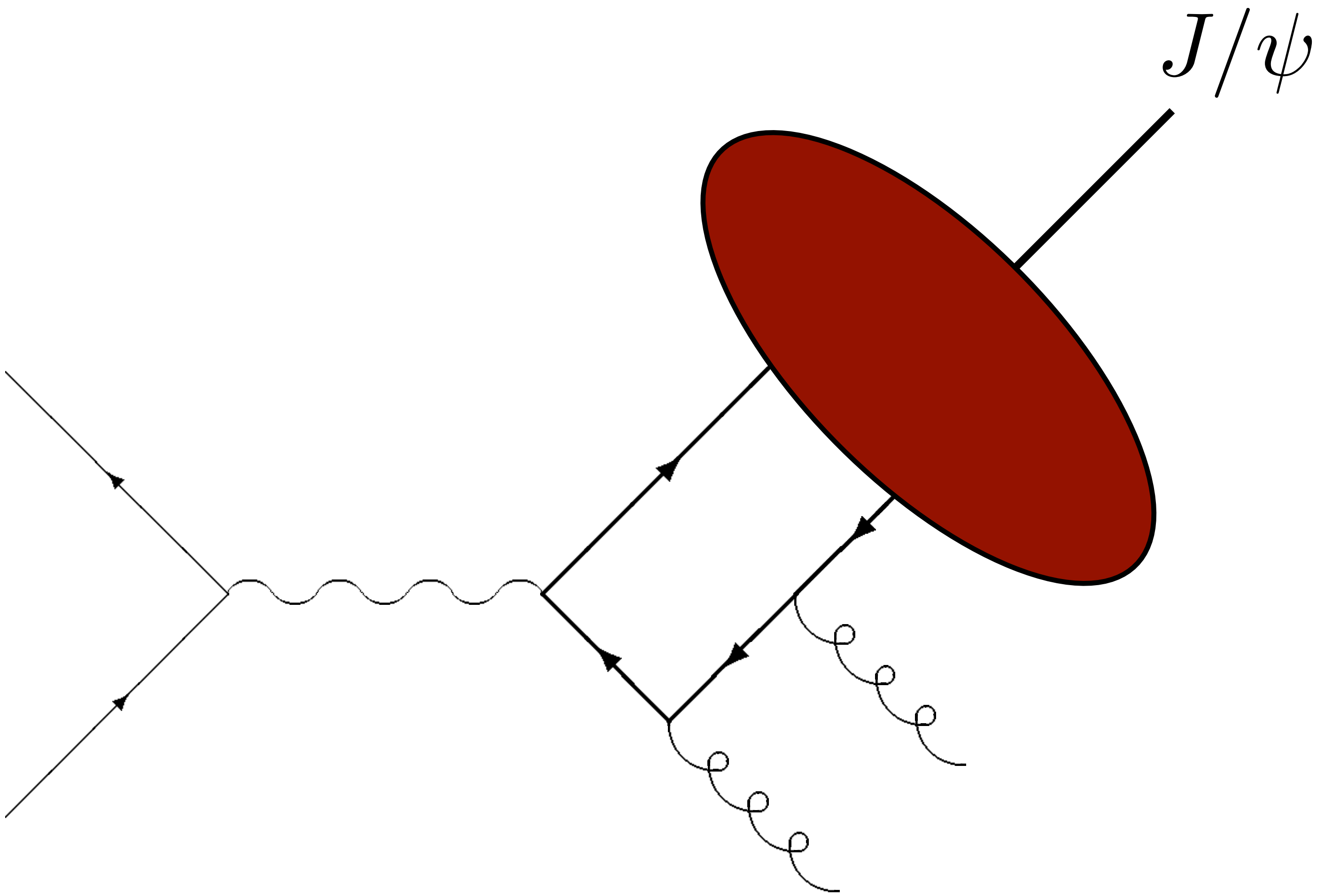

Again, if one of the gluons is soft?

Review of NRQCD

- Effective Field Theory based on non-relativistic expansion $v \ll 1$
- Integrate out modes with $p^\mu \gtrsim m_Q, m_Q v$
- EFT has soft quarks and gluons, low-frequency components of heavy quarks $p^\mu \sim m_Q v^2$
- New channels for production/decay

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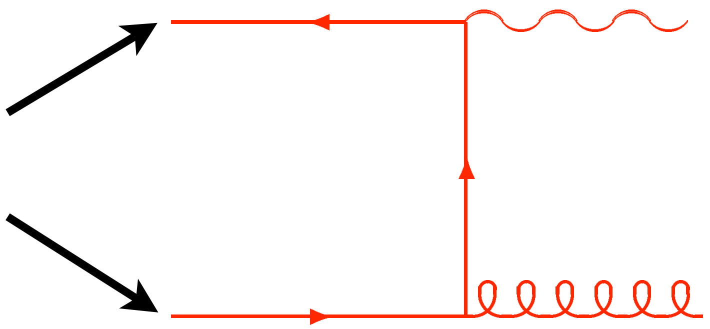
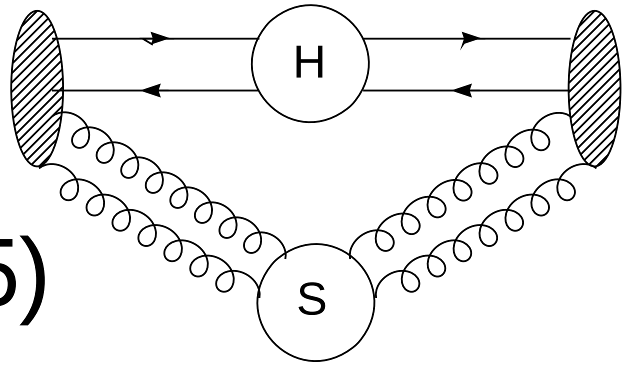
Color-octet ${}^1S_0$



Factorization in NRQCD

Decay



- Fully inclusive decay factorization presented in Bodwin, Braaten and Lepage, PRD51, 1125 (1995)
- Based on "topological factorization"

$$\Gamma(H \to \text{light hadrons}) = \sum_n 2\,\mathrm{Im} F_n(\Lambda)\langle H|\mathcal{O}_n(\Lambda)|H\rangle$$

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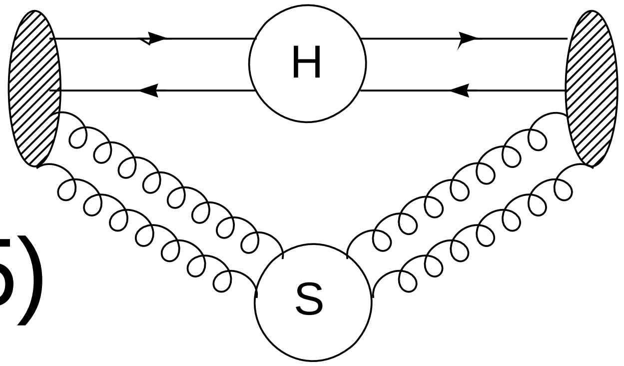

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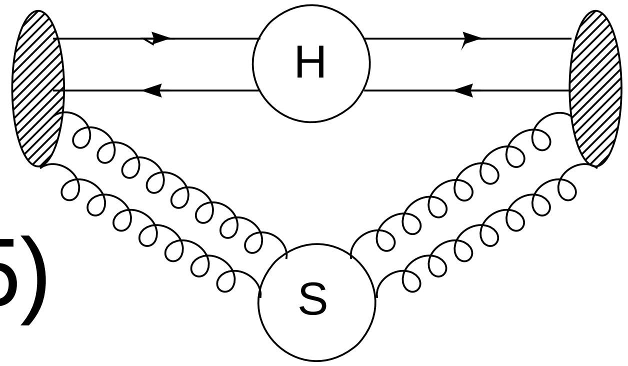

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Four-quark
NRQCD operators

$$\mathcal{O}_1(^3S_1) = \psi^\dagger \sigma^i \chi\, \chi^\dagger \sigma^i \psi$$

Factorization in NRQCD

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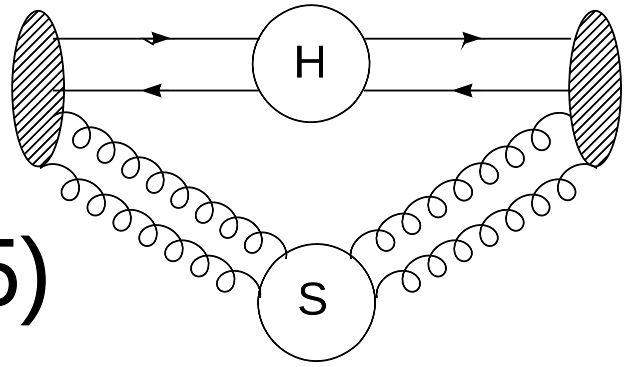

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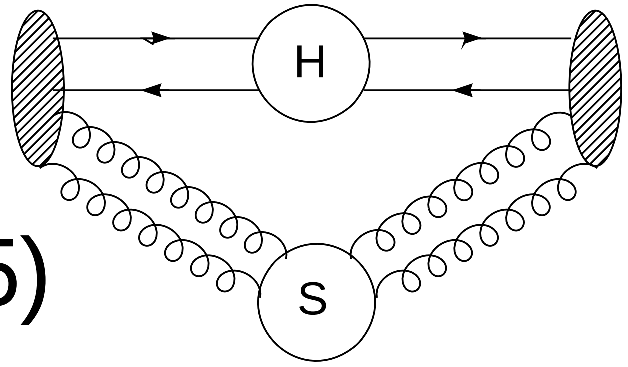

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Used without modification for all decays

Factorization in NRQCD

Production

- Inclusive production factorization argued in Bodwin, Braaten and Lepage, PRD51, 1125 (1995)
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$$\sigma(H+X)=\sum_n d\hat{\sigma}(c\bar{c}_n+X)\langle\mathcal{O}_n^H\rangle$$

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Failures of NRQCD?

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  - Open charm
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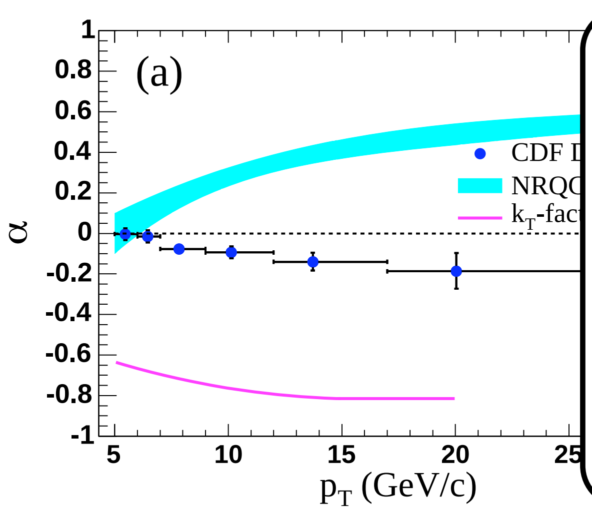

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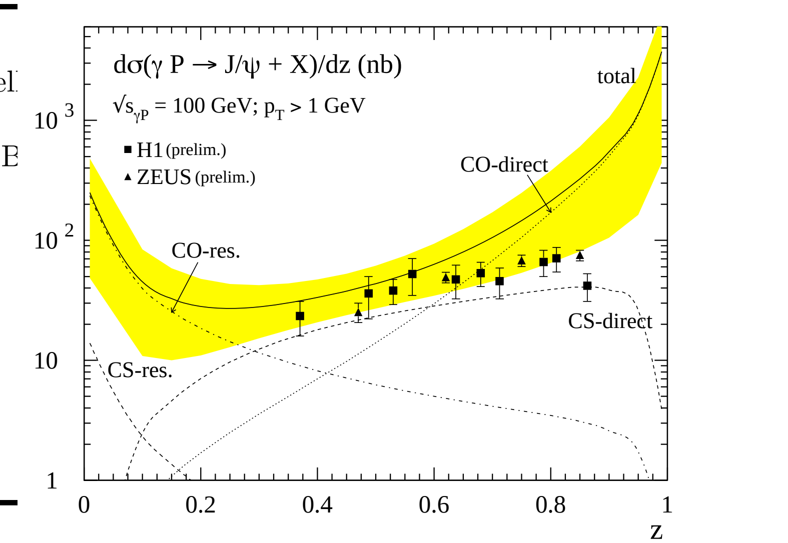

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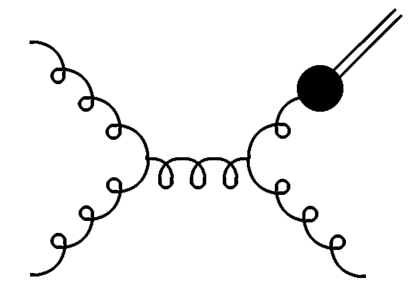

PRL99, 132001 (2007)

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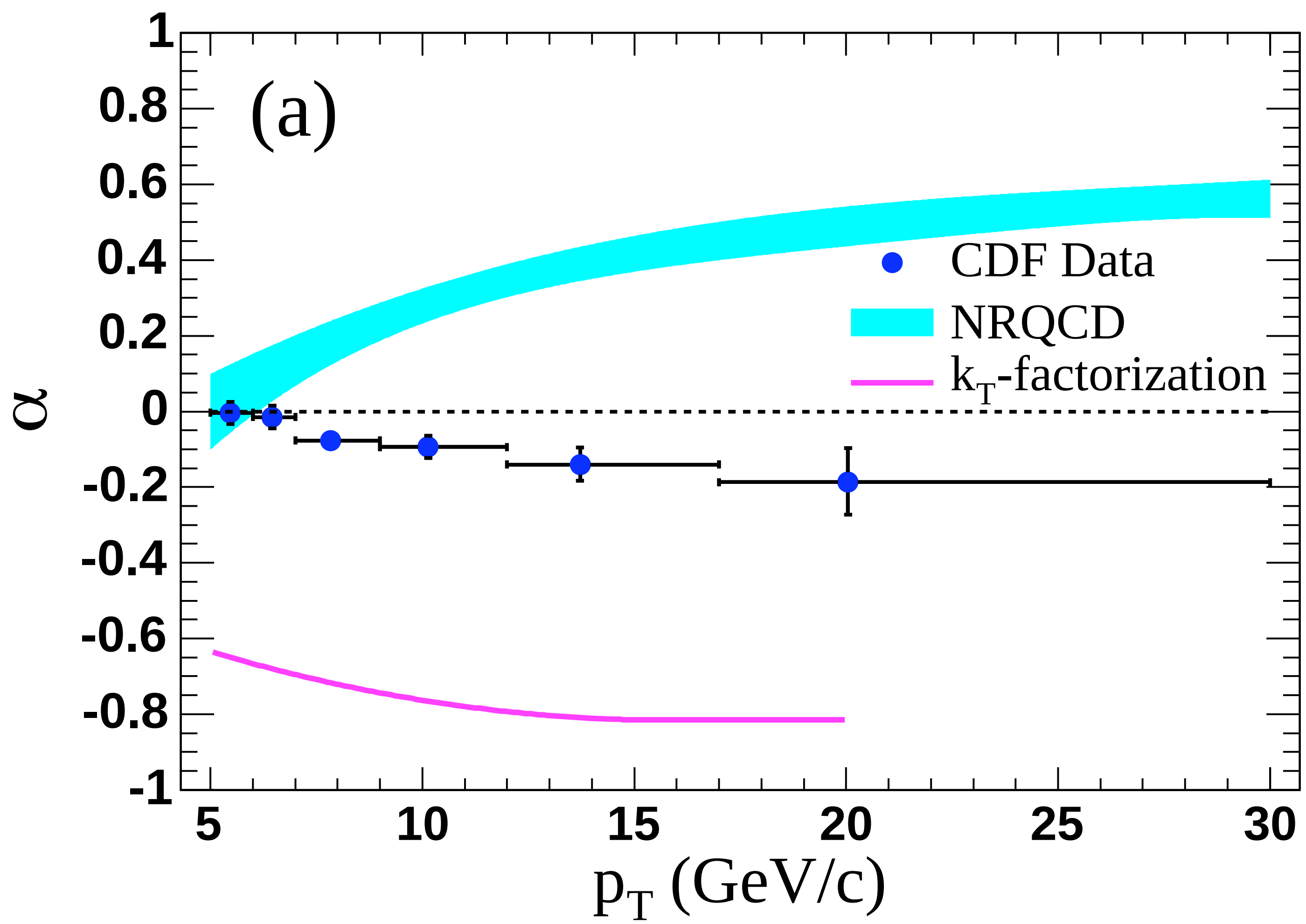

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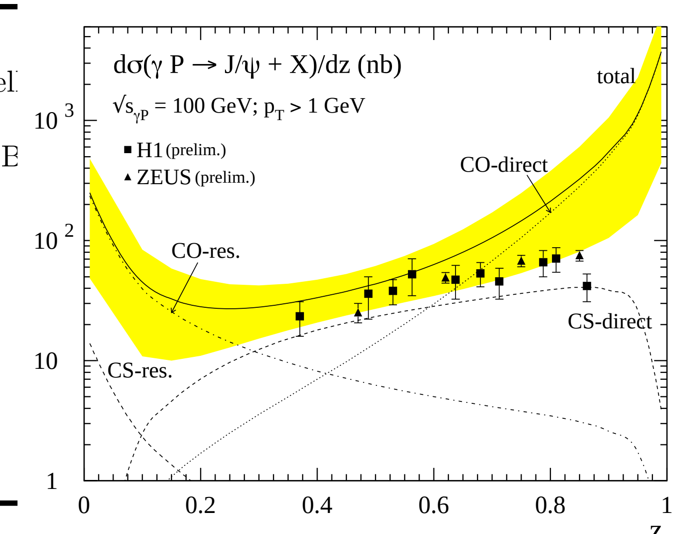

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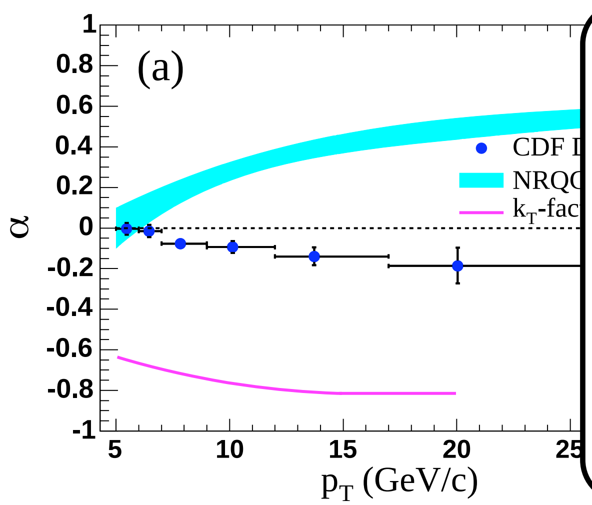

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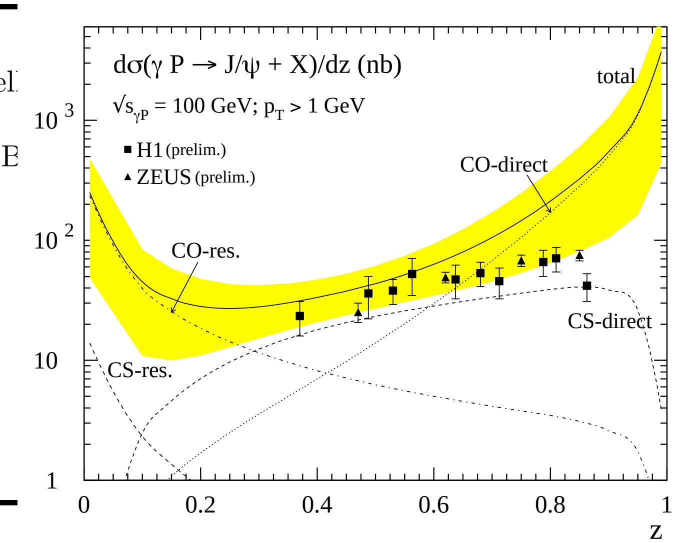

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numerous papers
0812.5106

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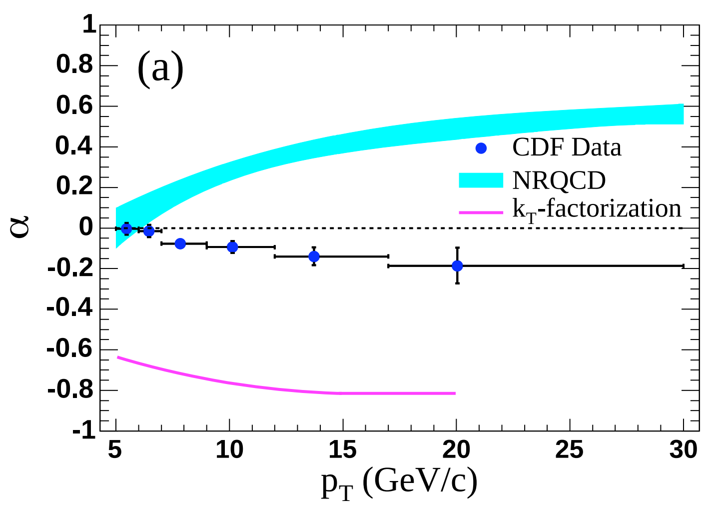

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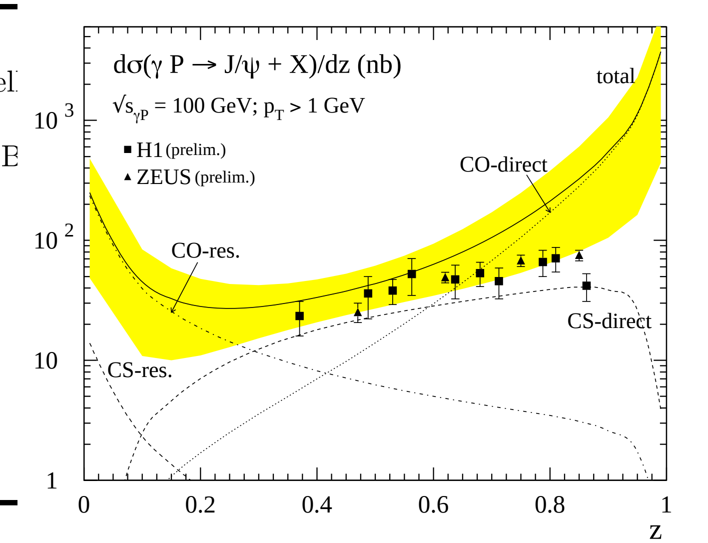

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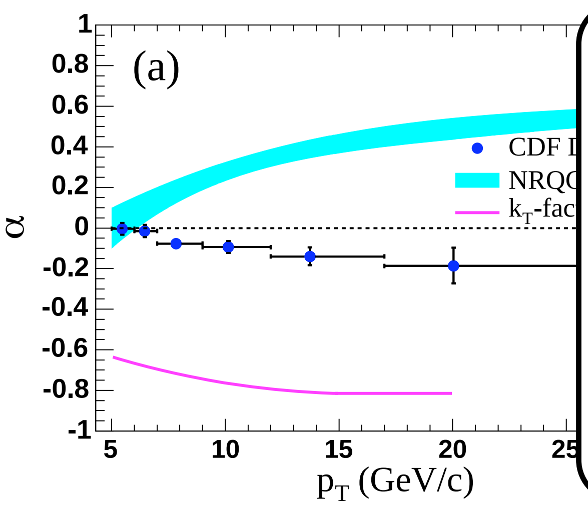

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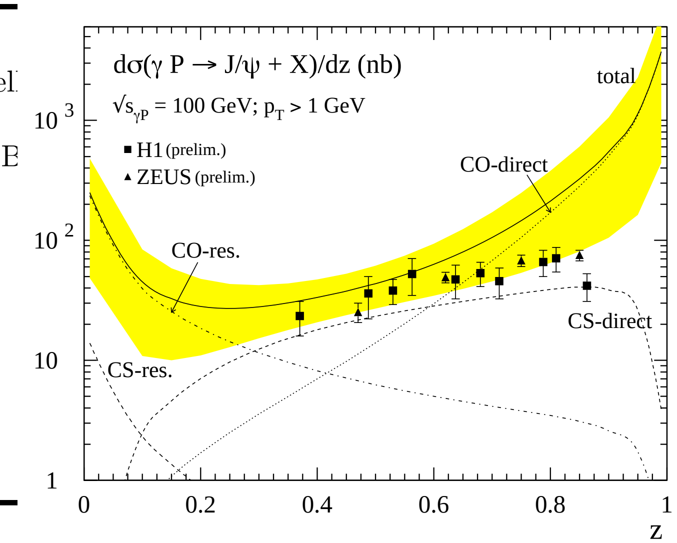

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PRD70, 071102 (2004), PRD72, 031101 (2005)
PLB557, 45 (2003)
PRD77, 094018 (2008)

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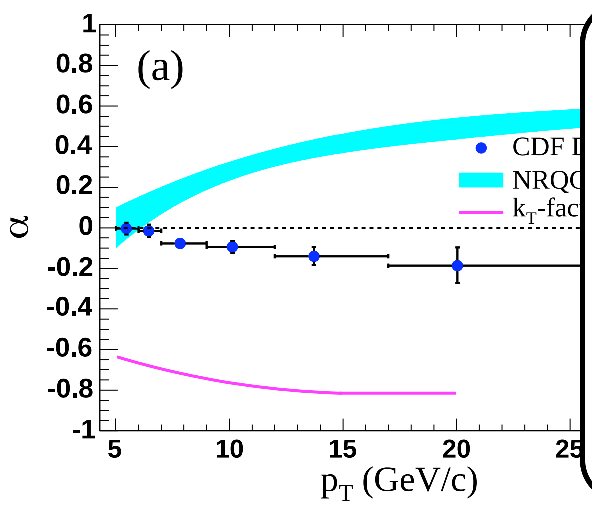

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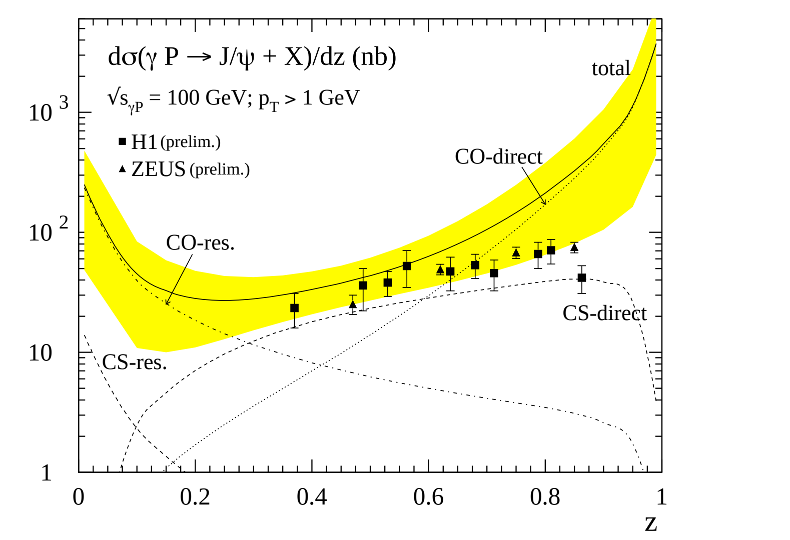

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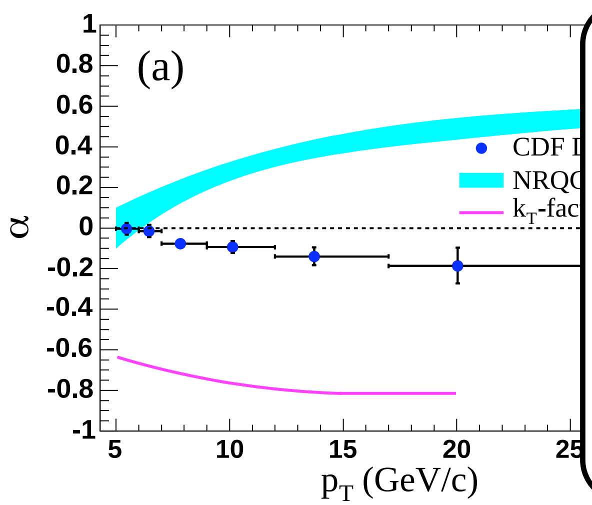

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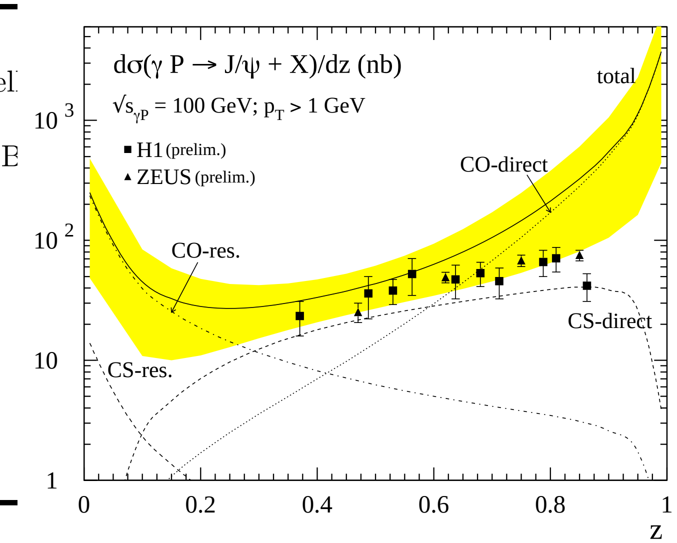

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Prog. Part. Nucl Phys. 47, 141 (2001)

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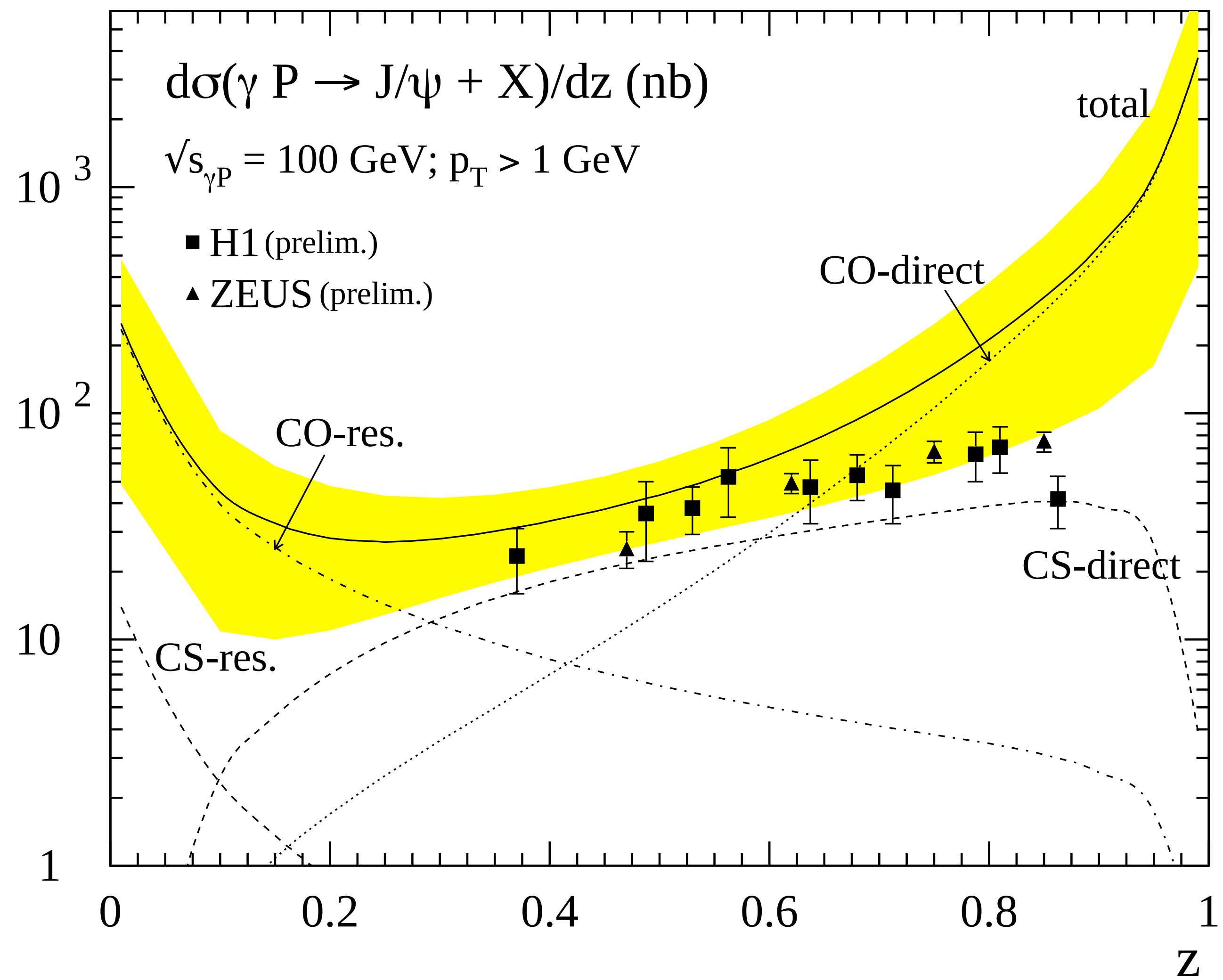

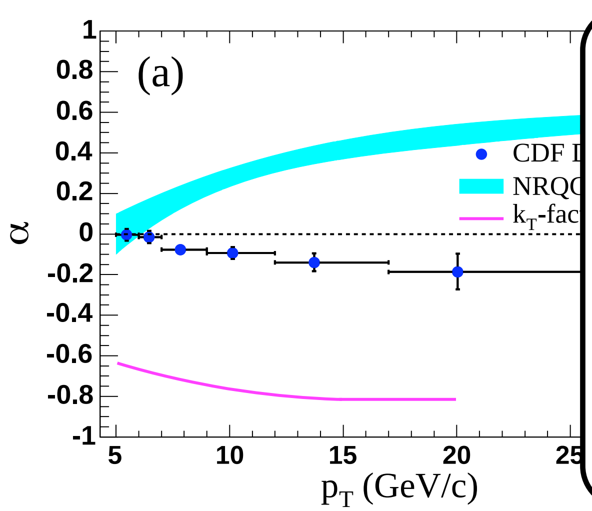

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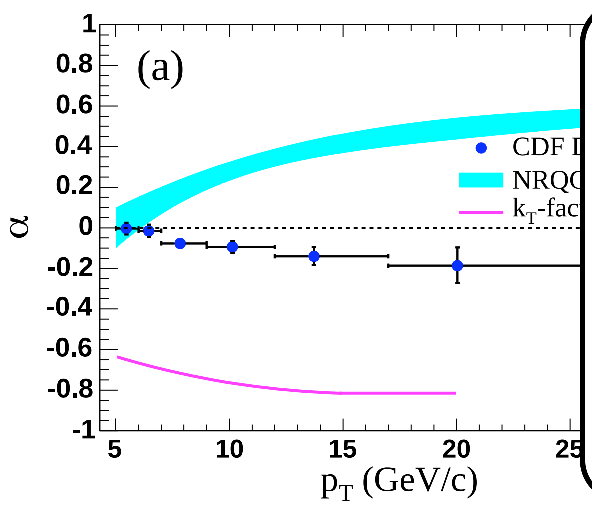

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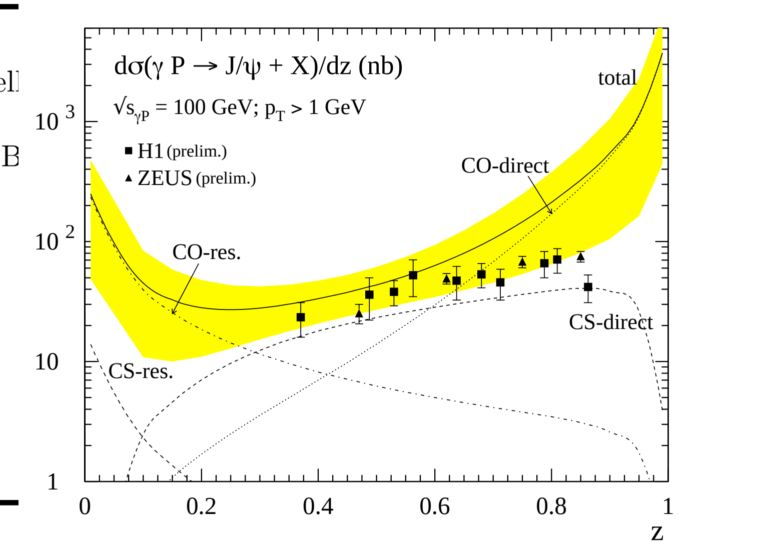

Factorization in NRQCD

Recent work

- Nayak, Qui, Sterman series of papers

  hep-ph/0501235, hep-ph/0509021, hep-ph/0608066, 0707.2973, 0711.3476

  - Importance of Wilson lines
  - Breakdown of EFT for $J/\psi + c\bar{c}$

- Bodwin, Tormo, Lee (next talk)
  - double charmonium production

    0805.3876

Quarkonium in SCET

Just like with HQET and B mesons,
combine NRQCD with SCET for quarkonium

Important when collinear quanta are present

- Radiative Υ and J/ψ decay (inclusive and exclusive)

  hep-ph/0106316, hep-ph/0211303, hep-ph/0212094, hep-ph/0401233, hep-ph/0406121, hep-ph/0407259, hep-ph/0411180, hep-ph/0507107, hep-ph/0511167, hep-ph/0701030, hep-ph/0702079

- $e^+e^- \to J/\psi + X$ at $\Upsilon(4S)$ energy

  hep-ph/0306139, 0705.3230

- J/ψ photoproduction

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  Use as example

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  hep-ph/0607121

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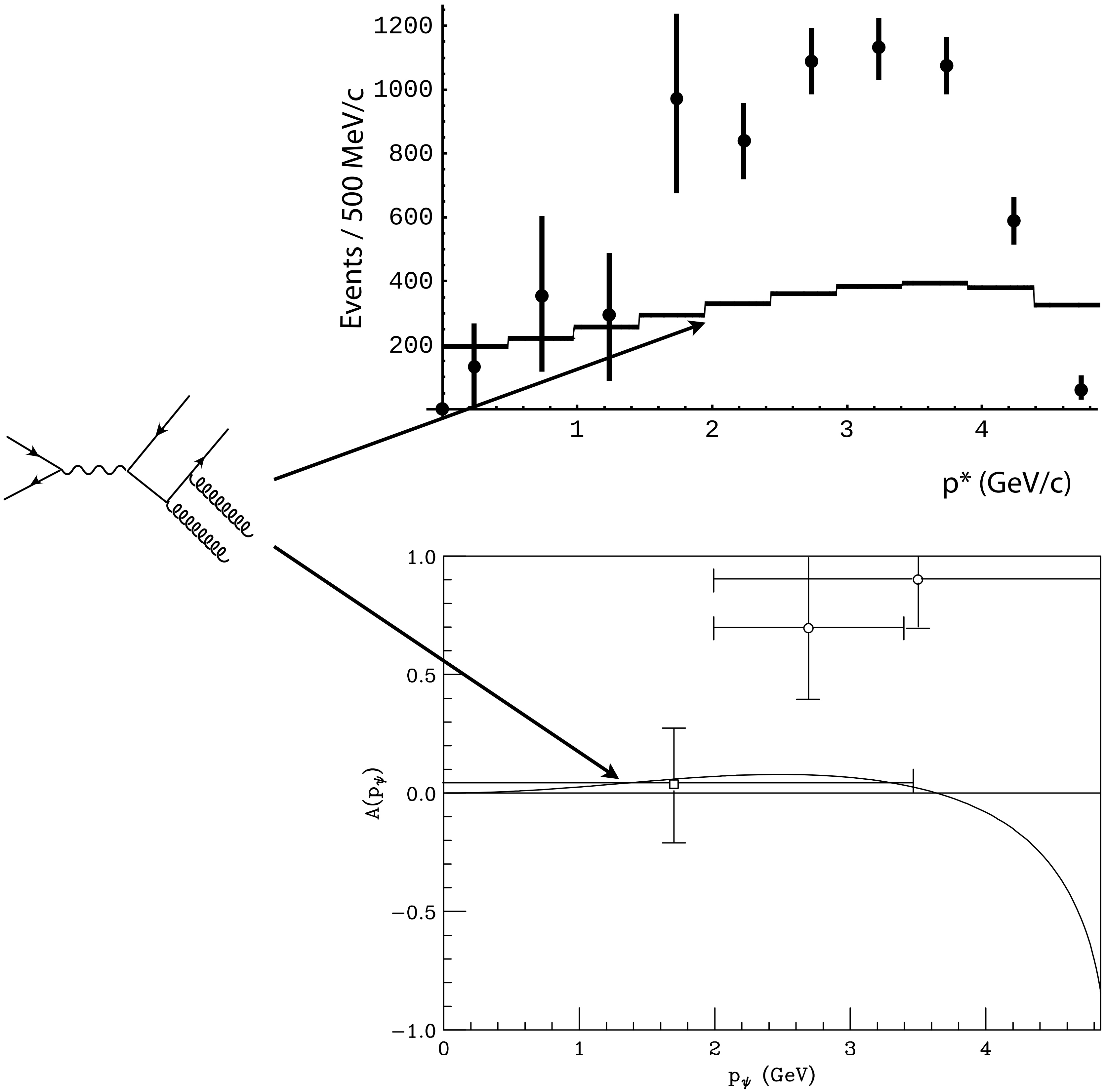
  0805.3876

Motivation $e^+e^- \to J/\psi X$

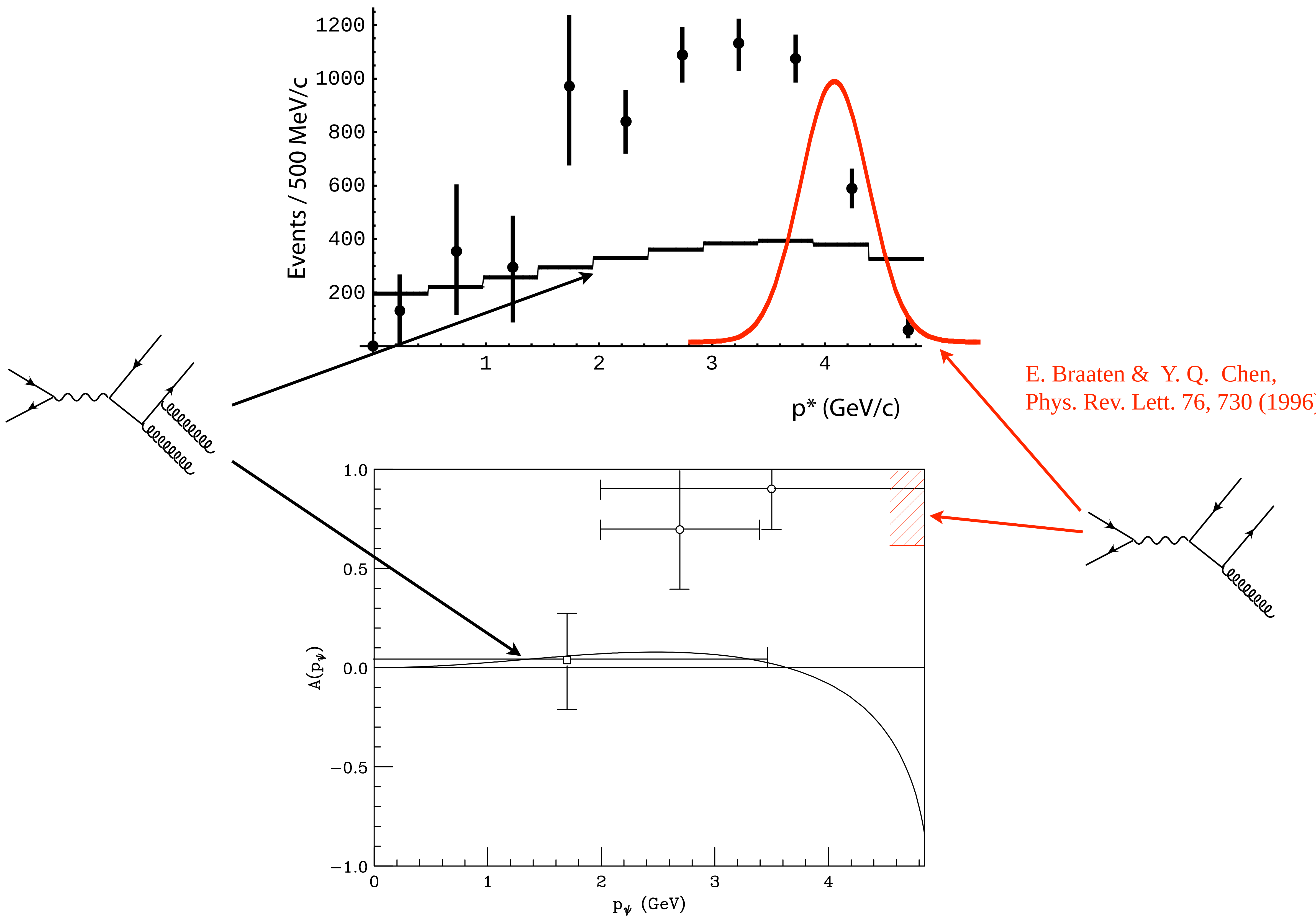
BaBar Data



BaBar and Belle Data

Motivation $e^+e^- \to J/\psi X$

BaBar Data

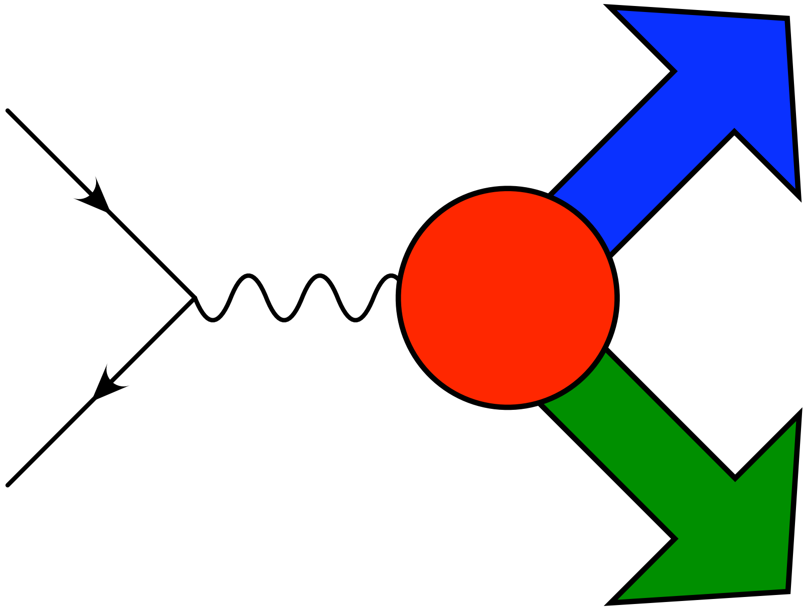


BaBar and Belle Data

Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example

Kinematics

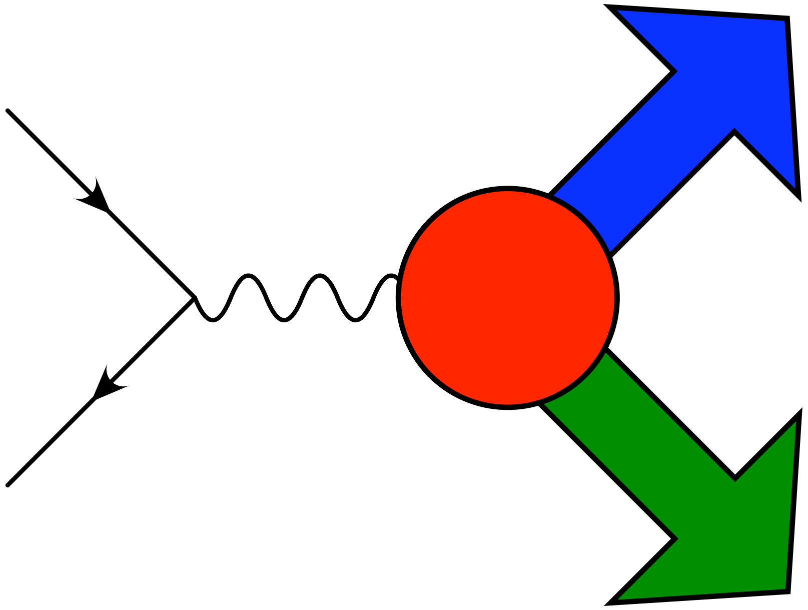


Quarkonium Factorization in SCET

$e^+e^- \longrightarrow J/\psi + X$ as an example

Kinematics

$p^\mu_{c\bar{c}} = Mv^\mu + \ell^\mu$



Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example

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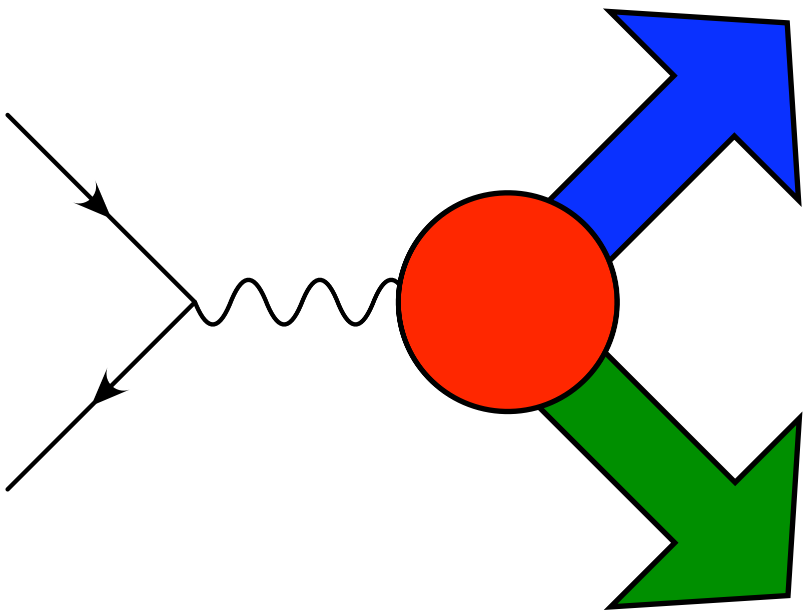


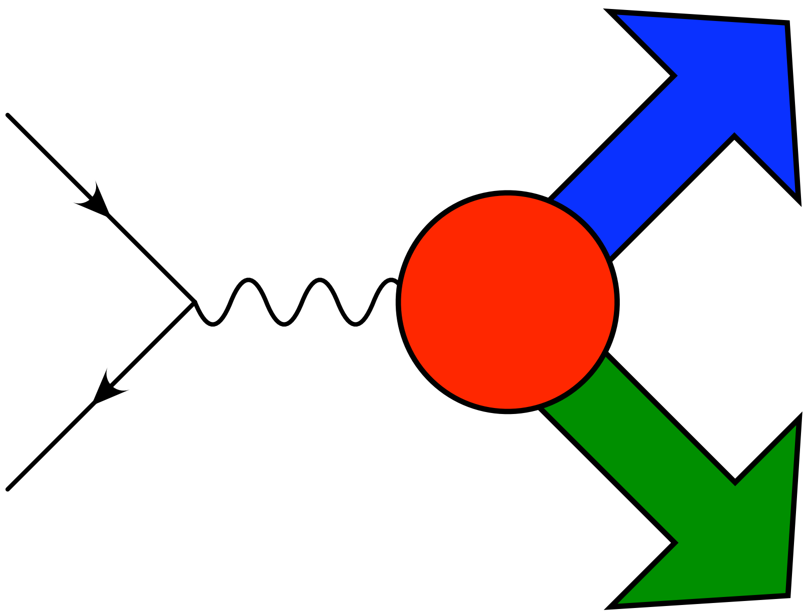
$$p^{\mu}_{c\bar{c}} = Mv^{\mu} + \ell^{\mu} \qquad v^{\mu} = \frac{1}{2}\left(\frac{M_\psi}{x\sqrt{s}}n^{\mu} + \frac{x\sqrt{s}}{M_\psi}\bar{n}^{\mu}\right)$$

Residual momentum of charm pair

$M = 2m_c$

Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example



Kinematics

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Residual momentum of charm pair

$$v^\mu = \frac{1}{2}\left(\frac{M_\psi}{x\sqrt{s}} n^\mu + \frac{x\sqrt{s}}{M_\psi}\bar{n}^\mu\right)$$

$$x = \frac{E_\psi + p_\psi}{\sqrt{s}}, \; \hat{x} = \frac{xM}{M_\psi}$$

$$p^\mu_X = \frac{\sqrt{s}}{2}\left[\left(1 - \frac{M^2}{s\hat{x}}\right) n^\mu + (1-\hat{x})\,\bar{n}^\mu\right] - \ell^\mu$$

Quarkonium Factorization in SCET

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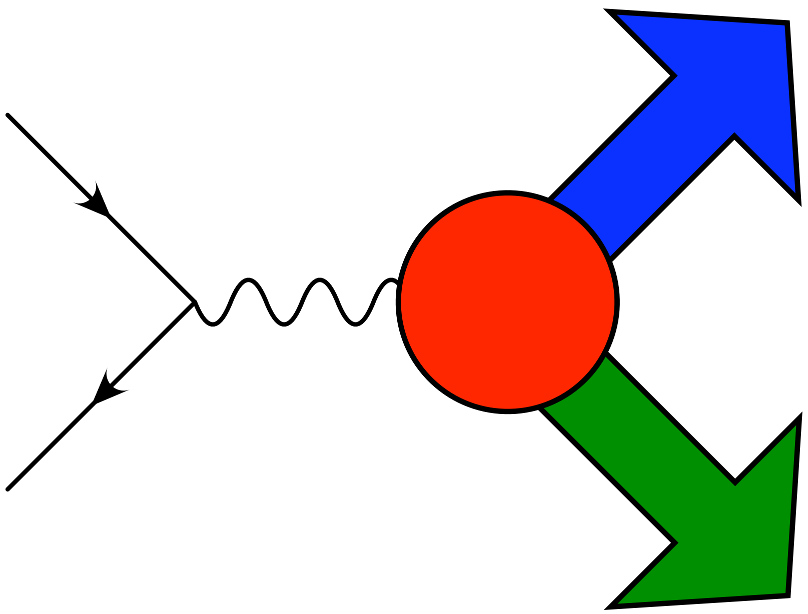

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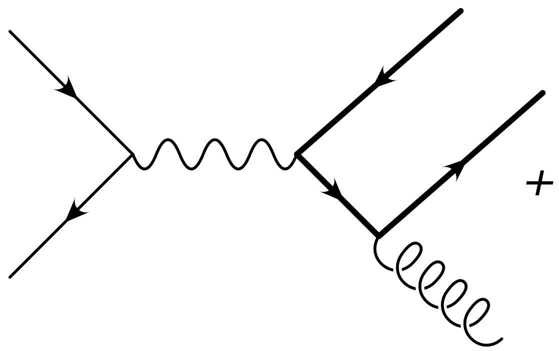
- As $x \to 1, \;\; v^\mu \propto \bar{n}^\mu, \;\; p^\mu_X \propto n^\mu$
  - Need collinear d.o.f. $\Rightarrow$ SCET

Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example



+ *crossed diagram*

Lowest order in α_s is color-octet

Quarkonium Factorization in SCET

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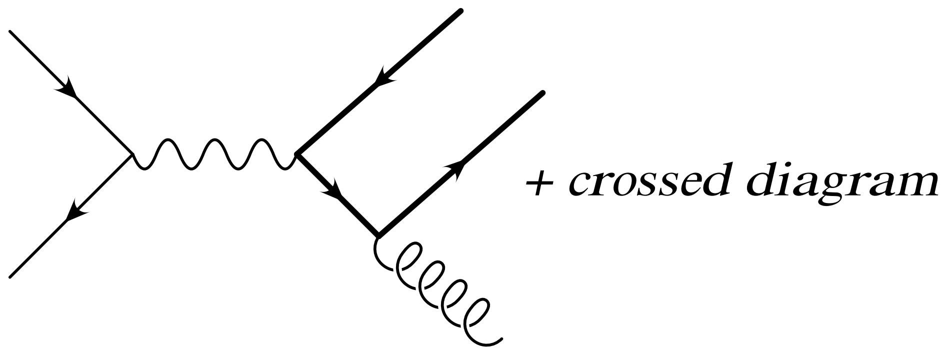

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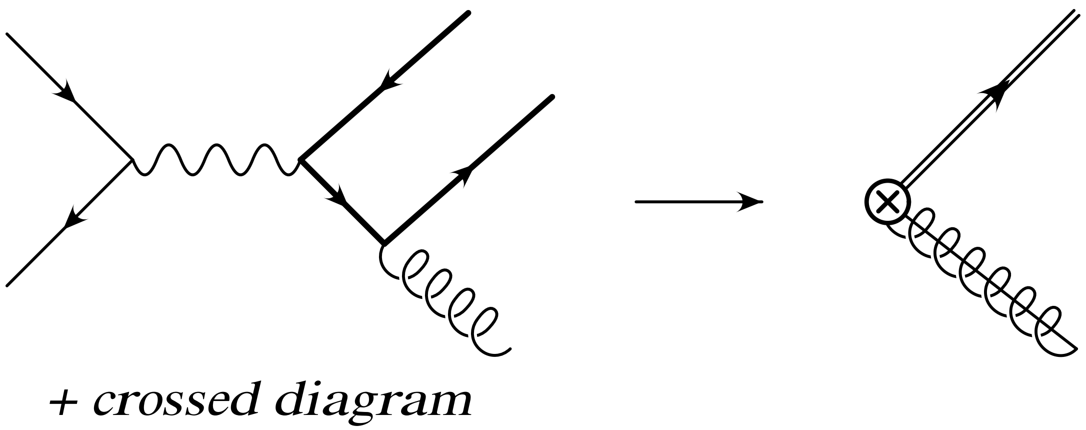
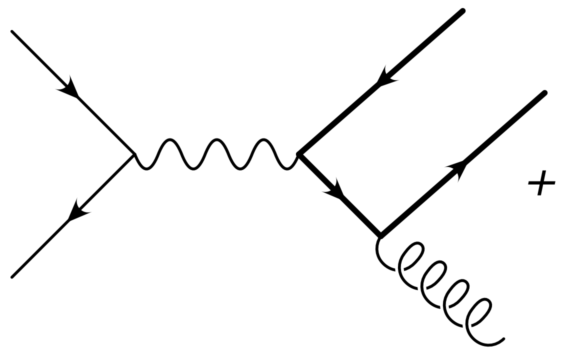
Matching onto NRQCD + SCET



$$J^\dagger_\mu({}^1S_0^{(8)}) = \psi^\dagger \Gamma^{(8,{}^1S_0)}_{\alpha\mu} B^\alpha_\perp \chi$$

Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example



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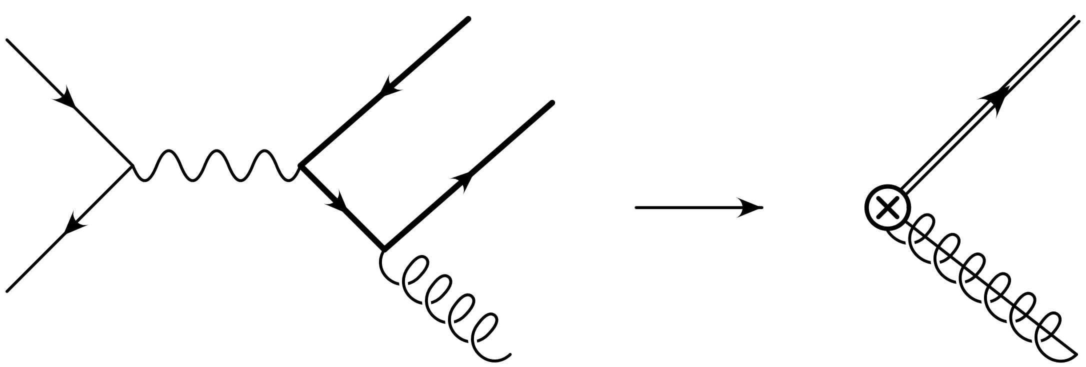

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NRQCD fields

Quarkonium Factorization in SCET

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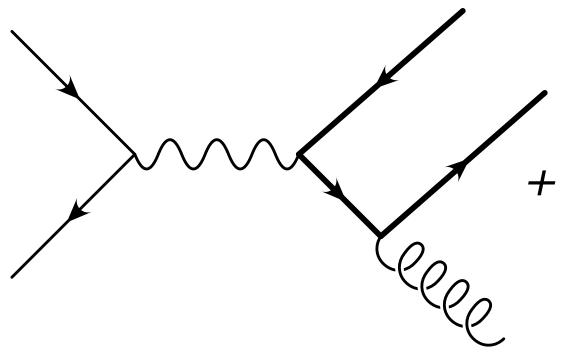

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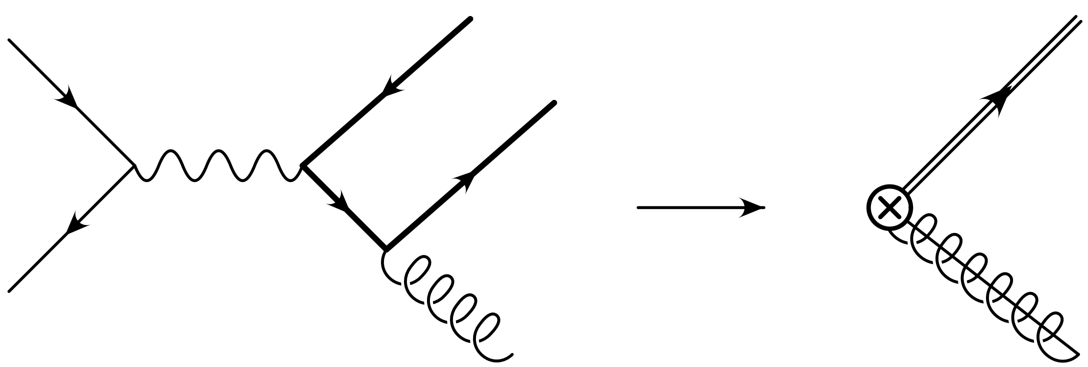

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Invariant collinear gauge field

$$J_\mu^\dagger({}^1S_0^{(8)}) = \psi^\dagger \Gamma_{\alpha\mu}^{(8,{}^1S_0)} B_\perp^\alpha \chi$$

NRQCD fields

Quarkonium Factorization in SCET

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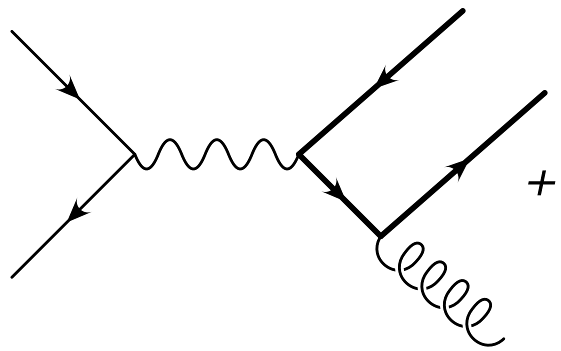

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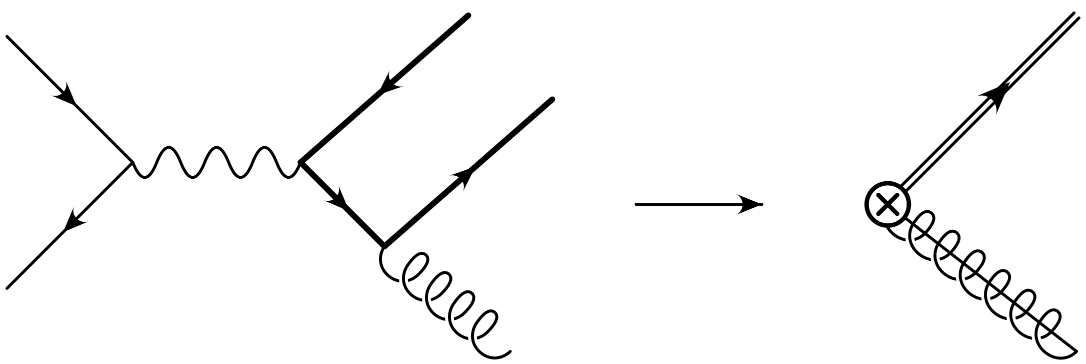

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Invariant collinear gauge field

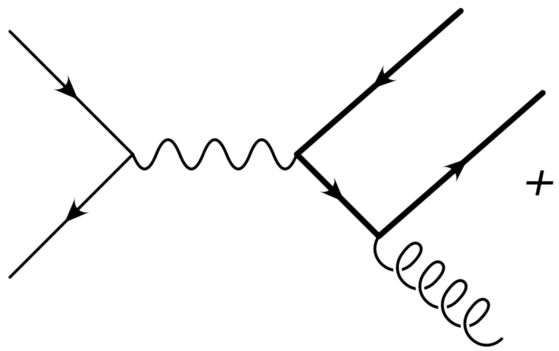
$$J_\mu^\dagger(^1S_0^{(8)}) = \psi^\dagger \Gamma_{\alpha\mu}^{(8,^1S_0)} B_\perp^\alpha \chi$$

NRQCD fields

LO matching gives $\Gamma_{\alpha\mu}^{(8,^1S_0)} = \dfrac{2g_s e e_c}{M}\epsilon_{\alpha\mu}^\perp$

Quarkonium Factorization in SCET

$e^+e^- \to J/\psi + X$ as an example



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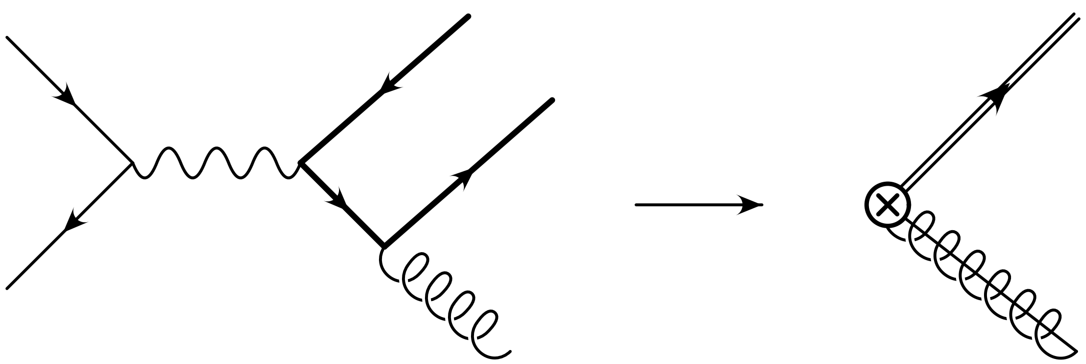
Matching onto NRQCD + SCET



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Invariant collinear gauge field

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$J^\dagger_\mu({}^3P_J^{(8)})$ is similar

Quarkonium Factorization in SCET

Write differential cross section as

$$2E_\psi \frac{d\sigma}{d^3p_\psi} = \frac{e^2}{16\pi^3 s^3} L^{\mu\nu} \int d^4y \, e^{-iq\cdot y} \sum_X \langle 0|J_\nu^\dagger(y)|J/\psi + X\rangle\langle J/\psi + X|J_\mu(0)|0\rangle$$

$$= \frac{e^2}{16\pi^3 s^3} L^{\mu\nu} T_{\mu\nu}$$

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Match QCD current onto SCET + NRQCD

$$J_\mu^\dagger(y) = e^{i(Mv - \bar{\mathcal{P}}n/2)\cdot y} \left[\psi^\dagger \Gamma^{\dagger(8,{}^1S_0)}_{\beta\nu} B_\perp^\beta \chi + \cdots\right]$$

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giving

$$T_{\mu\nu} = \int d^4y\, e^{-i\sqrt{s}/2(1-\hat{x})\bar{n}\cdot y} \Gamma_{\alpha\mu}^{(8,{}^1S_0)} \Gamma_{\beta\nu}^{\dagger(8,{}^1S_0)} \sum_X \langle 0|\chi^\dagger B_\perp^\beta \psi(y)|J/\psi{+}X\rangle\langle J/\psi{+}X|\psi^\dagger B_\perp^\alpha \chi(0)|0\rangle$$
$$= \Gamma_{\alpha\mu}^{(8,{}^1S_0)} \Gamma_{\beta\nu}^{\dagger(8,{}^1S_0)} T_{\text{eff}}^{\alpha\beta}$$

Quarkonium Factorization in SCET

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$$= \Gamma_{\alpha\mu}^{(8,{}^1S_0)}\Gamma_{\beta\nu}^{\dagger(8,{}^1S_0)} T_{\text{eff}}^{\alpha\beta}$$

and do standard decoupling of usofts

$$\psi^\dagger B_\perp^\beta \chi \to \psi^\dagger Y B_\perp^{(0)\beta} Y^\dagger \chi$$

Quarkonium Factorization in SCET

So far have

$$T_{\rm eff}^{\alpha\beta} = \int d^4y\, e^{-i\sqrt{s}/2(1-\hat{x})\bar{n}\cdot y} \sum_X \langle 0|\chi^\dagger Y B_\perp^{(0)\beta} Y^\dagger \psi(y)|J/\psi+X\rangle\langle J/\psi+X|\psi^\dagger Y B_\perp^{(0)\alpha} Y^\dagger \chi(0)|0\rangle$$

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J/ψ does not contain collinear quanta

$$T^{\alpha\beta}_{\rm eff} = \int d^4y\, e^{-i\sqrt{s}/2(1-\hat{x})\bar{n}\cdot y} \langle 0|\chi^\dagger Y T^A Y^\dagger \psi(y) \sum_{X_u} |J/\psi + X_u\rangle\langle J/\psi + X_u|\psi^\dagger Y T^A Y^\dagger \chi(0)|0\rangle$$
$$\times \frac{1}{2}\langle 0|\mathrm{Tr}\left[T^B B_\perp^{(0)\beta}(y)\right] \sum_{X_c} |X_c\rangle\langle X_c|\mathrm{Tr}\left[T^B B_\perp^{(0)\alpha}(0)\right]|0\rangle$$

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Completeness of states in usoft and collinear sectors

$$\sum_{X_u} |J/\psi + X_u\rangle\langle J/\psi + X_u| = a_\psi^\dagger \sum_{X_u} |X_u\rangle\langle X_u| a_\psi = a_\psi^\dagger a_\psi, \quad \sum_{X_c} |X_c\rangle\langle X_c| = 1$$

Quarkonium Factorization in SCET

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Define jet and shape functions

Quarkonium Factorization in SCET

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get $\delta(n\cdot y)$ and $\delta^2(y_\perp)$

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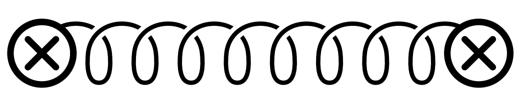

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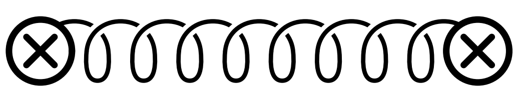


get $\delta(n\cdot y)$ and $\delta^2(y_\perp)$

$$S^{(8,{}^1S_0)}(\ell^+) = \int \frac{dy^-}{4\pi} e^{-i\ell^+ y^-/2} \frac{\langle 0|\chi^\dagger Y T^A Y^\dagger \psi(y^-) a_\psi^\dagger a_\psi \psi^\dagger Y T^A Y^\dagger \chi(0)|0\rangle}{2M\langle \mathcal{O}_8^\psi({}^1S_0)\rangle}$$

Quarkonium Factorization in SCET

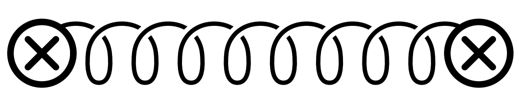
So far have

$$T^{\alpha\beta}_{\rm eff} = \int d^4y\, e^{-i\sqrt{s}/2(1-\hat{x})\bar{n}\cdot y} \langle 0|\chi^\dagger Y T^A Y^\dagger \psi(y) a^\dagger_\psi a_\psi \psi^\dagger Y T^A Y^\dagger \chi(0)|0\rangle$$

$$\times \frac{1}{2}\langle 0|\mathrm{Tr}\left[T^B B_\perp^{(0)\beta}(y)\right] \mathrm{Tr}\left[T^B B_\perp^{(0)\alpha}(0)\right]|0\rangle$$

Define jet and shape functions

$$\langle 0|\mathrm{Tr}\left[T^B B_\perp^{(0)\beta}(y)\right] \mathrm{Tr}\left[T^B B_\perp^{(0)\alpha}(0)\right]|0\rangle = -\frac{4\pi g_\perp^{\alpha\beta}}{s+M^2}\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot y} J(\bar{n}\cdot p\, n\cdot k + p_\perp^2)$$



get $\delta(n\cdot y)$ and $\delta^2(y_\perp)$

$$S^{(8,{}^1S_0)}(\ell^+) = \int \frac{dy^-}{4\pi} e^{-i\ell^+ y^-/2} \frac{\langle 0|\chi^\dagger Y T^A Y^\dagger \psi(y^-) a^\dagger_\psi a_\psi \psi^\dagger Y T^A Y^\dagger \chi(0)|0\rangle}{2M\langle \mathcal{O}_8^\psi({}^1S_0)\rangle}$$

Combine everything to get ...

Quarkonium Factorization in SCET

...after some change of variables

$$\frac{d\sigma^{(8,{}^1S_0)}}{dz} = \sigma_0^{(8,{}^1S_0)} P[r,z] \int_z^1 d\xi \, S^{(8,{}^1S_0)}(\xi) J(s(1-r)(\xi - z))$$

Kinematic factor

$$P[r,z] = \frac{\sqrt{(1+r)^2 z^2 - 4r}}{1-r}$$

$$r = \frac{M^2}{s}$$

Quarkonium Factorization in SCET

...after some change of variables

$$\frac{d\sigma^{(8,{}^1S_0)}}{dz} = \sigma_0^{(8,{}^1S_0)} P[r,z] \int_z^1 d\xi\, S^{(8,{}^1S_0)}(\xi) J(s(1-r)(\xi - z))$$

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- Rate factorized

Quarkonium Factorization in SCET

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Kinematic factor

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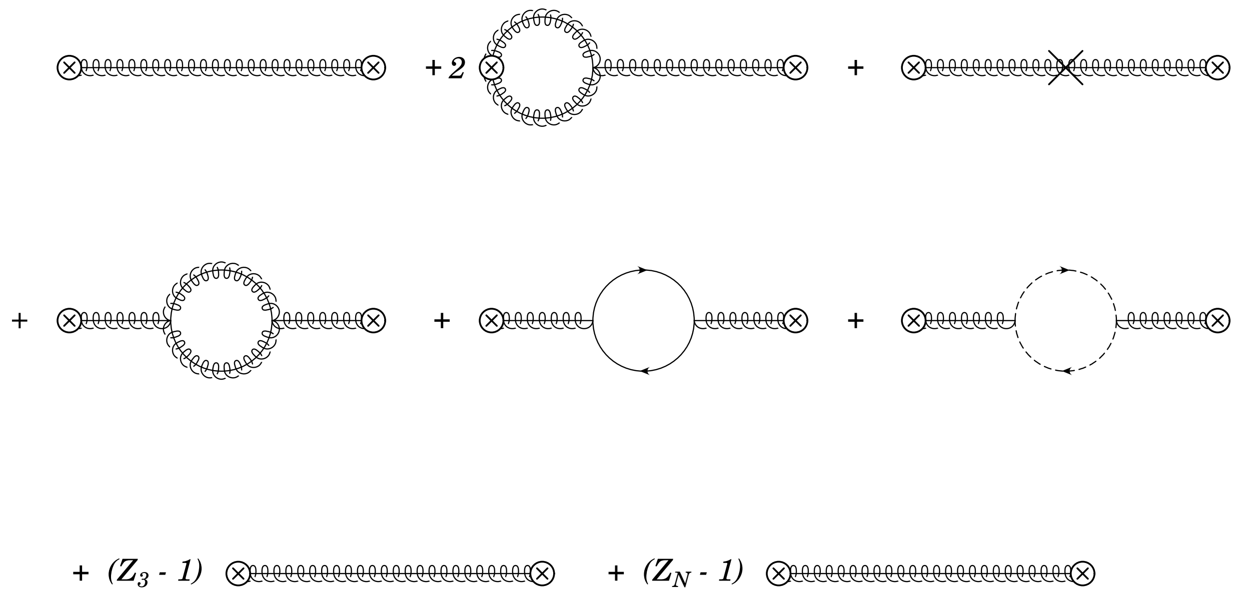
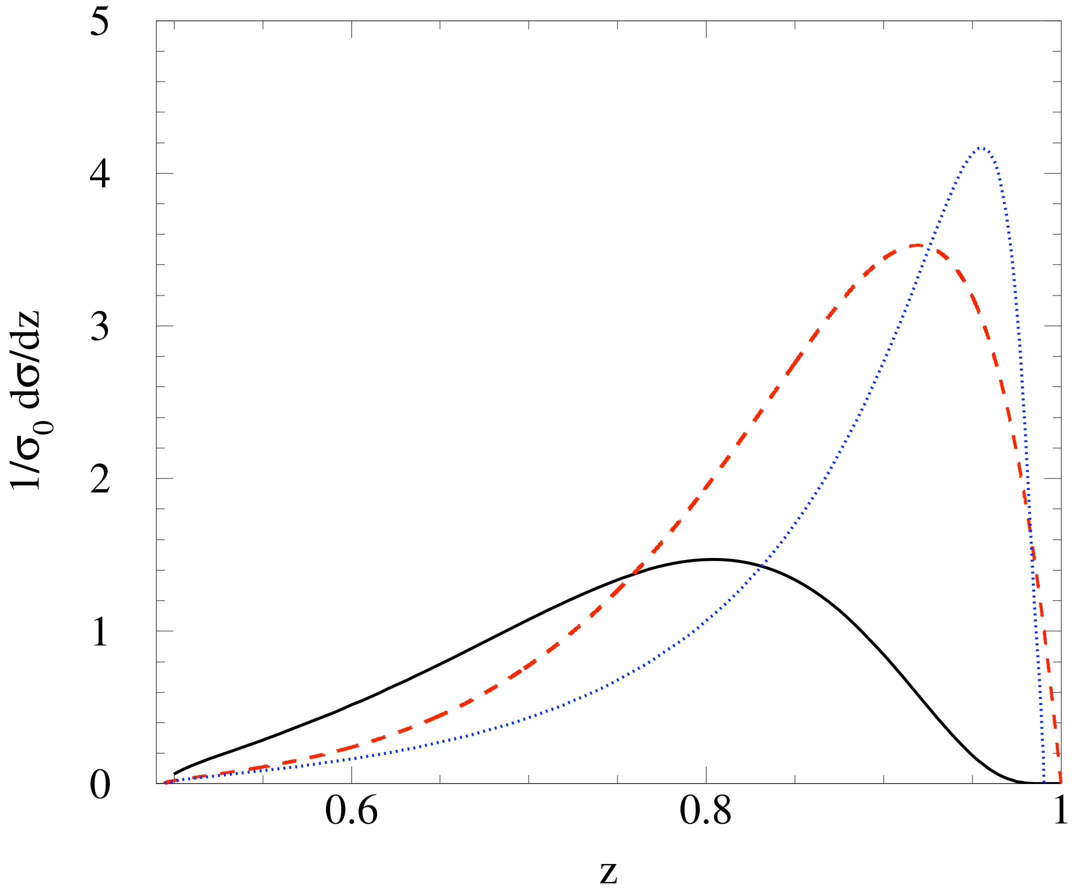
- Rate factorized
- Octet ${}^3P_J$ similar

Quarkonium Factorization in SCET

Can now run



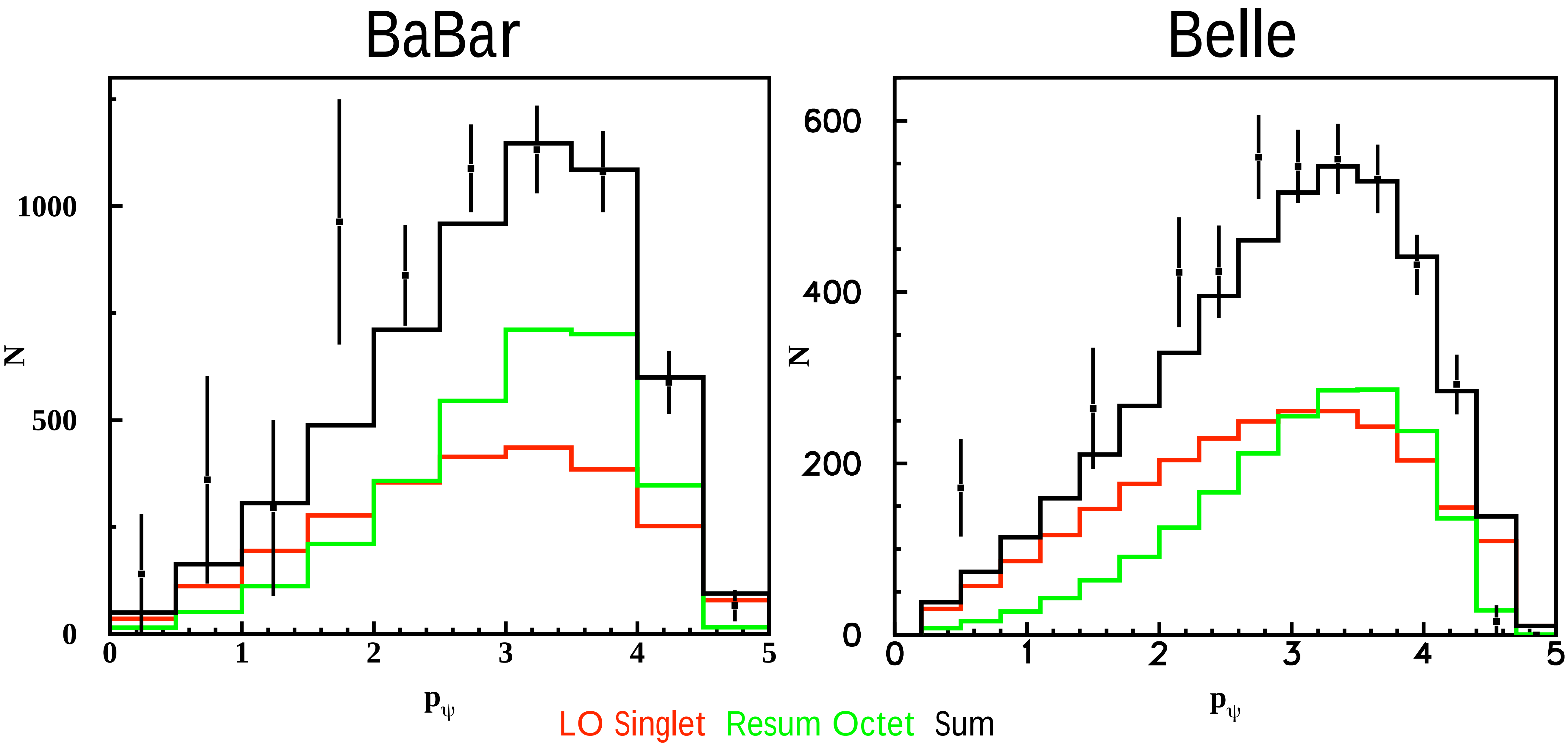
and model the shape function



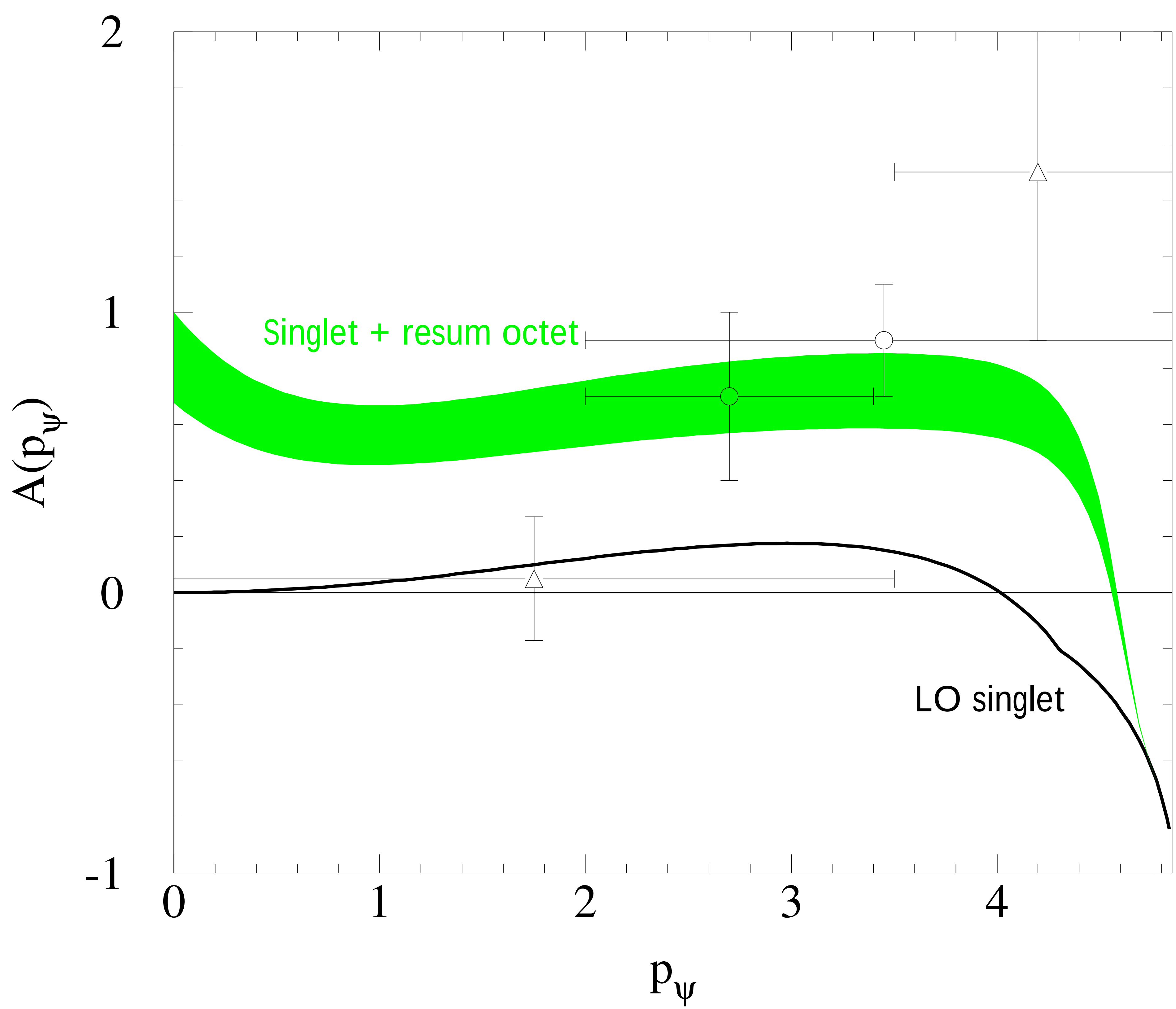
$$e^+e^- \to J/\psi X$$

After resummation and model of shape function

$$S(x) = \frac{1}{\bar{\Lambda}} \frac{a^{ab}}{\Gamma(ab)} (x-1)^{ab-1} e^{-a(x-1)}, \qquad x = \frac{k^+}{\bar{\Lambda}}$$



$e^+e^- \to J/\psi X$



Open Questions

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  - Endpoint resummation, soft functions

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Open Questions

- Factorization in hadroproduction
  - Endpoint resummation, soft functions
- Large number of exclusive processes
- How important are color-octet channels
- Treating charm quarks as collinear in SCET
- Can we solve all of NRQCD's problems

Color-octet $\gamma p \to J/\psi X$

hep-ph/0607121

Start with

$$2E_\psi \frac{d\sigma}{d^3 p_\psi} = \int d^3\boldsymbol{\omega}_1 \int d^3\boldsymbol{\omega}_2 \frac{-C^\dagger_{\beta\mu} C^\mu_\alpha}{32\pi^3 s} \sum_{X_n, X_s} (2\pi)^4 \delta^{(4)}(p_\gamma + p_P - p_\psi - p_s - p_n)$$
$$\times \frac{1}{2} \sum_{\text{spin}} \langle p_P | J^{\beta\dagger}(\boldsymbol{\omega}_1, 0) | J/\psi + X_s + X_n \rangle \langle J/\psi + X_s + X_n | J^\alpha(\boldsymbol{\omega}_2, 0) | p_P \rangle$$

where the J^α are the matched SCET+NRQCD currents

Color-octet $\gamma p \to J/\psi X$

hep-ph/0607121

After decoupling soft and collinear

$$
\begin{aligned}
2E_\psi \frac{d\sigma}{d^3 p_\psi} \;=\; & \int d^3\boldsymbol{\omega}_1 \int d^3\boldsymbol{\omega}_2 \frac{-C^\dagger_{\mu\beta} C^\mu_\alpha}{32\pi^3 s} \sum_{X_n, X_s} 2(2\pi)^4 \delta\big(\sqrt{s}(1-z) - n\cdot p_n - n\cdot p_s\big) \\
\times \; & \delta\Big(\frac{M_\psi^2}{z\sqrt{s}} - \bar{\omega}_2\Big) \delta^{(2)}(p_\perp - \omega_{2\perp}) \\
\times \; & 2 \sum_{\text{spin}} \langle p_P | \mathrm{Tr}\big[T^A B_\perp^\alpha(0)\big]\, \delta^{(3)}(\vec{\mathcal{P}}^\dagger - \boldsymbol{\omega}_2) | X_n \rangle \langle X_n | \delta^{(3)}(\vec{\mathcal{P}} - \boldsymbol{\omega}_1)\, \mathrm{Tr}\big[T^B B_\perp^\beta(0)\big] | p_P \rangle \\
\times \; & \langle 0 | \chi^\dagger_{-\mathbf{p}'} \tilde{Y} T^A \tilde{Y}^\dagger \psi_{\mathbf{p}'}(0) | J/\psi + X_s \rangle \langle J/\psi + X_s | \psi^\dagger_{\mathbf{p}} \tilde{Y} T^B \tilde{Y}^\dagger \chi_{-\mathbf{p}}(0) | 0 \rangle
\end{aligned}
$$

Color-octet $\gamma p \to J/\psi X$

hep-ph/0607121

Use delta function to shift position, and use completeness

$$\frac{d\sigma}{dz d^2 p_\perp} = \frac{-1}{16 s z}\left(\frac{2 e e_c g_s}{M}\right)^2 \int d^3\boldsymbol{\omega}_+ \int \frac{dx^-}{2}\, e^{\frac{i}{2}\sqrt{s}(1-z)x^-}\, \delta\Big(\frac{\bar{\omega}_+}{2} - \frac{M_\psi^2}{z\sqrt{s}}\Big)\, \delta^{(2)}\Big(p_\perp - \frac{\omega_{+\perp}}{2}\Big)$$
$$\times \frac{1}{2}\sum_{\text{spin}} \langle p_P | \mathrm{Tr}\big[B_\perp^\nu(x^-)\, \delta^{(3)}(\vec{\mathcal{P}}_+ - \boldsymbol{\omega}_+) B_{\perp\nu}(0)\big] | p_P\rangle$$
$$\times \langle 0 | \chi^\dagger_{-\mathbf{p}'} \tilde{Y} T^A \tilde{Y}^\dagger \psi_{\mathbf{p}'}(x^-) \mathcal{P}_\psi \psi^\dagger_{\mathbf{p}} \tilde{Y} T^A \tilde{Y}^\dagger \chi_{-\mathbf{p}}(0) | 0\rangle$$

Defining functions (soft and “jet”)
and integrate over everything possible

$$S^{(8,{}^1S_0)}(\ell^+) \equiv \int \frac{dx^-}{4\pi}\, e^{-\frac{i}{2}\ell^+ x^-} \frac{\langle 0 | \chi^\dagger_{-\mathbf{p}'} \tilde{Y} T^A \tilde{Y}^\dagger \psi_{\mathbf{p}'}(x^-) \mathcal{P}_\psi \psi^\dagger_{\mathbf{p}} \tilde{Y} T^A \tilde{Y}^\dagger \chi_{-\mathbf{p}}(0) | 0\rangle}{2M \langle \mathcal{O}_8^{J/\psi}({}^1S_0)\rangle}$$

$$\mathcal{J}_P(k^+) \equiv \int \frac{dy^-}{4\pi} e^{\frac{i}{2}k^+ y^-} \frac{1}{2}\sum_{\text{spin}} \langle p_P | \mathrm{Tr}\big[B_\perp^\nu(y^-)\, \delta\Big(\bar{\mathcal{P}} - \frac{2M_\psi^2}{\sqrt{s}}\Big) B_{\perp\nu}(0)\big] | p_P\rangle$$

Color-octet $\gamma p \to J/\psi X$

hep-ph/0607121

- Factorization formula for the endpoint, $z \to 1$ where $z = \frac{p_\psi \cdot p_P}{p_\gamma \cdot p_P}$

Gluon pdf

$$\rho = \frac{M^2}{s}$$

$$\frac{d\sigma}{dz} = \sigma_0 \rho |C_V(\bar{\mu})|^2 \int dk^+ S^{(8,{}^1S_0)}(-\sqrt{s}(1-z)+k^+) \underbrace{\int_\rho^1 \frac{d\xi}{\xi} C_{II}\left(\frac{\rho}{\xi}, k^+\right) f_{g/P}(\xi)}$$

Same shape function as $e^+e^- \to J/\psi + X$

show this by taking moments

$$\int \frac{dy^-}{4\pi} e^{ik^+y^-/2} \frac{1}{2} \sum_{spin} \langle p | Tr[B_\perp^\nu(y^-) \delta\left(\bar{P} - \frac{2M_\Psi^2}{\sqrt{s}}\right) B_{\perp\nu}(0)] | p \rangle$$

Color-octet $\gamma p \to J/\psi X$

- Resummation of logs leads to

$$\frac{d\sigma}{dz} = \rho\,\sigma_0 \int_z^1 \frac{du}{u} \hat{S}^{(8,{}^1S_0)}(z/u) \left(-u\frac{d}{du}\left\{\Theta(1-u)\frac{e^{lg_1(l)+g_2(l)}}{\Gamma[1-g_1(l)-lg_1'(l)]}\, f_{g/P}(\rho; M\sqrt{1-u})\right\}\right)$$

$$\rho = \frac{M^2}{s}$$

Logs are here

Convolution with pdf