The NLO Cascade

SCET09

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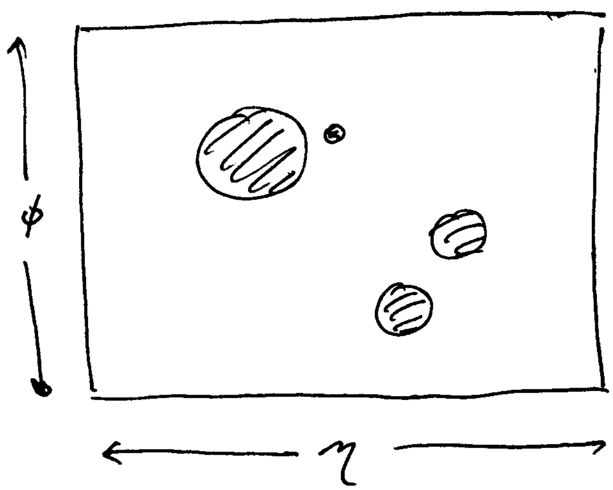
Motivation ~ LHC!   BSM ≃ SM   jets!



2 jet?

3 jet?

4 jet?

Measurements <u>define</u> Calculational Method

Physically: shouldn't phase space just have σ?

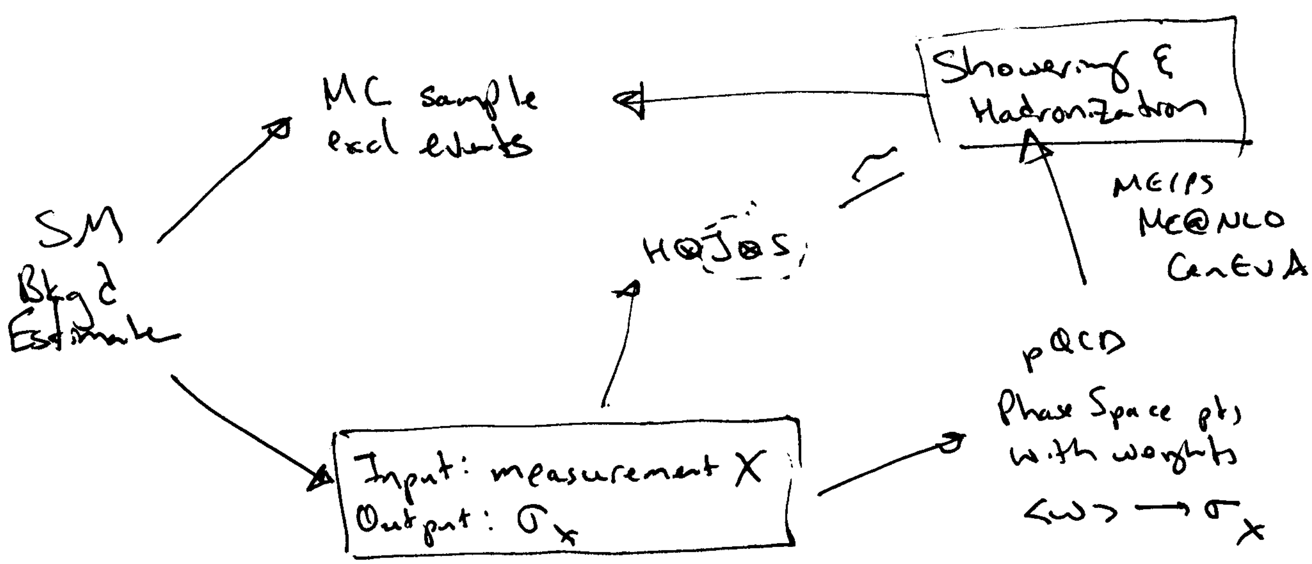
NLO Cascade

For any measurement X:

1) If σ_X known to NLO, get σ_X to NLO
2) σ_X includes LL resummation
3) No gratuitous numeric integrals!

Outline

1) The essential problem
2) What is a measurement?
3) MC measurements
4) The NLO Cascade ~~on~~ on computer



↳ key to NLO cascade

The Essential Problem

LO Cascade (ME/PS Matching)

	α_s^0	α_s^1	α_s^2
2	tree $\xrightarrow{\text{Sudakov}(\mu)}$		
3		tree $\Theta(\mu)$ $\xrightarrow{\text{Sudakov}}$	
4			tree $\Theta(\mu)$ $\rightarrow$

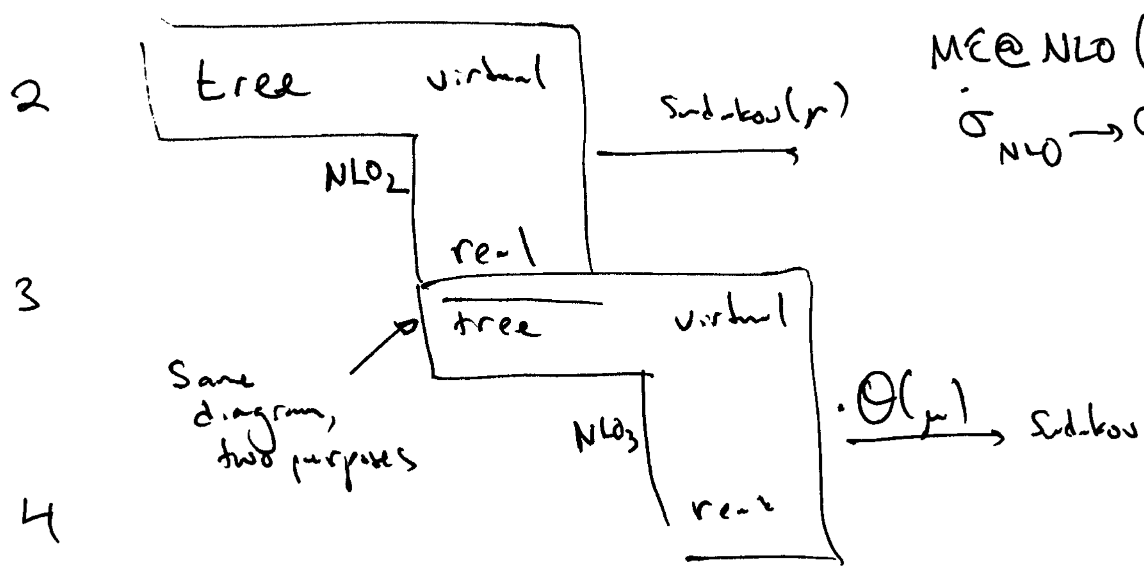
μ

$$\sigma_n = \sigma_{tree} \cdot \Delta_{Sudakov}(\mu) = \sigma_{tree} + \mathcal{O}(\alpha_s)$$

μ is vague

NLO Cascade

α_s^0 α_s^1 α_s^2



ME@NLO (POWHEG)

$\sigma_{NLO} \rightarrow \sigma_{NLO/LL}$

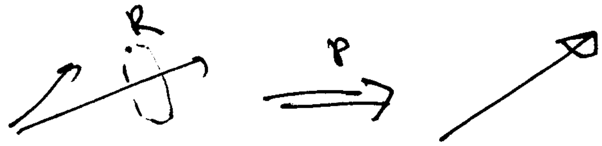
~~$g\mu_1$~~ μ is very specific

n vs. $n+1$ is a computational problem.

What is a measurement?

$$\sigma_X = \int\!\!\!\!\!\sum_k \sigma_k^{\text{parton}}(\Phi_k)\, X(\Phi_k)$$

$$\sigma_{n\text{-jet}} = \int\!\!\!\!\!\sum_k \sigma_k^{\text{parton}}\, \overbrace{P_n(\Phi_k \to \Phi_n)\, R_n(\Phi_k)}^{\text{Jet Algorithm}}$$



Cancel IR Divergences.

$$\sigma_n = \underbrace{\underset{\downarrow}{V_n}}_{\infty} \ldots$$

$$\sigma_n = B_n + V_n + \int d\Phi_{n+1}\, \cancel{R_n}\overset{B_{n+1}(\Phi_{n+1})}{}(\Phi_{n+1})\, J_{n+1\to n}$$

$V_n \to \infty$, $\int d\Phi_{n+1} B_{n+1} J_{n+1\to n} \to \infty \to 1.8 \times 10^{308}$; together: finite.

Subtractions.

$$\sigma_n = B_n + \underbrace{V_n + \int d\Phi_{n+1} S_{n+1} J_s}_{\text{finite}} + \underbrace{\int d\Phi_{n+1} \left[B_{n+1} J - S_{n+1} J_s\right]}_{\text{finite}}$$

$\int d\Phi_{n+1} S_{n+1} J_s$: has all singularities.

At NLO, with subtractions, measurement well-defined and calculable on computer.

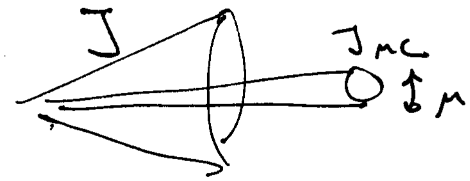
MC Measurements

Separate calculation for every J?

Cancel IR div. once and for all (Resum LL, too)

$\sigma_k^{MC}(\mu)$ ← finite by themselves.

$$\sigma_X = \sum_k \cancel{\sigma_k^{parton}}\, X(\Phi_k)$$



$$\left[\sigma_n^{MC}(\mu)\right]_{NLO} = B_n + V_n + \underbrace{\int B_{n+1}\, J_{MC}(\mu)}_{\text{gni!}}$$

$$\left[\sigma_{n+1}^{MC}(\mu)\right]_{LO} = B_{n+1}\, \bar{J}_{MC}(\mu)$$

$$\cancel{[}\sigma_n^{jet}\cancel{]_{NLO}} = \sigma_n^{MC}(\mu) + \int \sigma_{n+1}^{MC}(\mu)\, J$$

$$\sigma_{n+1}^{jet} = \sigma_{n+1}^{MC}(\mu) \simeq B_{n+1} \text{ in 3-jet region}$$

$$J_{MC} = P \cdot R \qquad \bar{J}_{MC} = 1 - R \equiv \bar{R}$$

Calculability

stand in for real emission up to power corrections.

$$[\sigma_n^{MC}(\mu)] = B_n + \underbrace{V_n + \int S_{n+1} J_S^{(\mu)}}_{\equiv V_n^S(\mu)} + \underbrace{\int B_{n+1} J_{MC}(\mu) - S_{n+1} J_S^{(\mu)}}_{\mathcal{J}^{ni}}$$

$$\mathcal{J}^{ni} \to \mathcal{O}(\mu/Q)$$

If $J_{MC} = J_S$

$\cancel{\mathcal{J}^{ni}} \Longrightarrow$ small μ $\Longrightarrow$ large μ/Q logs

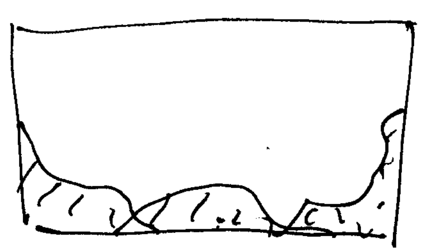
must resum!
(no harder than ME/PS)

Key subtlety:

$$J_S = \sum_i w_i P_i \cdot R_i$$

$$\bar{J}_S = \sum_i w_i (1 - R_i)$$

$$[\sigma^{MC}_{n+1}]_{LO} = B_{n+1} \sum_i w_i \bar{R}_i$$



Φ_{n+1}

Mess this up and get leading logs.

NLO Cascade

The final answer!

$$\frac{\mathrm{d}\sigma_n^{\mathrm{cascade}}(\mu)}{\mathrm{d}\Phi_n} \equiv \sum_i \left[\frac{\mathrm{d}\sigma_n^{\mathrm{cascade}}(\mu)}{\mathrm{d}\Phi_n}\right]_i \big(1 - R_n^i(\Phi_n, \mu)\big)$$

$$\left[\frac{\mathrm{d}\sigma_n^{\mathrm{cascade}}(\mu)}{\mathrm{d}\Phi_n}\right]_i = \left(\frac{\mathrm{d}\sigma_{n-1}^{\mathrm{cascade}}(t_n^i)}{\mathrm{d}\Phi_{n-1}^i}\, Q_{n-1\to n}^i(\Phi_{n-1\to n}^i) + M_n^i(\Phi_n)\right) \Delta_n^Q(t_n^i, \mu)$$

$$Q_{n-1\to n}^i(\Phi_{n-1}^i) = w_n^i(\Phi_n)\frac{S_n(\Phi_n)}{B_{n-1}(\Phi_{n-1}^i)}$$

$$\Delta_n^Q(a,b) = \exp\left[-\sum_i \int d\Phi_{n\to n+1}^i\, Q_{n\to n+1}^i(\Phi_{n\to n+1}^i)\, \Theta(a < t_{n+1}^i < b)\right]$$

$$M_n^{i,(0)}(\Phi_n) \quad = \quad w_n^i(\Phi_n)\,[B_n(\Phi_n) - S_n(\Phi_n)]$$

$$M_n^{i,(1)}(\Phi_n) \quad = \quad w_n^i(\Phi_n) V_n^S(\Phi_n, t_n^i) - V_{n-1}^S(\Phi_{n-1}^i, t_n^i) Q_{n-1\to n}^i(\Phi_{n-1\to n}^i)$$

$$\sigma_n^{\mathrm{cascade}}(\mu) \equiv \sum_i \left[\sigma_n^{\mathrm{cascade}}(\mu)\right]_i \left(1 - R_n^i\right)$$

$$\left[\sigma_n^{\mathrm{cascade}}(\mu)\right]_i = \left(\sigma_{n-1}^{\mathrm{cascade}}(t^i)\, Q_{n-1\to n}^i + M_n^i\right) \Delta_n^Q(t^i, \mu)$$

$$Q_{n-1\to n}^i = w_n^i \frac{S_n}{B_{n-1}}$$

$$M_n^{i,(0)} = w_n^i \left(B_n - S_n\right)$$

$$M_n^{i,(1)} = w_n^i V_n^S - V_{n-1}^S Q_{n-1\to n}^i$$

$$\sigma_2^{\mathrm{MC}}(\mu) = M_2 \Delta_2(Q,\mu)$$

$$\begin{aligned}
\left[\sigma_2^{\mathrm{MC}}(\mu)\right]^{(0)} &= M_2^{(0)} &&= B_2 \\
\left[\sigma_2^{\mathrm{MC}}(\mu)\right]^{(1)} &= M_2^{(1)} + M_2^{(0)}\Delta_2^{(1)} &&= V_2^S(\mu)
\end{aligned}$$

$$M_2^{(0)} = B_2 \qquad M_2^1 = V_2^S(Q)$$

$$\sigma_3^{\mathrm{MC}}(\mu) = \left(\sigma_2^{\mathrm{MC}}(t^A) Q^A_{2\to 3} + M_3^A\right) \Delta_3(t^A,\mu)\overline{R_A} + (A \to B)$$

$$Q^A_{2\to 3} = w_A \frac{S_3}{B_2}$$

$$\left[\sigma_3^{\mathrm{MC}}(\mu)\right]^{(0)} = \left(\sigma_2^{\mathrm{MC}}(t^A)^{(0)} Q^A_{2\to 3} + M_3^{A,(0)}\right)\overline{R_A} + (A \to B)$$

$$= B_3 w_A \overline{R_A} + (A \to B)$$

$$M_3^{A,(0)} = w_A (B_3 - S_3)$$

$$\sigma_3^{\mathrm{MC}}(\mu) = \left(\sigma_2^{\mathrm{MC}}(t^A) Q^A_{2\to 3} + M_3^A\right) \Delta_3(t^A, \mu) \overline{R_A} + (A \to B)$$

$$\begin{aligned}
\left[\sigma_3^{\mathrm{MC}}(\mu)\right]^{(1)} \;=\; & \left(\sigma_2^{\mathrm{MC}}(t^A)^{(1)} Q^A_{2\to 3} + M_3^{A,(1)}\right) \overline{R_A} \\
& + \left(\sigma_2^{\mathrm{MC}}(t^A)^{(0)} Q^A_{2\to 3} + M_3^{A,(0)}\right) \overline{R_A} \Delta_3^{(1)} \\
& + (A \to B)
\end{aligned}$$

$$= V_3^S(\mu) w_A \overline{R_A} + (A \to B)$$

$$M_3^{A,(1)} = w_A V_3^S(t^A) - V_2^S(t^A) Q^A_{2\to 3}$$