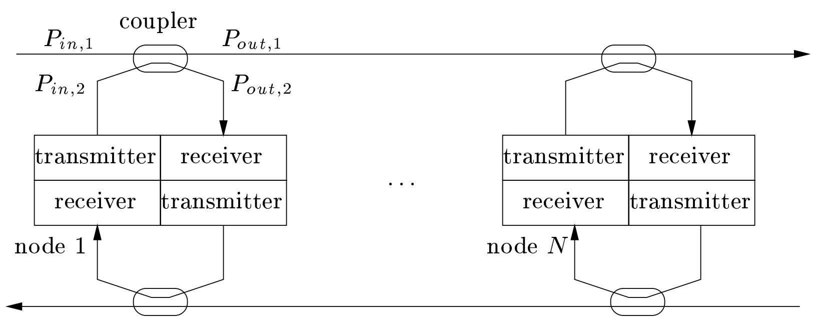
6.972 Optical Networks — 10 October 2001

MIT, Fall 2001 — Due date: 17 October 2001

Problem set #3

Problem 3.1 (Double bus): Consider the double bus architecture shown in figure 3.1.



Double bus

For each coupler, the relationship between input and output power is given by

$$\begin{bmatrix} P_{out,1} \\ P_{out,2} \end{bmatrix} = \gamma \begin{bmatrix} 1-\alpha & \alpha \\ \alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} P_{in,1} \\ P_{in,2} \end{bmatrix}.$$

Assume that we can choose the value α. Find the optimal value of α which minimizes the worst-case power loss among all pairs of nodes. Show that the worst-case power loss in the limit as $N \to \infty$ is given by $\frac{4}{N^2} e^{-2} \gamma^N$.

Problem 3.2 (Slotted Aloha for WDM networks):

(i) Do problem 7.7 in the book [RS98,p.324].

(ii) Do problem 7.8 in the book [RS98,p.324].

(iii) Do parts (a), (d), and (e) of problem 7.11 in the book [RS98,p.324].

Problem 3.3 (Maximum throughput of unslotted Aloha): Consider the use of unslotted Aloha for the sharing of a single WDM channel. Assume the following.

1. All packets are 1 time unit long.
2. There are infinite number of users, and each new packet arrives at a new user. (This assumption allows the derivation of the upper bound on the average packet delay. For 6.972, the derivation of the average packet delay is beyond our scope.)
3. Arrivals of new packets form a Poisson process of rate λ.
4. After receiving a new packet, each user transmits immediately.
5. Two packet collide when their transmission times overlap.
6. A collided packet is backlogged and is retransmitted after a random time which is exponentially distributed with mean $1/r$.

It follows that the initiation times of attempted transmissions form a nonhomogeneous Poisson process of instantaneous rate $G(n) = \lambda + nr$, where n is the backlog at a given time.

(a) Let T_i be the duration between the initiations of the i^{th} and the $(i+1)^{\text{th}}$ transmission attempts. The i^{th} transmission attempt is successful if both T_{i-1} and T_i exceed 1. Assuming a backlog of n in both intervals, argue that the i^{th} transmission is successful with probability $P_{succ} = e^{-2G(n)}$.

However, the backlog in T_i may be smaller than the backlog in T_{i-1} by 1. But for small r, we can approximate $P_{succ} \approx e^{-2G(n)}$.

(b) The throughput $\gamma(n)$ is defined as the expected number of successful transmissions per unit time. Based on the approximation of P_{succ} in part (a), find the expression for $\gamma(n)$ in terms of $G(n)$ and show that the maximum throughput of $1/(2e)$ is achieved when $G(n) = 1/2$.