Factorization at the LHC, Taking the Beam into Account

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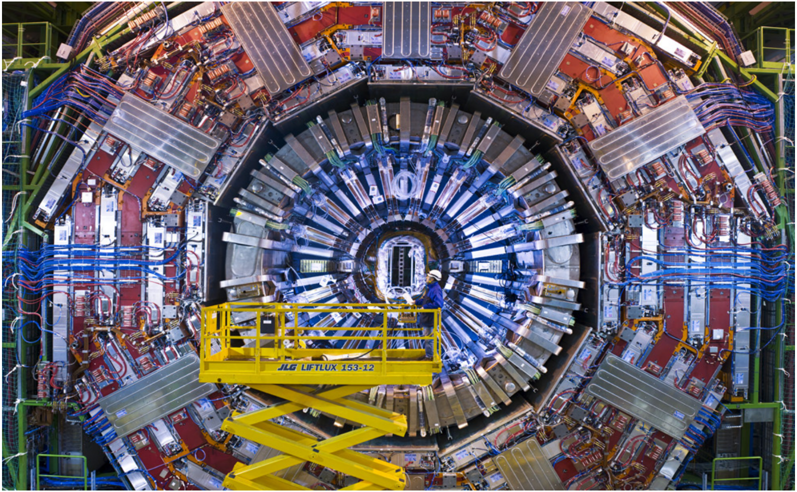
SCET Workshop ’09

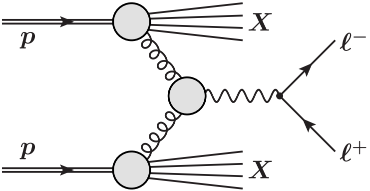
Motivation: the LHC!



$$\hat{\sigma} \xrightarrow{??} \frac{d\sigma}{dO}$$

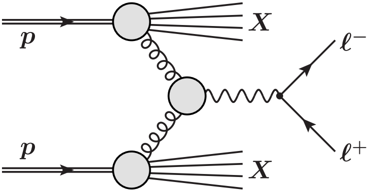
Outline

Example: Factorization for Drell-Yan



$$\frac{\mathrm{d}\sigma}{\mathrm{d}O}(E_{\mathrm{CM}}) = \sum_{i,j} \int \mathrm{d}x_a \, \mathrm{d}x_b \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}o}(x_a, x_b, \mu) \, f_i(x_a; \mu) \, f_j(x_b; \mu)$$

Example: Factorization for Drell-Yan



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Proven for invariant mass of the lepton pair, $O = M^2$. [Collins, Soper, Sterman; Bodwin]

Why it works:

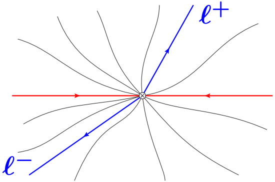
- Purely leptonic observable: $O = o$
- Completely inclusive in and independent of X

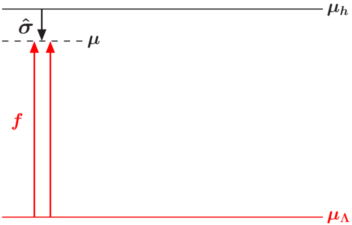
At the LHC: isolation cuts, jet algorithm...

Factorization Theorems We Know

- Drell-Yan, fully inclusive

$$\sigma = \hat{\sigma} \otimes f \otimes f$$





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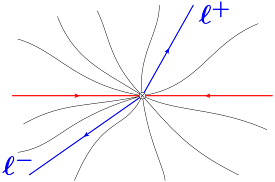
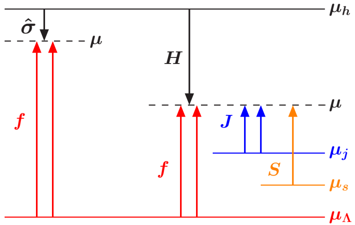
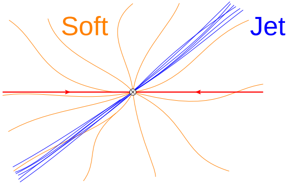




- Dijet production at threshold
  $1 \geq x_a\, x_b \geq M_{JJ}^2 / E_{\mathrm{CM}}^2 \to 1$

$$\boldsymbol{\sigma = H \otimes f \otimes f \otimes J \otimes J \otimes S}$$



$\mu_h \sim$ scale of hard interaction

$\mu_j \sim$ invariant mass of a jet

$\mu_s \sim$ energy of soft radiation

$\mu_\Lambda \sim$ low scale (Λ_{QCD})

[Kidonakis, Oderda, Sterman; Berger]

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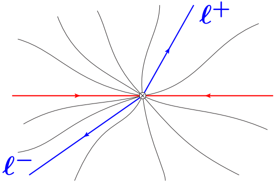

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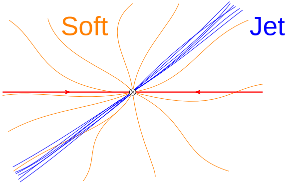

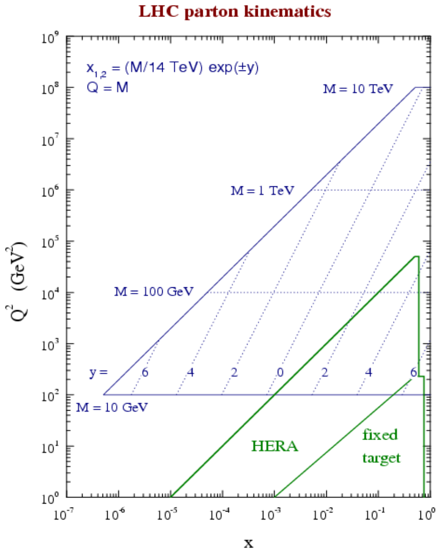

[Campbell, Huston, Stirling, LHC Physics Primer]

The Physical Picture

- We find: $\sigma = H$

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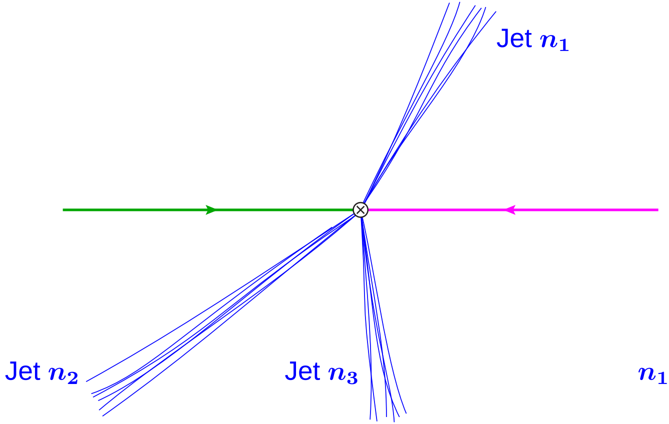

$n_1 \neq n_2 \neq n_3$

The Physical Picture

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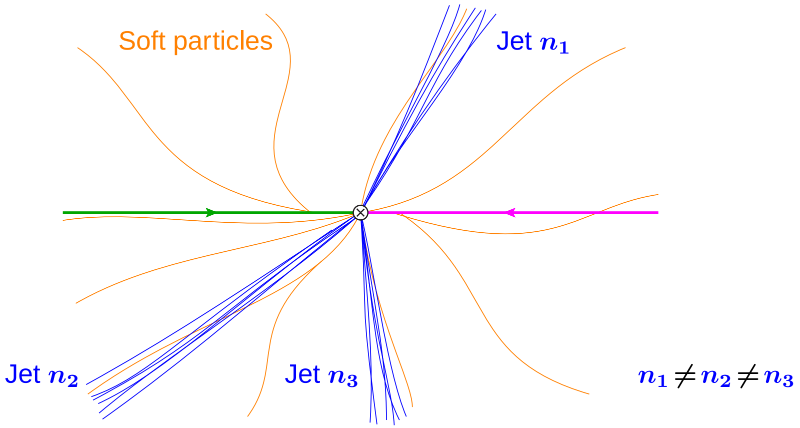
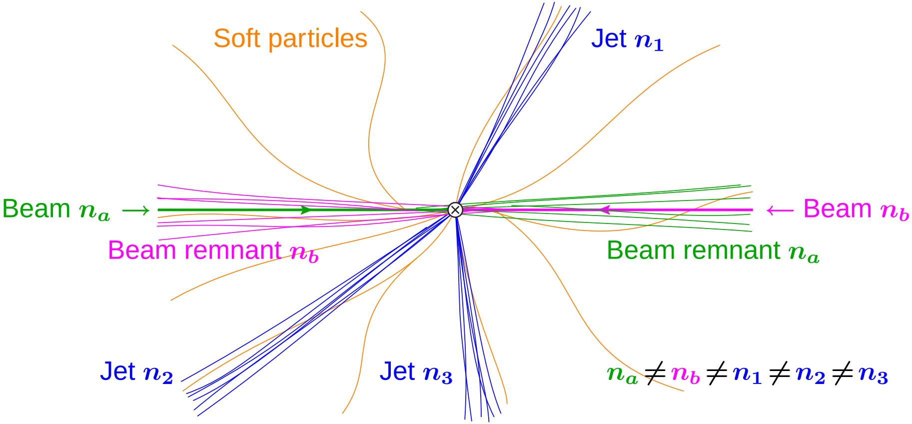


The Physical Picture

- We find: $\boldsymbol{\sigma = H \otimes J \otimes J \otimes J \otimes S \otimes B \otimes B}$



- Initial state and beam remnant in same collinear direction
- Soft modes in the jets and beam remnant "talk" to each other
- Isolating jets from the beam is indirect measurement of beam remnant

New Beam Function

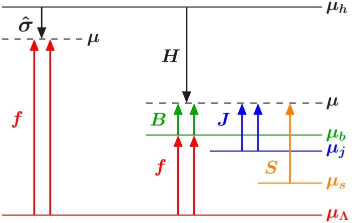
- In general we find:

$$\sigma = B \otimes B \otimes \Big(\sum_{N \text{ jets}} H_N \otimes \underbrace{J \otimes \ldots \otimes J}_{N \text{ times}} \otimes S_N \Big)$$

- H_N is the same as in threshold resummation!
- The new beam function B is still universal, like the PDF f
- The beam function B factorizes:

$$B = \mathcal{I} \otimes f$$

  - calculate $\mathcal{I}$ perturbatively
  - at tree level $B = f$ ($\mathcal{I} = \delta$)
  - natural scale $\sim \mu_b$
  - below μ_b just running of f, $x_{a,b}$ freeze out at μ_b



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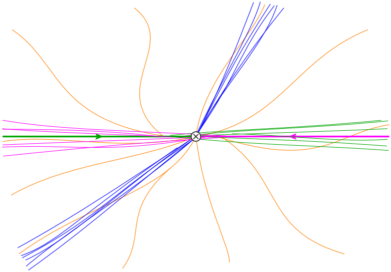
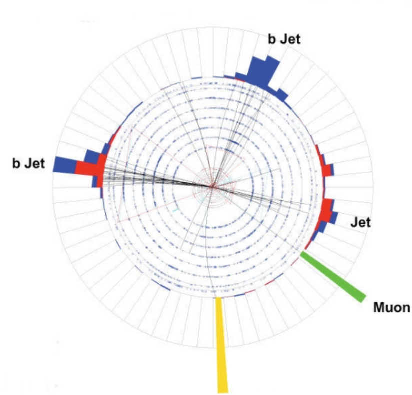


- Lets now look at the 2 jet case...

Energy Flow Operator

- Lets derive the factorization theorem to see where B comes from



- Most observables only depend on the energy distribution $\rho_X(\Omega)$
- For example:
  X has n particles with $p_i = (E_i, \vec{p}_i)$

$$\rho_X(\Omega) = \sum_i E_i\, \delta(\Omega - \Omega_i)$$

- Energy flow operator:

$$\mathcal{E}(\Omega)|X\rangle = \rho_X(\Omega)|X\rangle$$

[Korchemsky, Oderda, Sterman; Lee, Sterman, Bauer, Fleming;
We follow: Bauer, Hornig, Tackmann]

Energy Flow & Factorization

- QCD cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{1}{2E_{\mathrm{CM}}^2} \sum_X \big|\mathcal{M}(pp \to X)\big|^2 \, \delta[O - f_O(X)] \times (2\pi)^4 \delta^4(P_a + P_b - P_X)$$

Energy Flow & Factorization

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- Match onto SCET

$$\mathcal{M}(pp \to X) = \langle X | \mathcal{Q} | pp \rangle$$

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- Match onto SCET

$$\mathcal{M}(pp \to X) = \big\langle X \big| \mathcal{Q} \big| pp \big\rangle$$

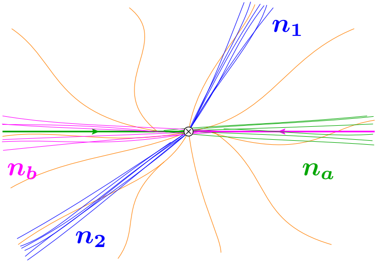
- Use energy flow to do sum over the final states

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{1}{2E_{\mathrm{CM}}^2} \sum_X \langle pp|\mathcal{Q}^\dagger|X\rangle\langle X|\mathcal{Q}|pp\rangle \,\delta(O - f_O[\rho_X]) \\ \times (2\pi)^4 \delta^4(P_a + P_b - P_X) \\ = \frac{1}{2E_{\mathrm{CM}}^2} \int \mathcal{D}\rho \int d^4x \,\langle pp|\mathcal{Q}^\dagger(x)\,\delta[\rho - \mathcal{E}]\,\mathcal{Q}(0)|pp\rangle \,\delta(O - f_O[\rho])$$

Soft-Collinear Factorization

- SCET Lagrangian: [Bauer et. al.]

$$\mathcal{L} = \mathcal{L}_{n_a} + \mathcal{L}_{n_b} + \sum_i \mathcal{L}_{n_i} + \mathcal{L}_s$$



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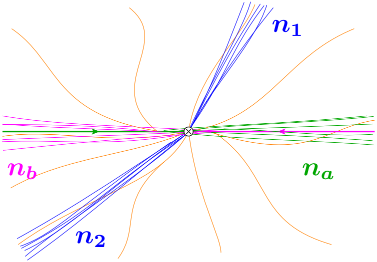
- Decouple collinear and soft [BPS]

$$\xi_{n,p}(x) \to Y_n(x)\,\xi^{(0)}_{n,p}(x)$$
$$A^\mu_{n,p}(x) \to Y_n(x)\,A^{(0)\mu}_{n,p}(x)\,Y_n^\dagger(x)$$



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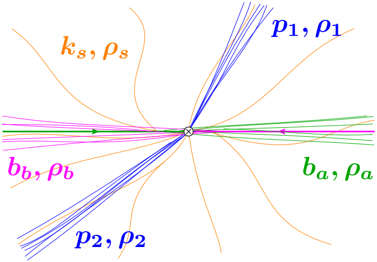

- Energy flow factorizes [Bauer, Hornig, Tackmann]

$$\mathcal{E}(\Omega) = \mathcal{E}_a(\Omega) + \mathcal{E}_b(\Omega) + \sum_i \mathcal{E}_i(\Omega) + \mathcal{E}_s(\Omega)$$

$$\int \mathcal{D}\rho\, \delta[\rho - \mathcal{E}] = \int \mathcal{D}\rho_a\, \delta[\rho_a - \mathcal{E}_a]\, \mathcal{D}\rho_b\, \delta[\rho_b - \mathcal{E}_b] \prod_i \mathcal{D}\rho_i\, \delta[\rho_i - \mathcal{E}_i]$$
$$\times\, \mathcal{D}\rho_s\, \delta[\rho_s - \mathcal{E}_s]$$

Starting Point for Factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \frac{1}{2E_{\mathrm{CM}}^2} \int \mathcal{D}\rho \, \mathrm{d}^4x \, \langle pp | \mathcal{Q}^\dagger(x) \, \delta[\rho - \mathcal{E}] \, \mathcal{Q}(0) | pp \rangle \, \delta[O - f_O[\rho]]$$

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$$= \int \mathrm{d}^4 b_{a,b} \, \mathcal{D}\rho_{a,b} \, \mathrm{d}^4 p_{1,2} \, \mathcal{D}\rho_{1,2} \, \mathrm{d}^4 k_s \, \mathcal{D}\rho_s \, \delta[O - f_O[\rho]]$$

$$\times H(\hat{s}, \ldots) \, B(b_a, \rho_a) \, B(b_b, \rho_b) \, J(p_1, \rho_1) \, J(p_2, \rho_2) \, S(k_s, \rho_s)$$

$$\times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s)$$

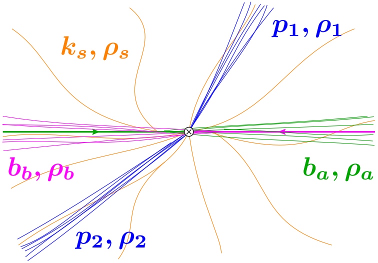
with

$H(\hat{s}, \ldots) =$ hard function

$B(b_a, \rho_a) =$ beam distribution

$J(p_i, \rho_i) =$ jet distribution

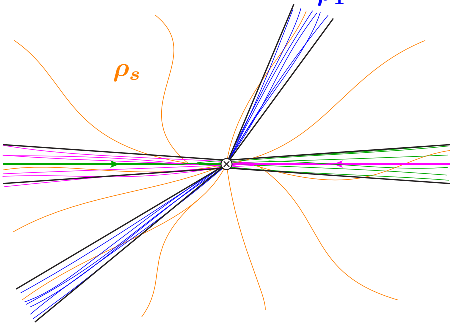
$S(k_s, \rho_s) =$ soft distribution



- These distributions are not inclusive, they depend on ρ_i

Kinematics

- $P^\mu_{i\triangleleft}[\rho] \equiv P^\mu_\triangleleft(\vec{n}_i, R)[\rho] =$ momentum in cone of radius R around $\vec{n}_i$



$$\begin{aligned} P^\mu_1 &= P^\mu_{1\triangleleft}[\rho] \\ &= P^\mu_{1\triangleleft}[\rho_1] + P^\mu_{1\triangleleft}[\rho_s] \\ &= q^\mu_1 + \ell^\mu_1 \end{aligned}$$

$$\begin{aligned} B^\mu_a &= P^\mu_{a\triangleleft}[\rho_a] + P^\mu_{a\triangleleft}[\rho_s] \\ &= q^\mu_a + \ell^\mu_a \end{aligned}$$

Kinematics

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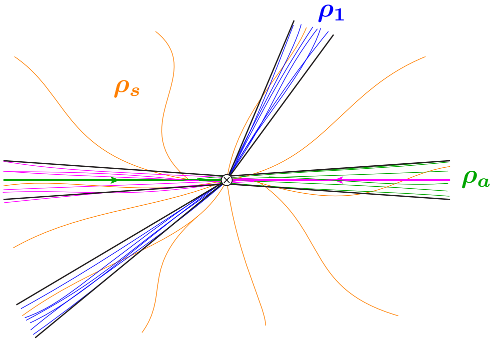

$$
\begin{aligned}
P_1^\mu &= P^\mu_{1\triangleleft}[\rho] \\
&= P^\mu_{1\triangleleft}[\rho_1] + P^\mu_{1\triangleleft}[\rho_s] \\
&= q_1^\mu + \ell_1^\mu
\end{aligned}
$$

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\begin{aligned}
B_a^\mu &= P^\mu_{a\triangleleft}[\rho_a] + P^\mu_{a\triangleleft}[\rho_s] \\
&= q_a^\mu + \ell_a^\mu
\end{aligned}
$$

- Expanding:

$$P_i^- = \bar{n}_i \cdot P_i = q_i^- \qquad P_i^+ = n_i \cdot P_i = q_i^+ + \ell_i^+$$

$$B_a^- = \bar{n}_a \cdot B_a = q_a^- \qquad B_a^+ = n_a \cdot B_a = q_a^+ + \ell_a^+$$

- $b_a^- = (1 - x_a)\, E_{\mathrm{CM}}$

Factorization

$$\mathrm{d}\sigma = \int \mathrm{d}^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, \mathrm{d}^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, \mathrm{d}^4 k_s\, \mathcal{D}\rho_s$$
$$\times H(\hat{s},\dots)\, B(b_a,\rho_a)\, B(b_b,\rho_b)\, J(p_1,\rho_1)\, J(p_2,\rho_2)\, S(k_s,\rho_s)$$
$$\times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s)$$

Factorization

$$\begin{aligned} \mathrm{d}\sigma = \int & \mathrm{d}^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, \mathrm{d}^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, \mathrm{d}^4 k_s\, \mathcal{D}\rho_s \\ & \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\ & \times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s) \\ & \times \mathrm{d}\ell^+_{a,b,1,2}\, \delta(\ell^+_{a,b} - P^+_{\triangleleft a,b}[\rho_s])\, \delta(\ell^+_{1,2} - P^+_{\triangleleft 1,2}[\rho_s]) \end{aligned}$$

Factorization

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& \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\
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& \times \mathrm{d}q^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, \mathrm{d}B^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b})
\end{aligned}
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Factorization

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& \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\
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& \times \mathrm{d}P^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}])
\end{aligned}
$$

Factorization

$$
\begin{aligned}
d\sigma = \int & d^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, d^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, d^4 k_s\, \mathcal{D}\rho_s \\
& \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\
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& \times dq^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, dB^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b}) \\
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& \times dP^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}]) \\
= \int & d\ell^+_{a,b,1,2}\, S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2)
\end{aligned}
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& \times \mathrm{d}\ell^+_{a,b,1,2}\, \delta(\ell^+_{a,b} - P^+_{\triangleleft a,b}[\rho_s])\, \delta(\ell^+_{1,2} - P^+_{\triangleleft 1,2}[\rho_s]) \\
& \times \mathrm{d}q^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, \mathrm{d}B^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b}) \\
& \times \mathrm{d}q^+_{1,2}\, \delta(q^+_{1,2} - P^+_{\triangleleft}[\rho_{1,2}])\, \mathrm{d}P^+_{1,2}\, \delta(P^+_{1,2} - q^+_{1,2} - \ell^+_{1,2}) \\
& \times \mathrm{d}P^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}]) \\
= \int & \mathrm{d}\ell^+_{a,b,1,2}\, S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2) \\
& \times \mathrm{d}x_{a,b}\, \mathrm{d}B^+_{a,b}\, B_{\triangleleft}(x_a, B^+_a - \ell^+_a)\, B_{\triangleleft}(x_b, B^+_b - \ell^+_b)
\end{aligned}
$$

Factorization

$$
\begin{aligned}
\mathrm{d}\sigma = \int & \mathrm{d}^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, \mathrm{d}^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, \mathrm{d}^4 k_s\, \mathcal{D}\rho_s \\
& \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\
& \times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s) \\
& \times \mathrm{d}\ell^+_{a,b,1,2}\, \delta(\ell^+_{a,b} - P^+_{\triangleleft a,b}[\rho_s])\, \delta(\ell^+_{1,2} - P^+_{\triangleleft 1,2}[\rho_s]) \\
& \times \mathrm{d}q^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, \mathrm{d}B^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b}) \\
& \times \mathrm{d}q^+_{1,2}\, \delta(q^+_{1,2} - P^+_{\triangleleft}[\rho_{1,2}])\, \mathrm{d}P^+_{1,2}\, \delta(P^+_{1,2} - q^+_{1,2} - \ell^+_{1,2}) \\
& \times \mathrm{d}P^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}]) \\
= \int & \mathrm{d}\ell^+_{a,b,1,2}\, S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2) \\
& \times \mathrm{d}x_{a,b}\, \mathrm{d}B^+_{a,b}\, B_{\triangleleft}(x_a, B^+_a - \ell^+_a)\, B_{\triangleleft}(x_b, B^+_b - \ell^+_b) \\
& \times \mathrm{d}^4 p_{1,2}\, J_{\triangleleft}(p^-_1, P^+_1 - \ell^+_1)\, J_{\triangleleft}(p^-_2, P^+_2 - \ell^+_2)
\end{aligned}
$$

Factorization

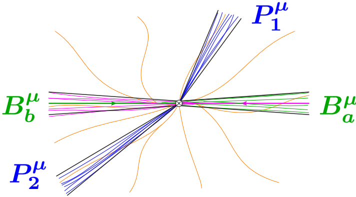
$$
\begin{aligned}
d\sigma = \int & d^4 b_{a,b}\, \mathcal{D}\rho_{a,b}\, d^4 p_{1,2}\, \mathcal{D}\rho_{1,2}\, d^4 k_s\, \mathcal{D}\rho_s \\
& \times H(\hat{s}, \dots)\, B(b_a, \rho_a)\, B(b_b, \rho_b)\, J(p_1, \rho_1)\, J(p_2, \rho_2)\, S(k_s, \rho_s) \\
& \times (2\pi)^4 \delta^4(P_a + P_b - b_a - b_b - p_1 - p_2 - k_s) \\
& \times d\ell^+_{a,b,1,2}\, \delta(\ell^+_{a,b} - P^+_{\triangleleft a,b}[\rho_s])\, \delta(\ell^+_{1,2} - P^+_{\triangleleft 1,2}[\rho_s]) \\
& \times dq^+_{a,b}\, \delta(q^+_{a,b} - P^+_{\triangleleft}[\rho_{a,b}])\, dB^+_{a,b}\, \delta(B^+_{a,b} - q^+_{a,b} - \ell^+_{a,b}) \\
& \times dq^+_{1,2}\, \delta(q^+_{1,2} - P^+_{\triangleleft}[\rho_{1,2}])\, dP^+_{1,2}\, \delta(P^+_{1,2} - q^+_{1,2} - \ell^+_{1,2}) \\
& \times dP^-_{1,2}\, \delta(P^-_{1,2} - P^-_{\triangleleft}[\rho_{1,2}]) \\
= \int & d\ell^+_{a,b,1,2}\, S_{\triangleleft}(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2) \\
& \times dx_{a,b}\, dB^+_{a,b}\, B_{\triangleleft}(x_a, B^+_a - \ell^+_a)\, B_{\triangleleft}(x_b, B^+_b - \ell^+_b) \\
& \times d^4 p_{1,2}\, J_{\triangleleft}(p^-_1, P^+_1 - \ell^+_1)\, J_{\triangleleft}(p^-_2, P^+_2 - \ell^+_2) \\
& \times H(x_a, x_b, p^-_1, p^-_2)\, dO\, \delta[O - f_O(x_a, x_b, B^+_a, B^+_b, p^-_1, p^-_2)] \\
& \times (2\pi)^4 \delta^4(\tfrac{1}{2} x_a E_{\mathrm{CM}}\, n_a + \tfrac{1}{2} x_b E_{\mathrm{CM}}\, n_b - p_1 - p_2)
\end{aligned}
$$

Factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O} = \int \mathrm{d}\ell^+_{a,b,1,2}\, S_\triangleleft(\ell^+_a, \ell^+_b, \ell^+_1, \ell^+_2)$$
$$\times\, \mathrm{d}x_{a,b}\, \mathrm{d}B^+_{a,b}\, B_\triangleleft(x_a, B^+_a - \ell^+_a)\, B_\triangleleft(x_b, B^+_b - \ell^+_b)$$
$$\times\, \mathrm{d}^4 p_{1,2}\, J_\triangleleft(p^-_1, P^+_1 - \ell^+_1)\, J_\triangleleft(p^-_2, P^+_2 - \ell^+_2)$$
$$\times\, H(x_a, x_b, p^-_1, p^-_2)\, \delta[O - f_O(x_a, x_b, B^+_a, B^+_b, p^-_1, p^-_2)]$$
$$\times\, (2\pi)^4 \delta^4(\tfrac{1}{2} x_a\, E_{\mathrm{CM}}\, n_a + \tfrac{1}{2} x_b\, E_{\mathrm{CM}}\, n_b - p_1 - p_2)$$

- Pick an observable, e.g.

$$M^2_{JJ} = 2P_1 \cdot P_2 = \frac{P^-_1\, P^-_2}{p^-_1\, p^-_2}\, x_a\, x_b\, E^2_{\mathrm{CM}}$$



Does not depend on $B^+_{a,b}$

- Cannot integrate away $B^+_{a,b}$ to give PDF: $B \to f$

Factorization

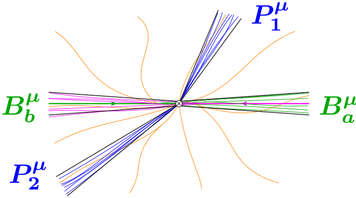
- Simple example that demonstrates that B is important:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}P_1^- \, \mathrm{d}P_2^-} = \int \mathrm{d}x_{a,b}\, H(x_a, x_b, p_1^-, p_2^-; \mu) \int \mathrm{d}\ell^+_{a,b,1,2}\, S_\triangleleft(\ell_a^+, \ell_b^+, \ell_1^+, \ell_2^+; \mu)$$
$$\times \tilde{B}_\triangleleft(x_a, B^+_{a\,\max} - \ell_a^+; \mu)\, \tilde{B}_\triangleleft(x_b, B^+_{b\,\max} - \ell_b^+; \mu)$$
$$\times \tilde{J}_\triangleleft(p_1^-, P^+_{1\,\max} - \ell_1^+; \mu)\, \tilde{J}_\triangleleft(p_2^-, P^+_{2\,\max} - \ell_2^+; \mu)$$

- Pick an observable, e.g.

$$M_{JJ}^2 = 2P_1 \cdot P_2 = \frac{P_1^- \, P_2^-}{p_1^- \, p_2^-}\, x_a\, x_b\, E_{\mathrm{CM}}^2$$

Does not depend on $B^+_{a,b}$

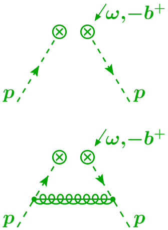


- Cannot integrate away $B^+_{a,b}$ to give PDF: $B \longrightarrow f$
- Can only integrate $B^+_{a,b}$ up to $B^+_{a,b\,\max}$, e.g. $B^+_{a,b}$ is restricted by exp. cuts $\mu_b^2 \sim B^+_{\max}(xE_{\mathrm{CM}}) \sim e^{-2y_{\mathrm{cut}}} x(1-x) E_{\mathrm{CM}}^2 \sim (50 - 200\ \mathrm{GeV})^2$

Beam Function

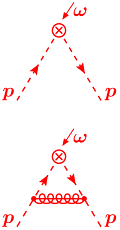
- We now consider the inclusive B (drop the cone restriction $B_{\triangleleft}$)

- Quark beam function $B_q(b^+\omega, z; \mu)$



$$B_q(b^+\omega, \omega/P_a^-; \mu) = \frac{1}{\omega} \int \frac{\mathrm{d}x^-}{4\pi} \, e^{\mathrm{i} b^+ x^-/2}$$
$$\times \langle p_n | \bar{\chi}_n(x^- n/2) \, \frac{\bar{n}\!\!\!/}{2} \, \delta(\omega - \bar{\mathcal{P}}) \, \chi_n(0) | p_n \rangle$$

- Compare with PDF $f_q(z; \mu)$

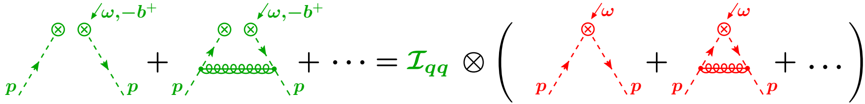


$$f_q(\omega/P_a^-; \mu) = \langle p_n | \bar{\chi}_n(0) \, \frac{\bar{n}\!\!\!/}{2} \, \delta(\omega - \bar{\mathcal{P}}) \, \chi_n(0) | p_n \rangle$$

Beam Function Factorizes

- B factorizes (matching SCET$_I \to$ SCET$_{II}$)

$$B_q(b^+\omega, z; \mu) = \sum_{i=q,g,\bar{q}} \int \mathrm{d}z'\, \mathcal{I}_{qi}(b^+\omega, z - z'; \mu)\, f_i(z'; \mu)$$

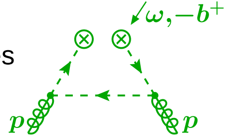


- At tree level $\mathcal{I}_{qq}^{\text{tree}}(b^+\omega, z) = \delta(b^+\omega)\,\delta(z)$ and $\mathcal{I}_{qg}^{\text{tree}} = \mathcal{I}_{q\bar{q}}^{\text{tree}} = 0$ so

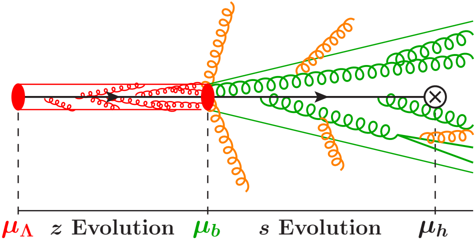
$$B_q^{\text{tree}}(b^+\omega, z) = \delta(b^+\omega)\, f_q(z)$$

- At one-loop $\mathcal{I}_{qg} \neq 0$

gluon PDF f_g mixes into B_q



Beam Function Running

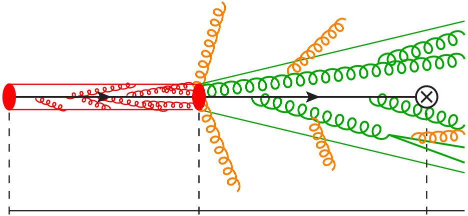


- running up $\mu_\Lambda \longrightarrow \mu_b$

$$\mu \frac{d}{d\mu} f(z;\mu) = \int dz'\, \gamma^f(z,z') f(z';\mu)$$

$$f(z;\mu_b) = \int dz'\, U^f(z,z';\mu_b,\mu_\Lambda)\, f(z';\mu_\Lambda)$$

Beam Function Running



- running up $\mu_\Lambda \to \mu_b$

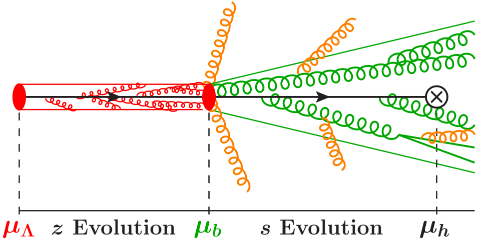
$$\mu \frac{d}{d\mu} f(z;\mu) = \int dz'\, \gamma^f(z,z') f(z';\mu)$$

$$f(z;\mu_b) = \int dz'\, U^f(z,z';\mu_b,\mu_\Lambda)\, f(z';\mu_\Lambda)$$

- matching at μ_b, with $s = b^+\omega$ = inv. mass of initial state jet

$$B(s,z;\mu_b) = \int \mathrm{d}z'\, \mathcal{I}(s,z-z';\mu_b)\, f(z';\mu_b)$$

Beam Function Running



- running up $\mu_\Lambda \longrightarrow \mu_b$

$$\mu \frac{d}{d\mu} f(z;\mu) = \int dz' \, \gamma^f(z,z') f(z';\mu)$$

$$f(z;\mu_b) = \int dz' \, U^f(z,z';\mu_b,\mu_\Lambda) \, f(z';\mu_\Lambda)$$

- matching at μ_b, with $s = b^+\omega =$ inv. mass of initial state jet

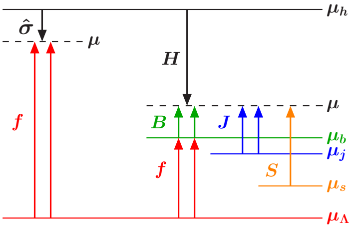
$$B(s,z;\mu_b) = \int \mathrm{d}z' \, \mathcal{I}(s, z - z';\mu_b) \, f(z';\mu_b)$$

- running up $\mu_b \longrightarrow \mu$

$$B(s,z;\mu) = \int ds' \, U^B(s,s';\mu,\mu_b) \, B(s',z;\mu_b)$$

Beam Function Running

- Scales and running:



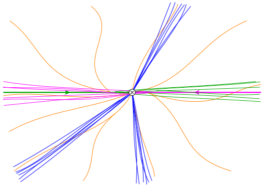
μ_h ~ scale of hard interaction
μ_j ~ inv. mass of final state jet
μ_b ~ inv. mass of initial state jet
μ_s ~ energy of soft radiation
μ_Λ ~ low scale (Λ_{QCD})

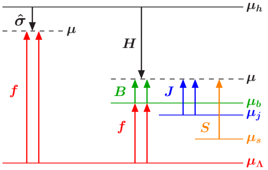
- Unlike f, the RGE for B includes Sudakov double logs

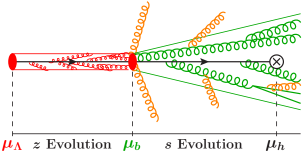
$$B(s, z; \mu) = \int ds' \, U^B(s, s'; \mu, \mu_b) \, B(s', z; \mu_b)$$

Invariant mass restrictions on the real radiation yield terms $\gamma^B(s, s'; \mu) \propto \ln(\mu/s)$, which sum the Sudakov double logs

Summary & Outlook







- One needs to take the beam into account
- The beam function enters factorization theorems in a universal way

$$\sigma = B \otimes B \otimes \Big(\sum_{N \text{ jets}} H_N \otimes \underbrace{J \otimes \ldots \otimes J}_{N \text{ times}} \otimes S_N \Big)$$

  - PDF f replaced by B
  - H_N as in threshold resummation
- B factorizes at a scale μ_b

$$B_q(b^+\omega;\mu) = \int \mathrm{d}z'\, \mathcal{I}_{qi}(b^+\omega, z - z';\mu)\, f_i(z';\mu)$$

- Below μ_b: usual running of f involving z
- Above μ_b: the evolution affects the inv. mass $b^+\omega$, but keeps z fixed!