From FQHE to new field-theoretic dualities

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CTP50, MIT, 3/24/2018

Plan

- Fractional quantum Hall effect as a nonperturbative problem
- “Old” composite fermion
- Missing symmetries
- New composite fermion and dualities

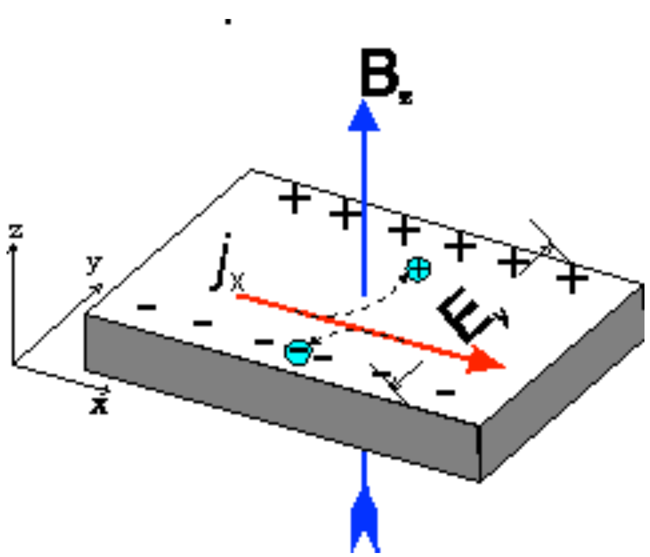
Refs: DTS, PRX 2015
D.X. Nguyen, S.Golkar, M.Roberts, DTS arxiv:1709.07855

New dualities suggested by FQHE

- Duality: equivalence between two field theories, which often look completely different from each other
- A famous example: duality between sine-Gordon theory and Thirring model in (1+1)D Coleman 1975
  - boson = fermion
- Another well known example: duality between complex scalar and abelian Higgs model in (2+1)D
  - particle on one side = vortex on the other side
    Peskin; Dasgupta, Halperin1970s
- FQHE: duality between two fermionic theories

The Hall effect

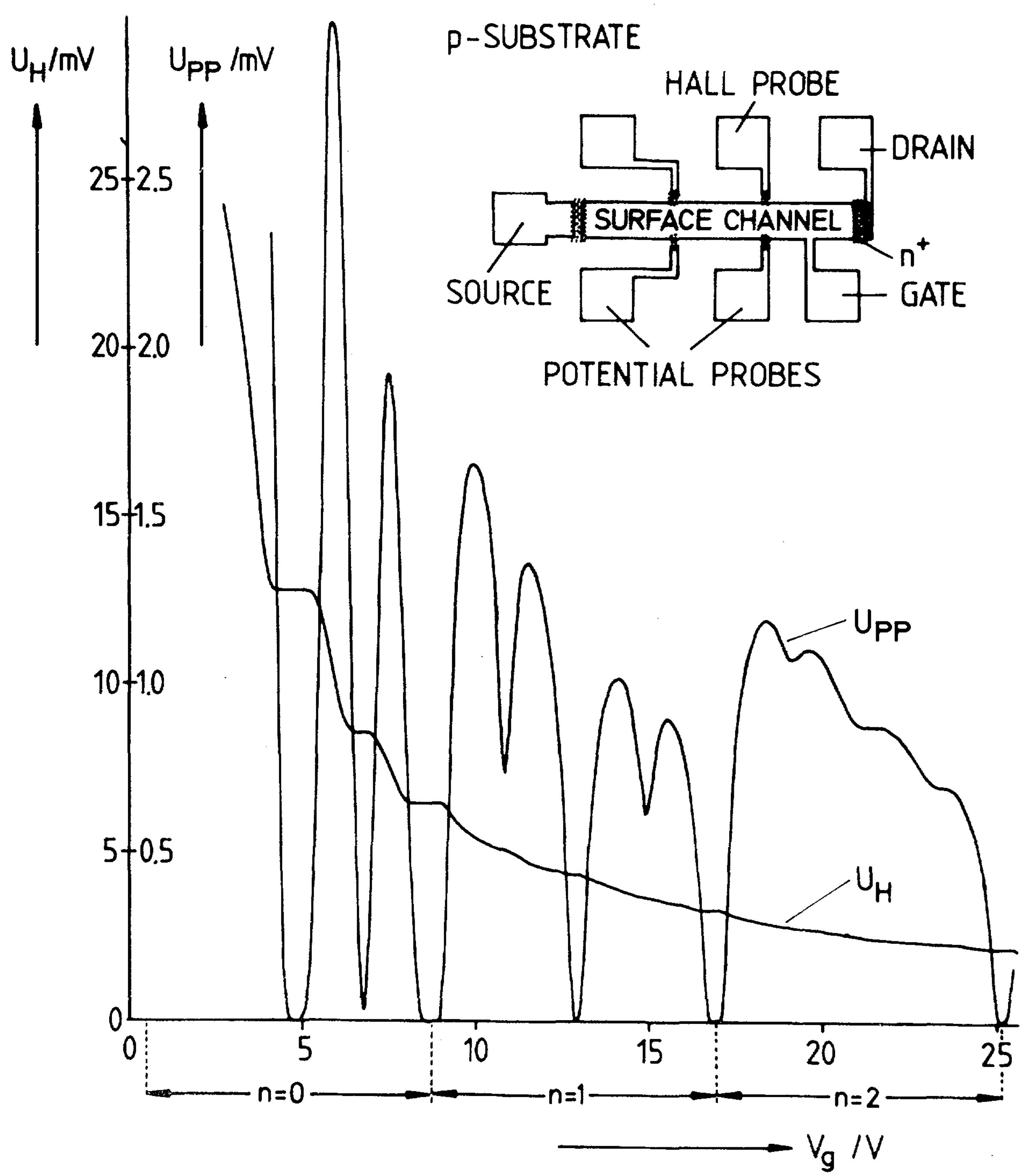
- Quantum Hall effects are among the most surprising discoveries in physics
- In 1879 Edwin Hall discovered voltage perpendicular to electric current in a magnetic field (the Hall effect)



$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y$$

Integer quantum Hall effect

Dorda, Pepper, von Klitzing 1980



Plateaux with quantized Hall conductivity

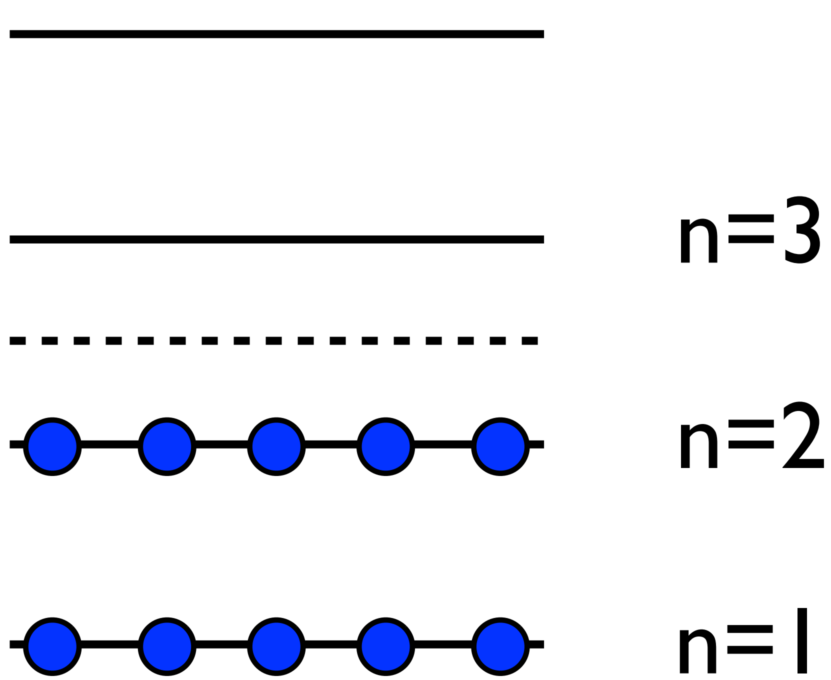
$$\sigma_{xy} = n\frac{e^2}{h}$$

$$n = 1, 2, 3, \ldots$$

precision ~ 10^{-9}!

Landau level and IQHE

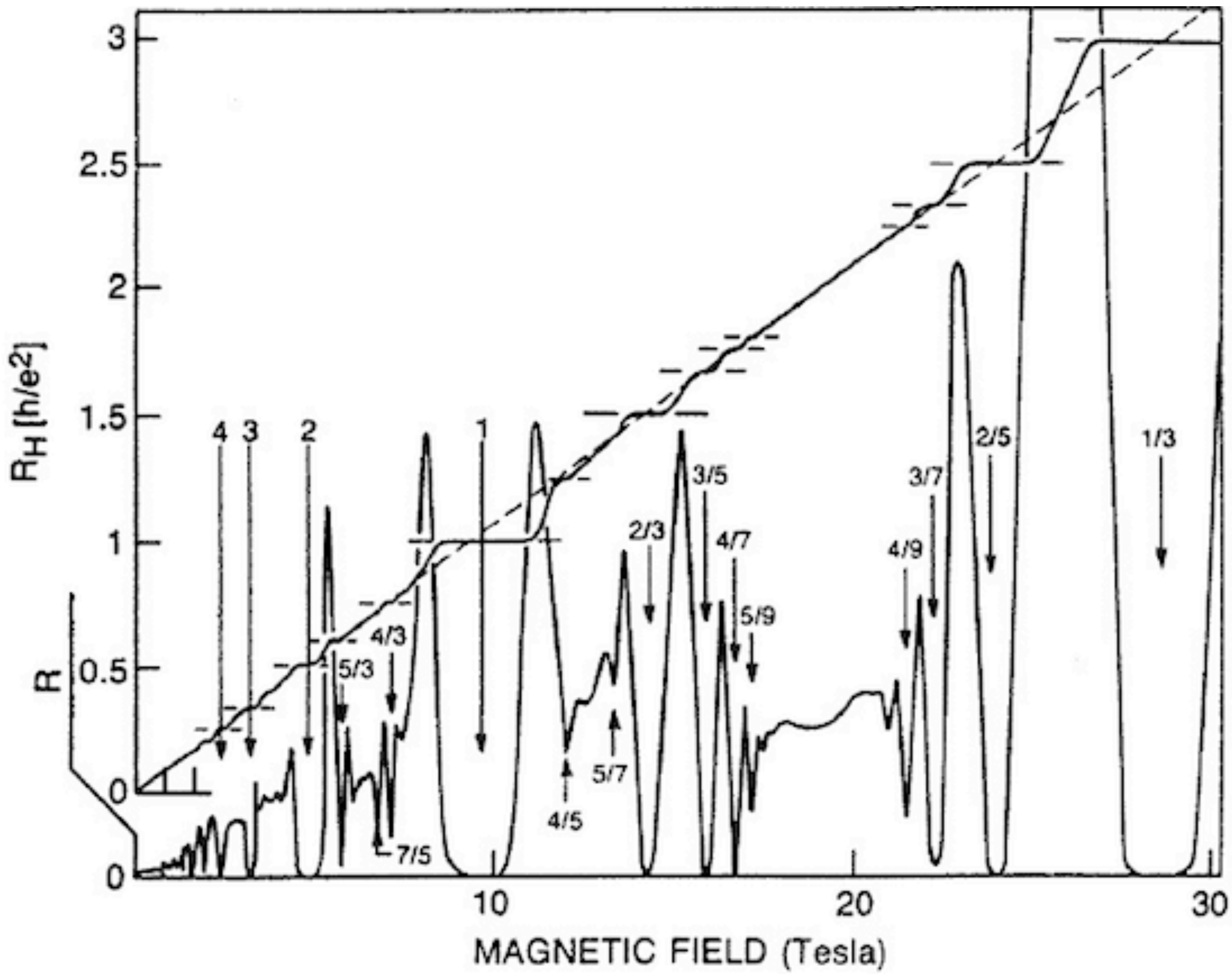
- Landau 1930: discrete energy levels in B field
- IQHE: electrons filling n Landau levels



$$\sigma_{xy} = n\frac{e^2}{h}$$

- Tsui, Stormer, Gossard (1982): quantum Hall effect when a fraction of a Landau level is filled

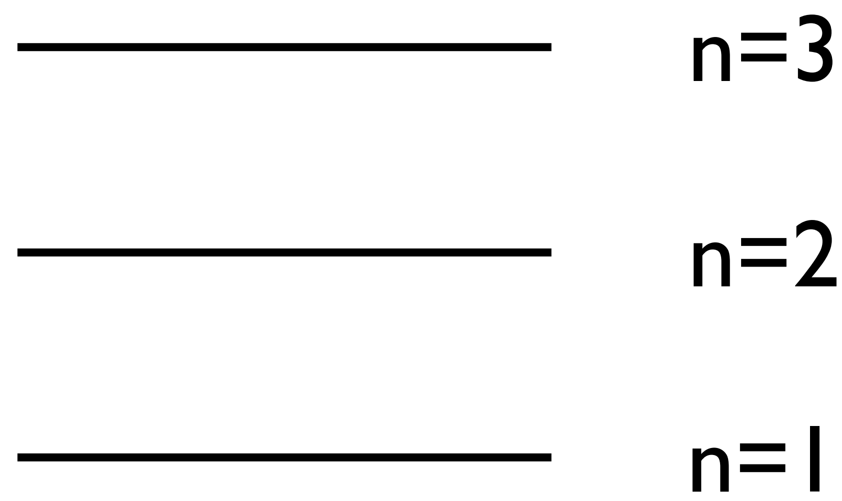
Fractional quantum Hall effect



Stormer 1992

Fractional QHE

Increasing B field: more states available
in lowest Landau level



Filling fraction

$$\nu = \frac{\text{number of particles}}{\text{number of magnetic flux quanta}}$$

Fractional QHE

Increasing B field: more states available
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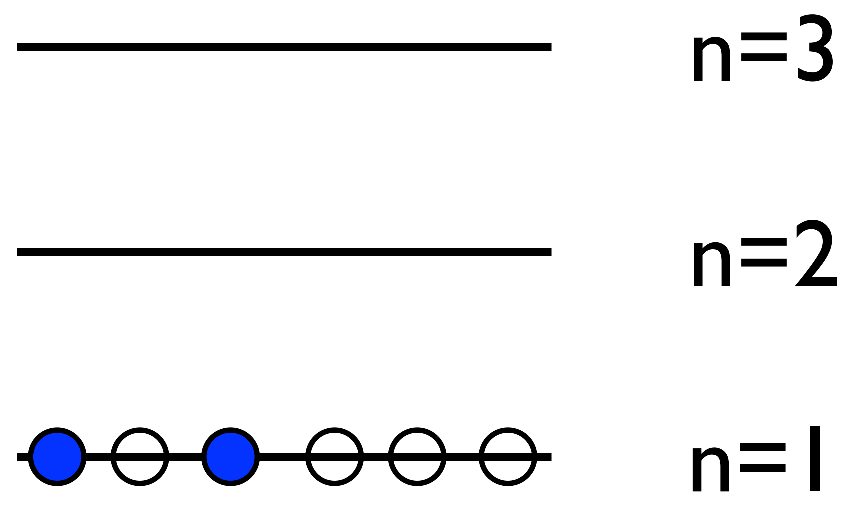

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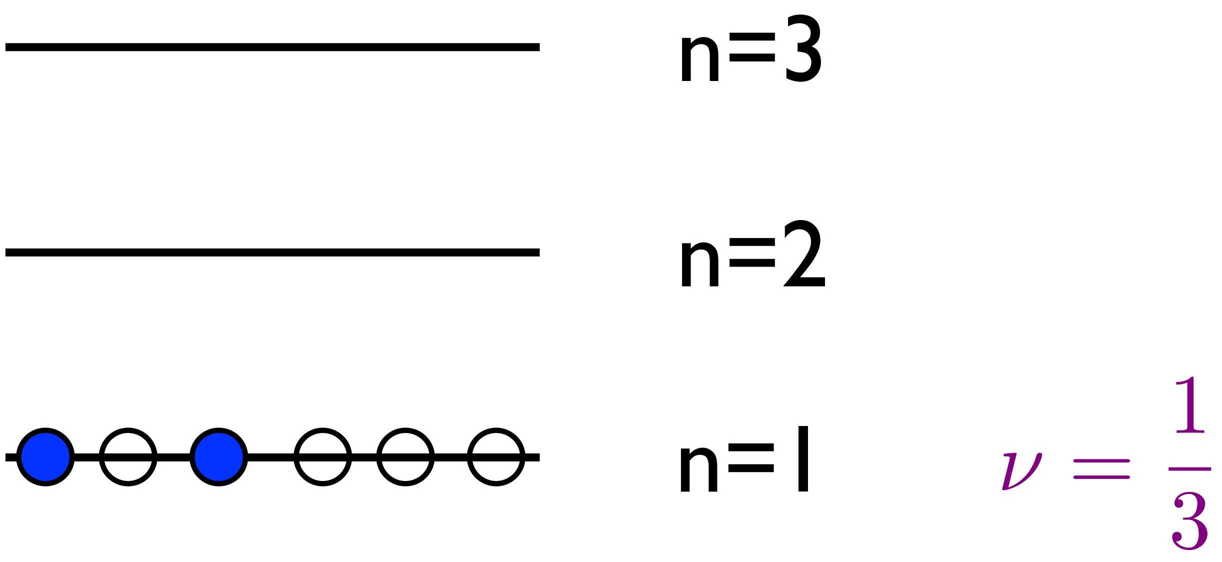


Filling fraction

$$\nu = \frac{\text{number of particles}}{\text{number of magnetic flux quanta}}$$

- The fractional quantum Hall effect was not predicted by theory
  - first theory Laughlin 1983
- Rephrasing Weisskopf: a group of theorists in closed building would not be able to predict FQHE

A nonperturbative problem

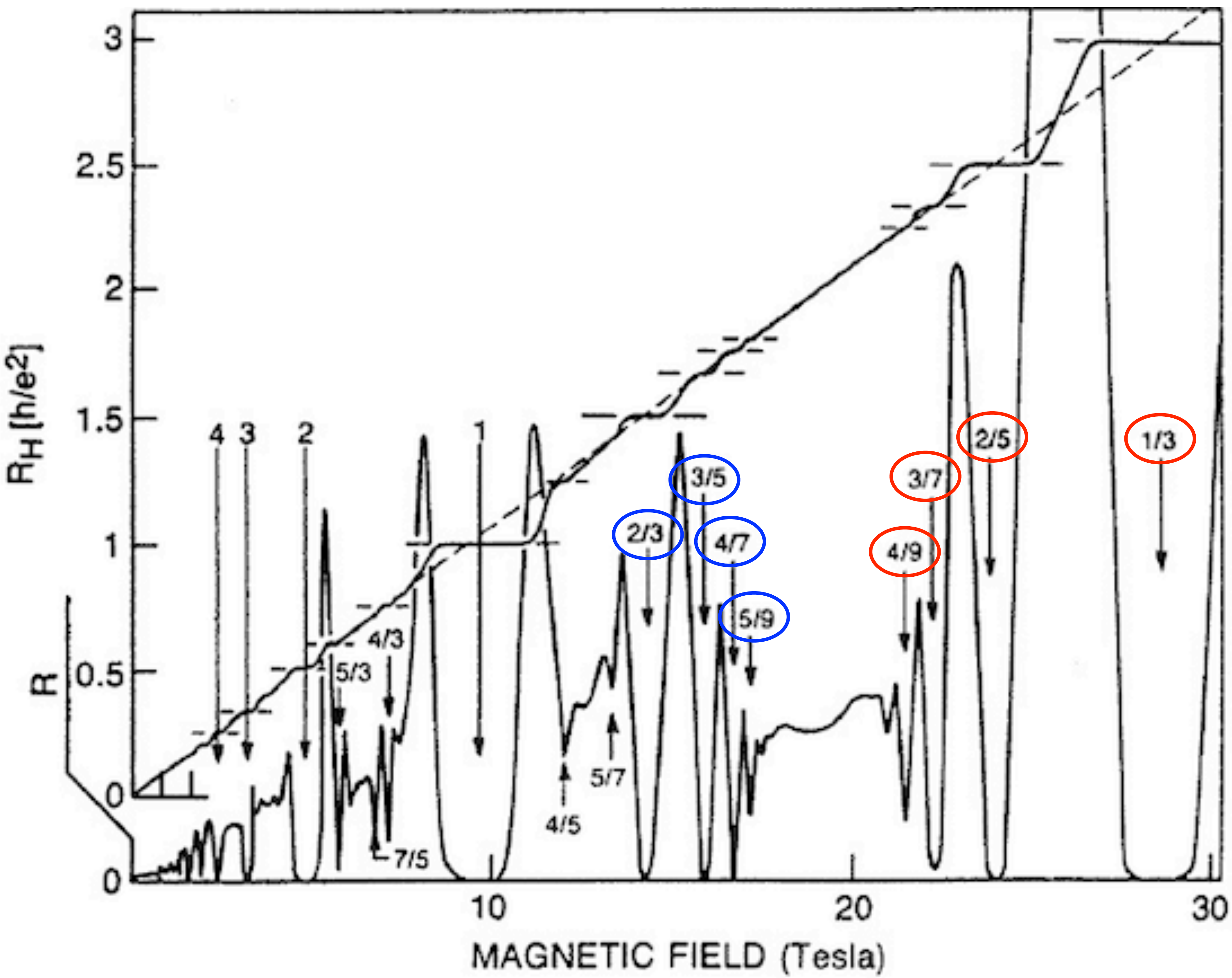
$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b\rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

$$E_{\mathrm{cyclotron}} = \frac{\hbar e B}{mc} \quad \gg \quad E_{\mathrm{Coulomb}} = \frac{e^2}{\ell_B}$$

No perturbation theory

only one Landau level is important

Nevertheless, some systematics



$$\nu = \frac{n+1}{2n+1} \qquad \nu = \frac{n}{2n+1}$$

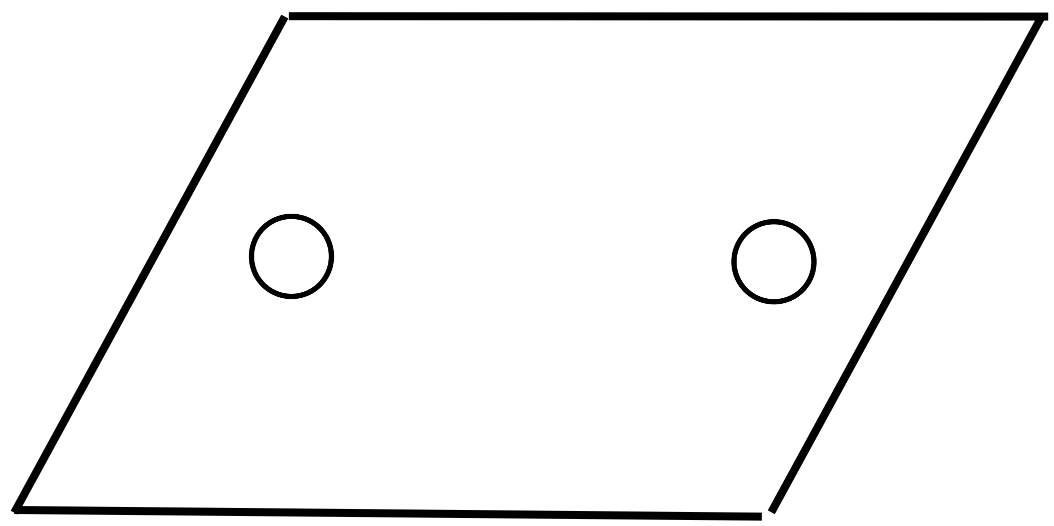
Half filled Landau level

- Most surprisingly, when the Landau level is half filled (ν=1/2), the ground state seems to be a Fermi liquid!
- Fermionic quasiparticle moves in straight line
  - so has to be electrically neutral
  - what is the nature of this quasiparticle?

“Old” composite fermion

Composite fermion

Flux attachment Wilczek 1982, Jain 1989



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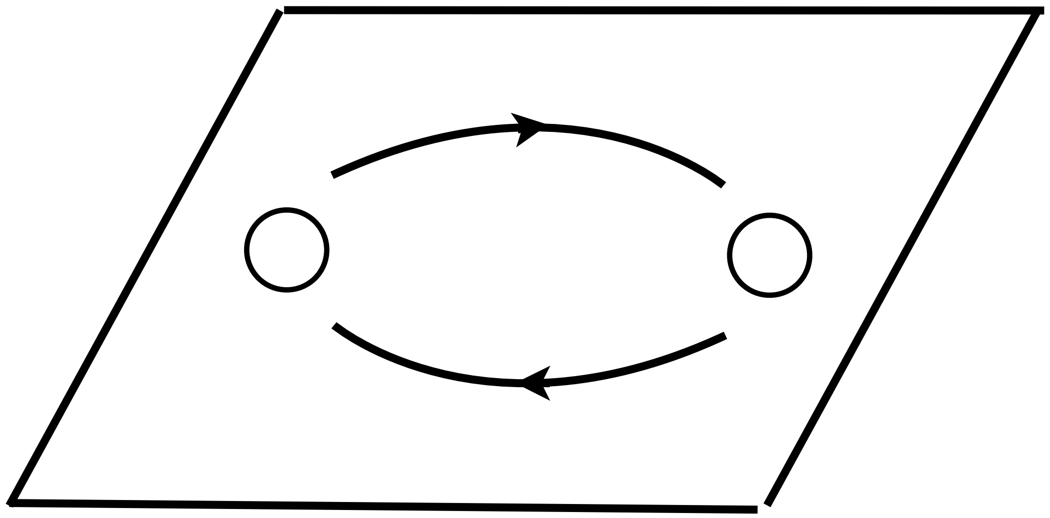

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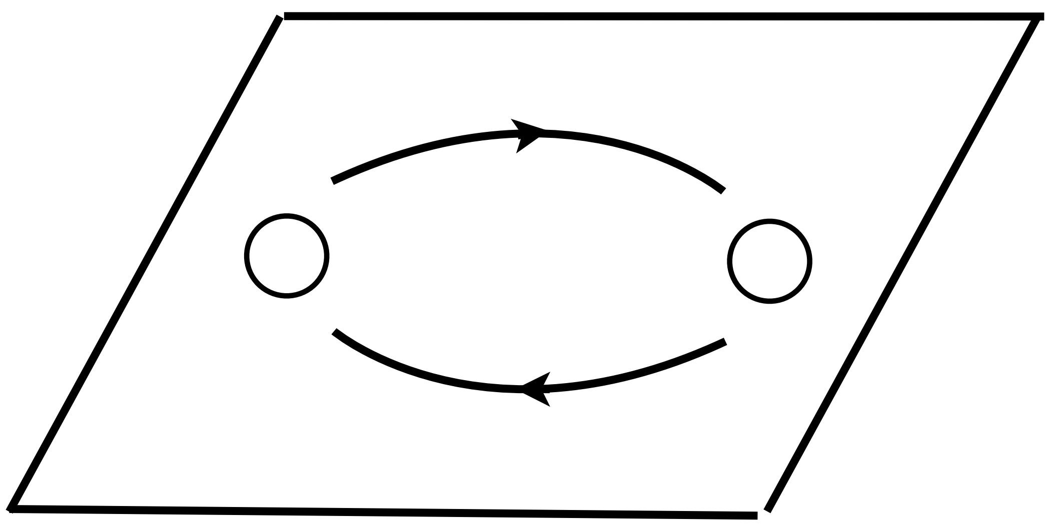

(-1)

Composite fermion

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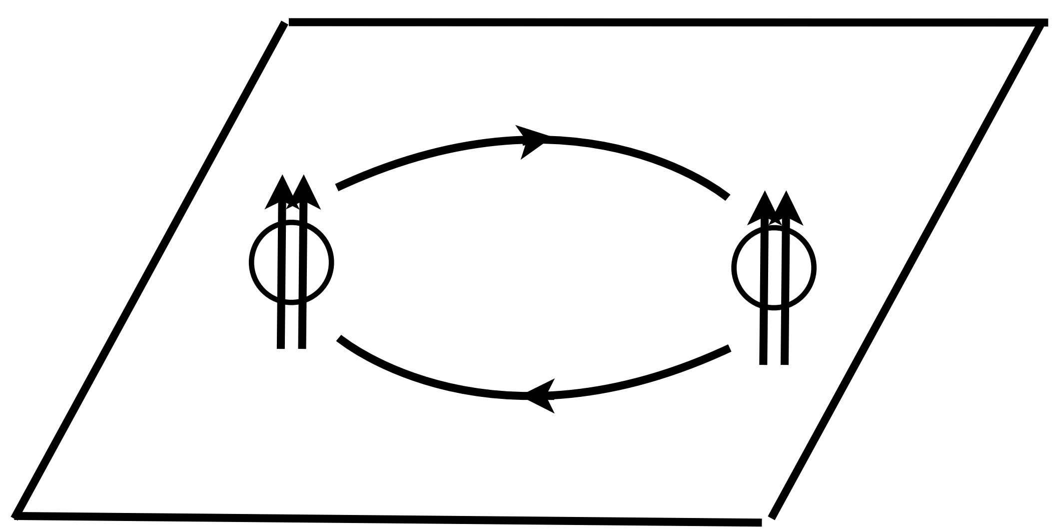

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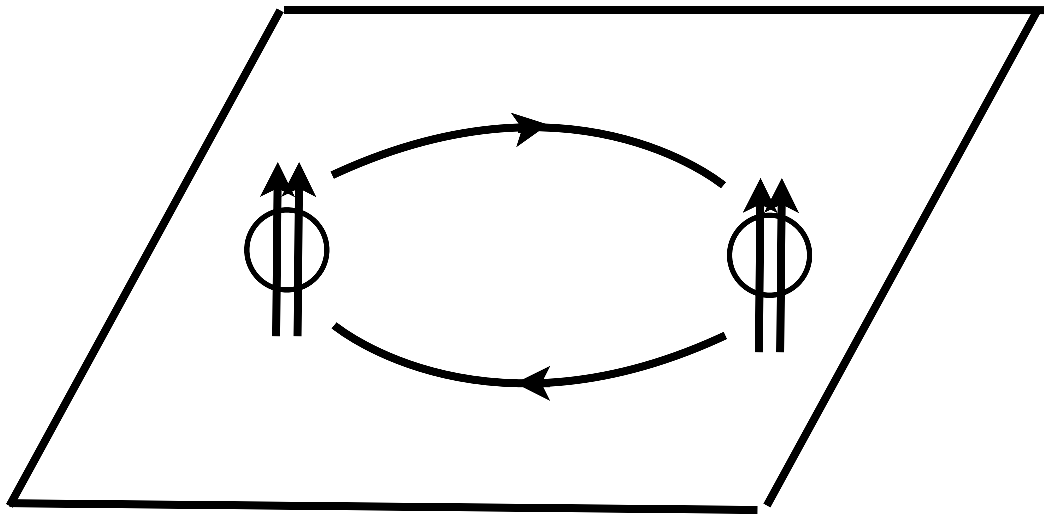

$$(-1)\exp(2i\pi) = (-1)$$

Composite fermion

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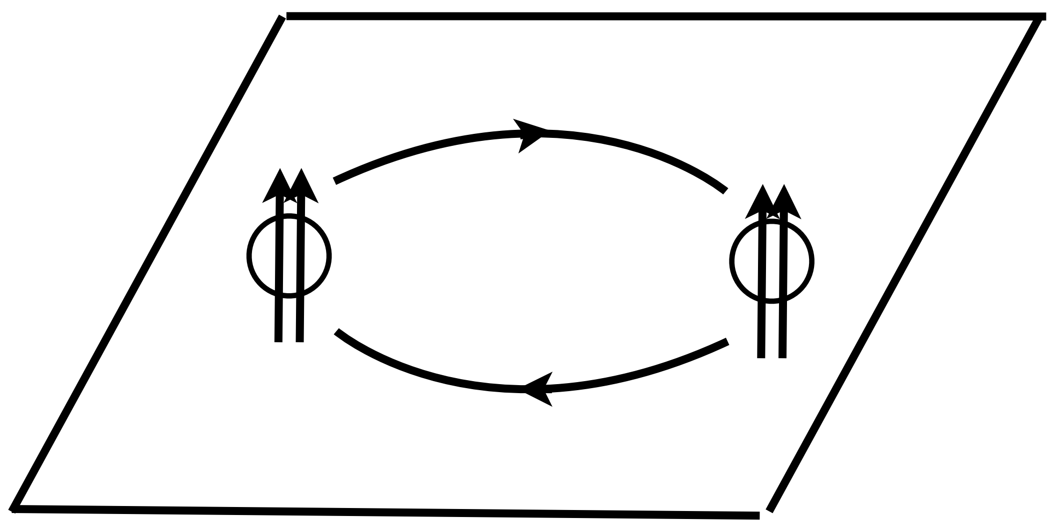

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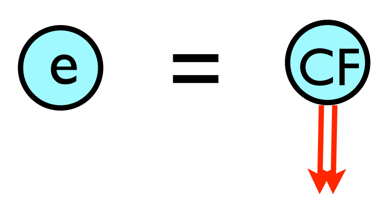
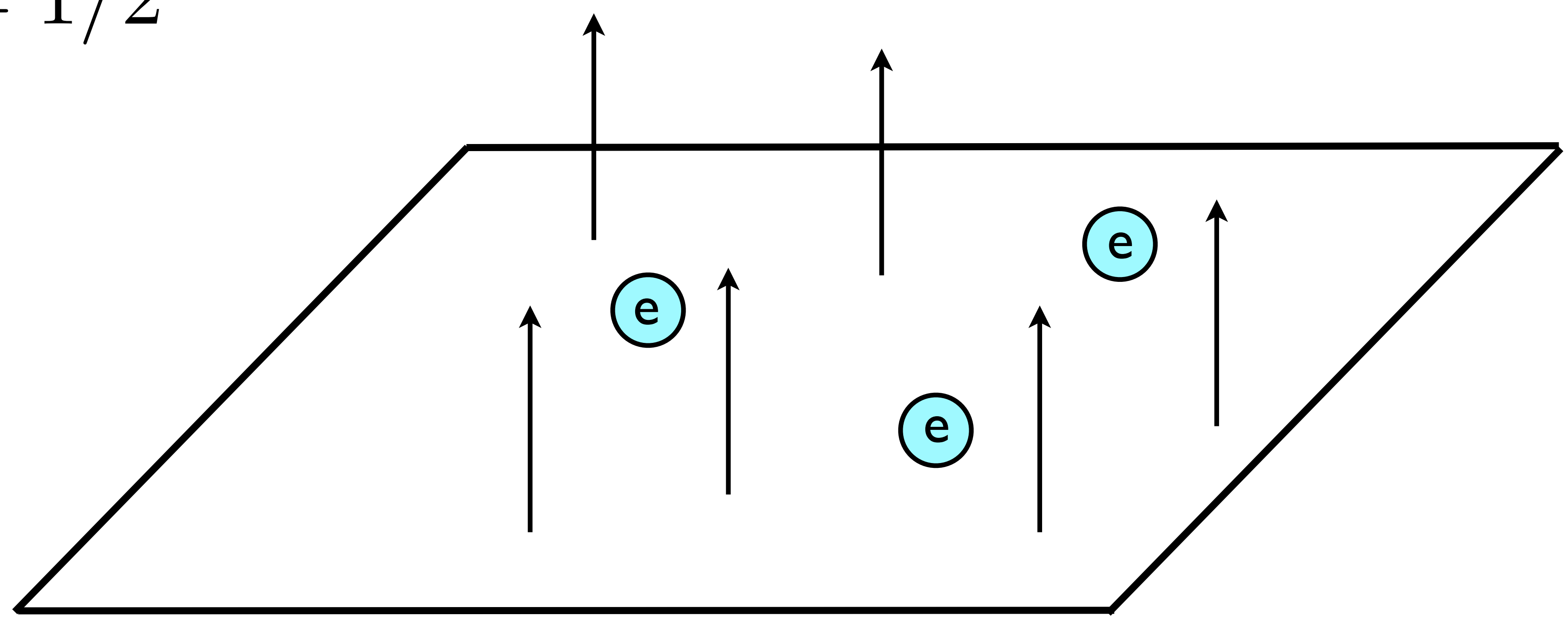
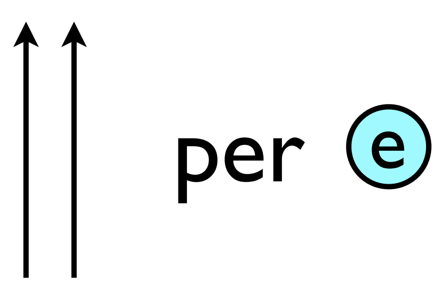


Composite fermion

$\nu = 1/2$





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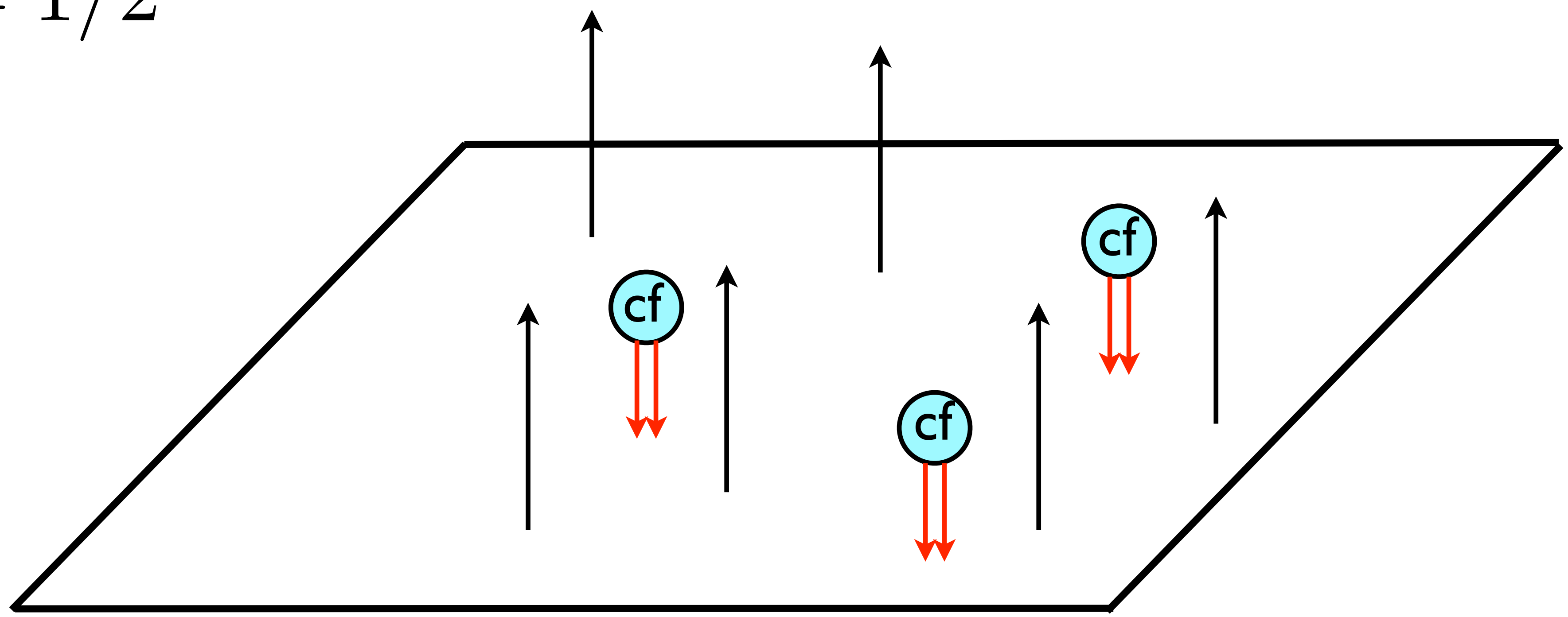

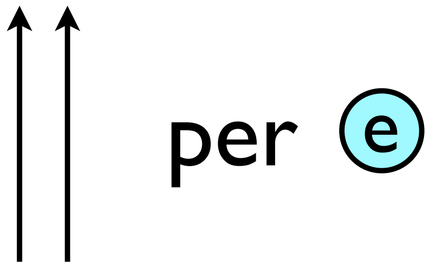

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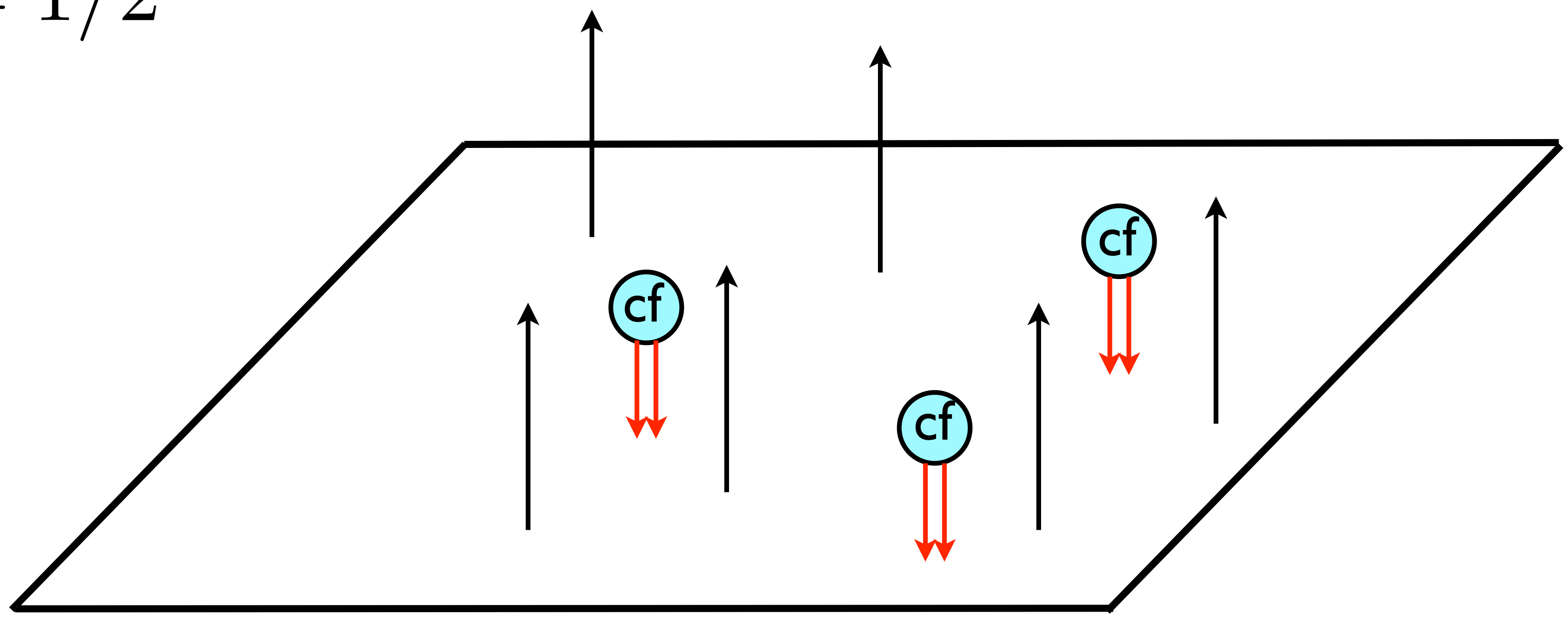
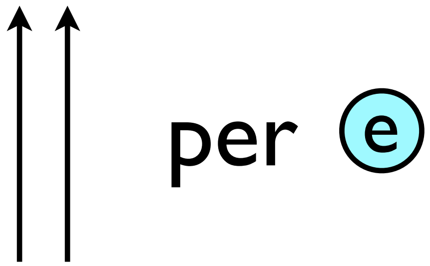
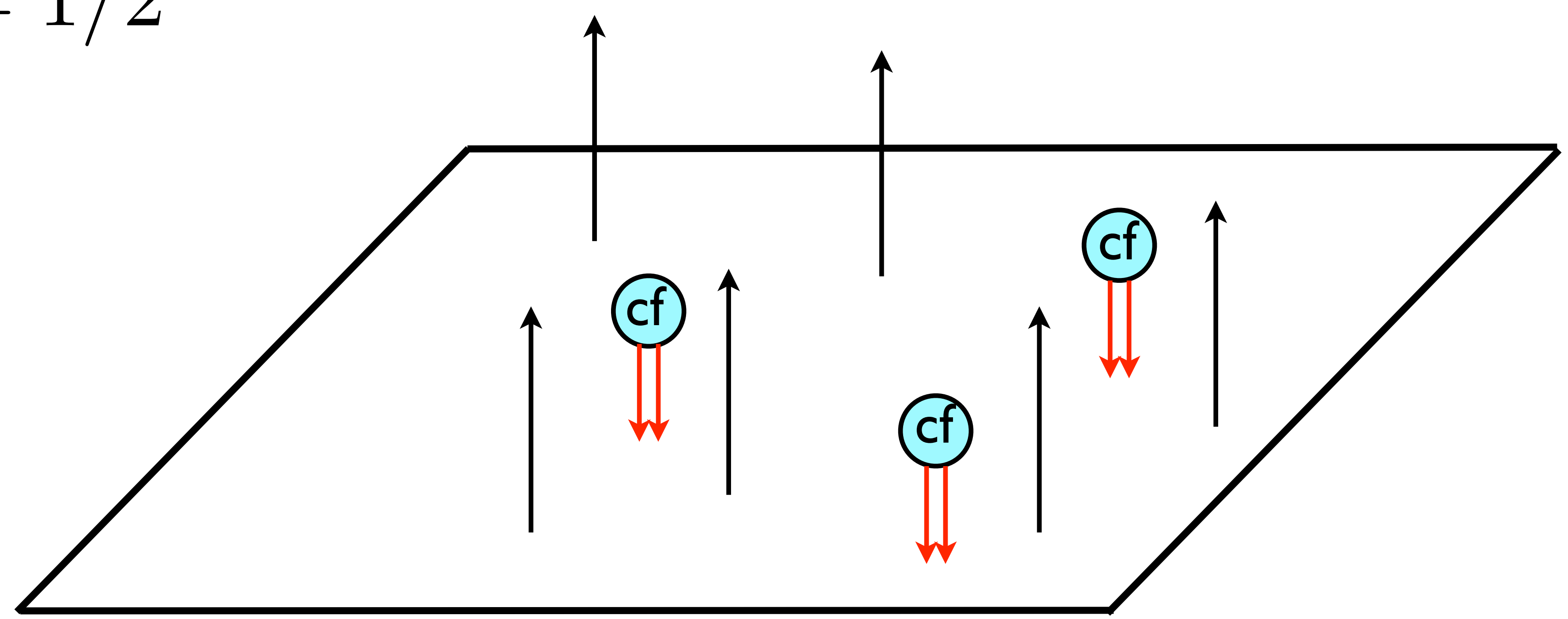


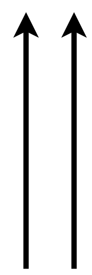


average zero field

Composite fermion

$\nu = 1/2$





per e

average zero field

Fermi liquid of CF

Jain's sequence of plateaux

- Composite fermion feels a reduced magnetic field

$$B_{\text{eff}} = B - 2\pi n$$

smaller magnetic field → larger filling factor

Electrons	Composite fermions
$\nu = \frac{n}{2n+1}$	n filled Landau levels
$\nu = \frac{n+1}{2n+1}$	n+1 filled Landau levels

Jain's sequence of plateaux

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Electrons	Composite fermions
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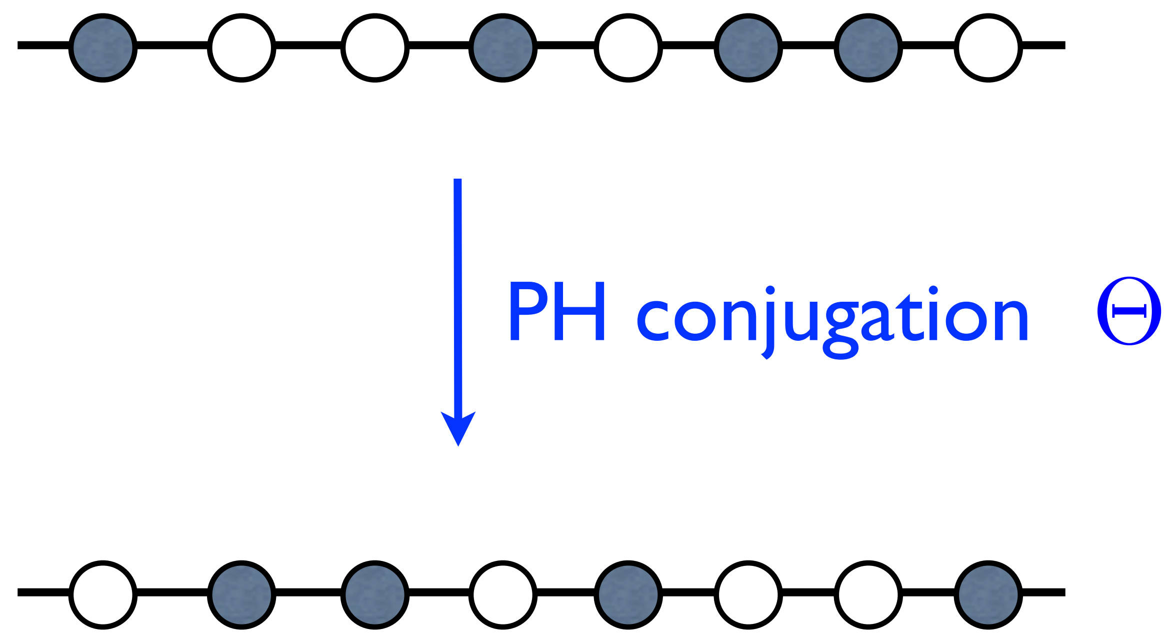
gapped

(end of the old story)

The mystery of a missing symmetry

Particle-hole symmetry

Girvin 1984



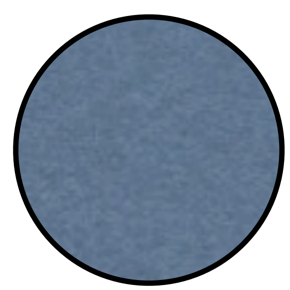
$$\nu \to 1 - \nu$$

$$\nu = \frac{1}{2} \to \nu = \frac{1}{2}$$

wikipedia.org

Half empty = half full

PH symmetry of a Fermi liquid?



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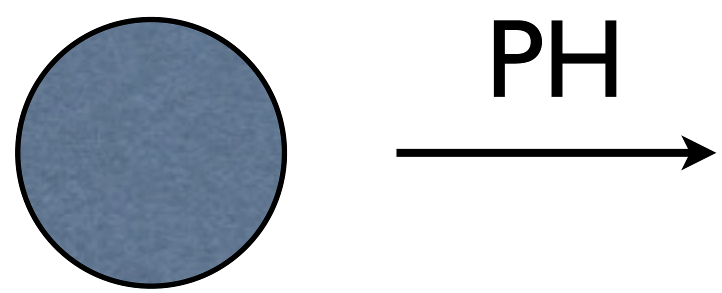

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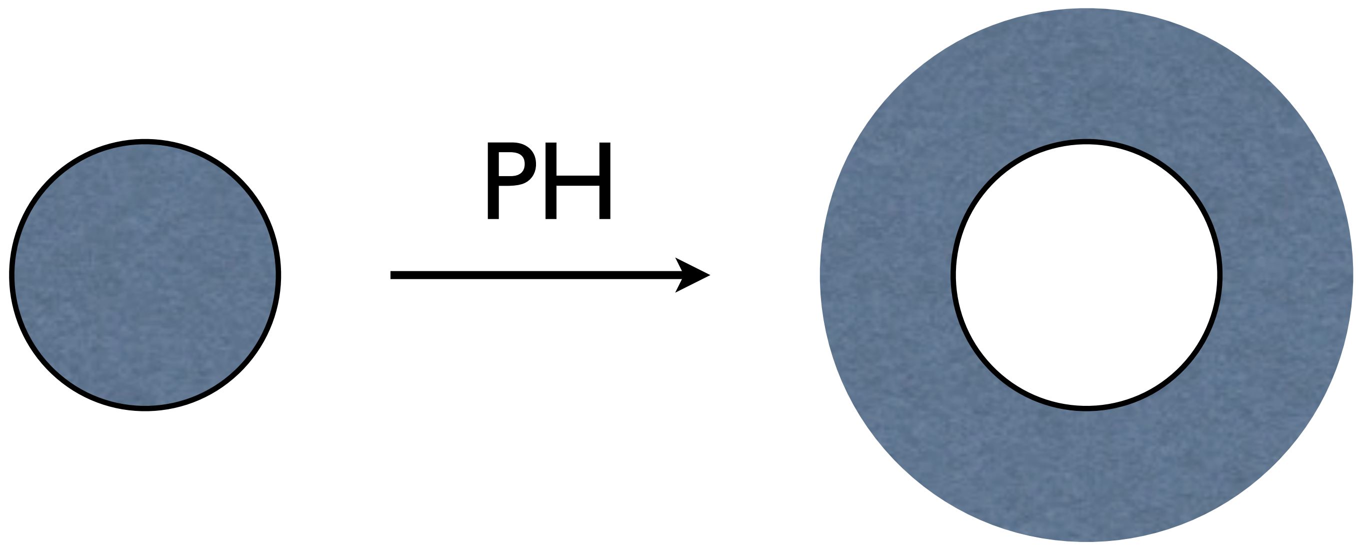


PH asymmetry in the CF theory

CF representation of PH conjugate pairs of FQH states

$$\nu = \frac{n}{2n+1}$$

$$\nu = \frac{n+1}{2n+1}$$

ν=1/3

ν=2/3

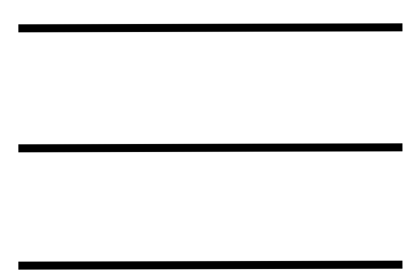
PH asymmetry in the CF theory

CF representation of PH conjugate pairs of FQH states

$$\nu = \frac{n}{2n+1}$$

$$\nu = \frac{n+1}{2n+1}$$

ν=2/5



ν=3/5

PH asymmetry in the CF theory

CF representation of PH conjugate pairs of FQH states

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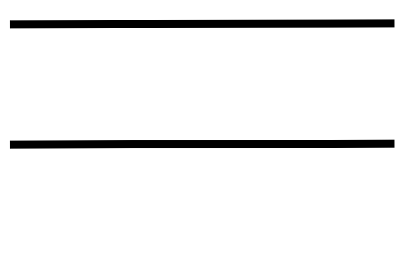
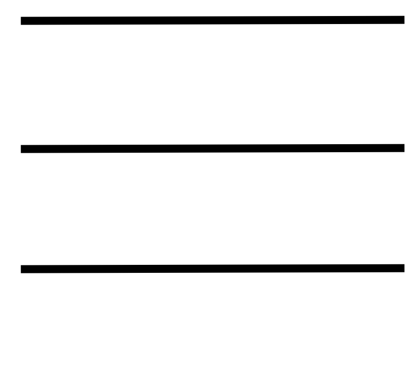


ν=3/7



ν=4/7

PH asymmetry in the CF theory

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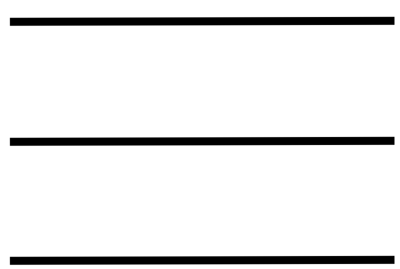
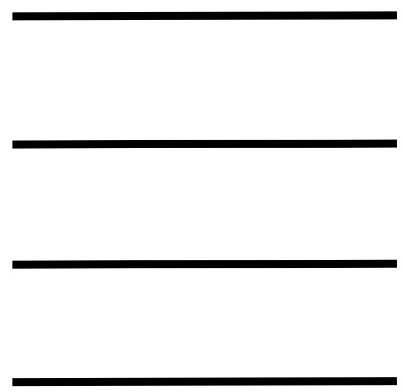


ν=3/7



ν=4/7

CF picture does not respect PH symmetry

- The particle-hole asymmetry of the CF picture has been known for a long time Kivelson et al 1997

- only recently a solution has appeared:
  - The composite fermion is a Dirac fermion

Fermionic particle-vortex duality

DTS; Metlitski, Vishwanath; Senthil, Wang 2015

Dirac fermion in (2+1)D $\mathcal{L} = i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$

QED in (2+1)D $\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu a_\lambda$

A fermionic version of od particle-vortex duality Peskin; Halperin Dasgupta 1970s

Particle-vortex duality

original fermion	dual fermion
magnetic field	density
density	magnetic field

$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda \right]$$

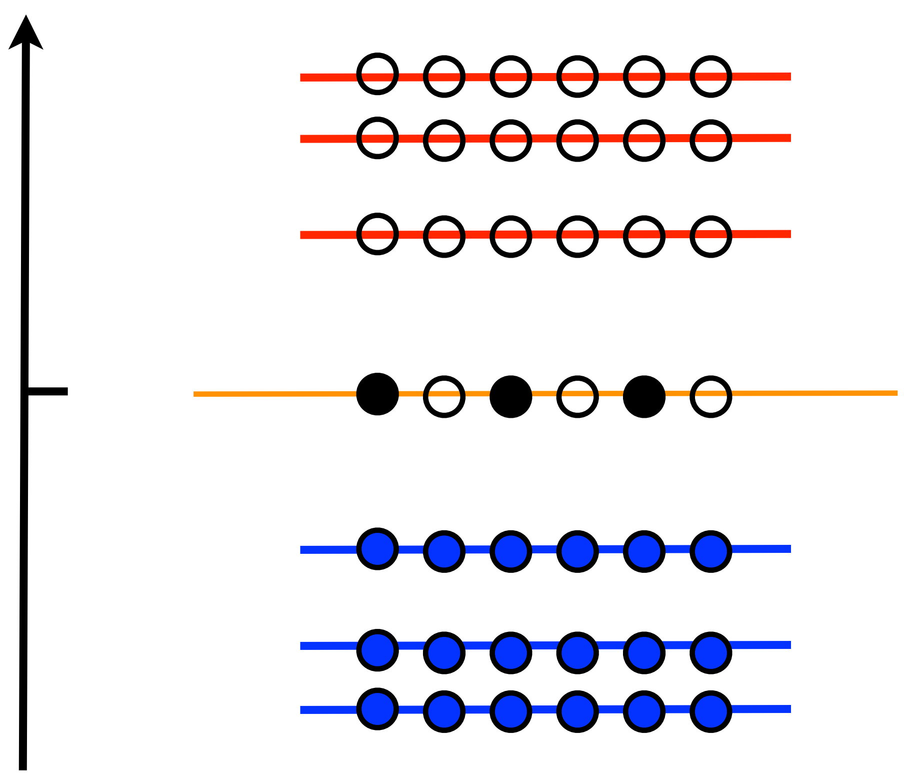
$$\rho = \frac{\delta S}{\delta A_0} = -\frac{b}{4\pi}$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^0\psi \rangle = \frac{B}{4\pi}$$

- In the FQHE the original fermion is the electron
- the dual fermion is the composite fermion
- Their numbers do not have to match

$$\Theta^2 = (-1)^{N_{\mathrm{CF}}} \neq (-1)^{N_e}$$

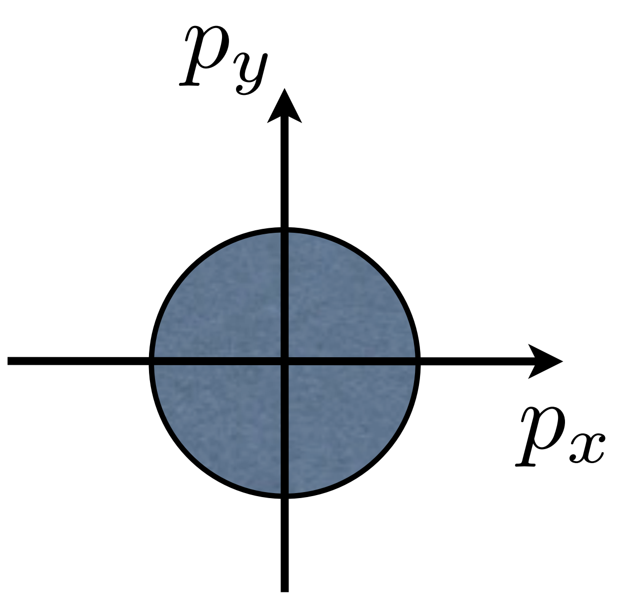
- We still don't have an explicit duality transformation



original fermion

B≠0

n=0



dual fermion

n≠0

b=0

PH-Pfaffian state

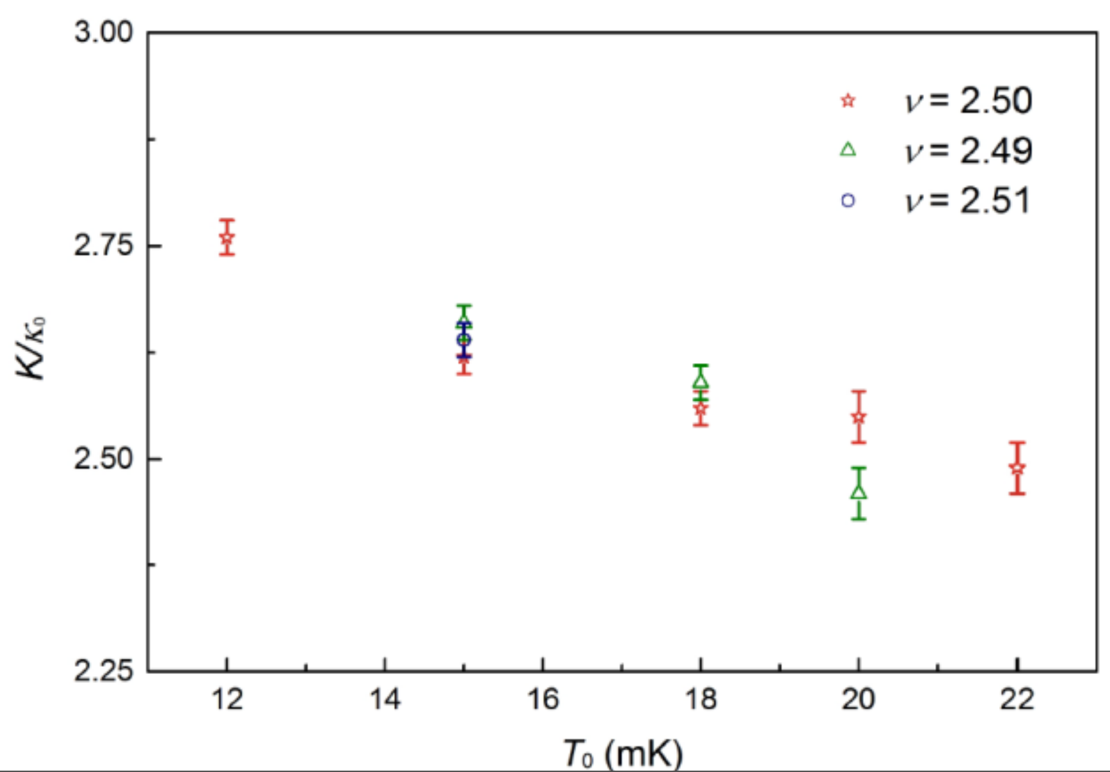
- Can dual fermions form Cooper pairs? seems to happen on the second Landau level
- Simplest pairing does not break particle-hole symmetry

$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

  - (cf quark pairing Rajagopal Wilczek)
- A new gapped state: PH-Pfaffian state
- Previously proposed Pfaffian and anti-Pfaffian states: pairing with orbital angular momentum ±2

Nature of nu=5/2 state?

- numerics prefers Pfaffian or antiPfaffian
- recent experiment prefers PH-Pfaffian
  - measures edge thermal Hall coefficient, which distinguishes PH-Pfaffian (κ=5/2) from Pfaffian (κ=7/2) or anti-Pfaffian (κ=3/2) Banerjee, Heiblum... 2017



Predictions of Dirac CF theory

- Transport predictions
  - Hall conductivity = 1/2, a longitudinal thermoelectric coefficient =0 at half filling Potter, Serbyn, Vishwanath 2015
- Absence of Friedel oscillations for PH-symmetric operators; verified numerically Geraedts et al 2015
-

Seed duality and web of dualities

- Karch and Tong, and independently Seiberg, Senthil, Wang and Witten, showed that both the bosonic and fermionic particle-vortex duality can be obtained from a single "seed" duality
    - fermion = boson + flux

Outlook

- Field-theoretic dualities in dimensions larger than 2 are still not well understood
- Fractional quantum Hall effect is a experimental window to these dualities
- Nontrivial relationship to spin liquids, interacting surfaces of topological insulators.

Extra slides

Seed duality

Karch and Tong PRX 2016

$$S_{\text{fermion}}[\psi; A] = \int d^3x \; i\bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi + \ldots \qquad S_{\text{scalar}}[\phi; A] = \int d^3x \; |(\partial_\mu - iA_\mu)\phi|^2 + \ldots$$

$$S_{CS}[A] = \frac{1}{4\pi}\int d^3x \; \epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \qquad S_{BF}[a; A] = \frac{1}{2\pi}\int d^3x \; \epsilon^{\mu\nu\rho}a_\mu\partial_\nu A_\rho$$

$$Z_{\text{fermion}}[A] = \int \mathcal{D}\psi \; \exp\left(iS_{\text{fermion}}[A]\right)$$

$$Z_{\text{scalar+flux}}[A] = \int \mathcal{D}\phi\mathcal{D}a \; \exp\left(iS_{\text{scalar}}[\phi; a] + iS_{CS}[a] + iS_{BF}[a; A]\right)$$

$$\boxed{Z_{\text{fermion}}[A]\, e^{-\frac{i}{2}S_{CS}[A]} = Z_{\text{scalar+flux}}[A]}$$

- from the seed duality, a whole "web of dualities" can be derived
- example: self-duality of 2 fermions coupled to a U(1) gauge field
  - can be verified on the lattice Kartik, Narayanan 2015-2018

Status

- The dualities have passed many self-consistency checks,
- but there are no proof
- The situation is increasingly common in physics but perhaps one should not get used to it

Consequences of PH symmetry

$$\mathbf{j} = \sigma_{xx}\mathbf{E} + \sigma_{xy}\mathbf{E}\times\hat{\mathbf{z}} + \alpha_{xx}\boldsymbol{\nabla}T + \alpha_{xy}\boldsymbol{\nabla}T\times\hat{\mathbf{z}}$$

conductivities

thermoelectric coefficients

- At exact half filling, in the presence of particle-hole symmetric disorders

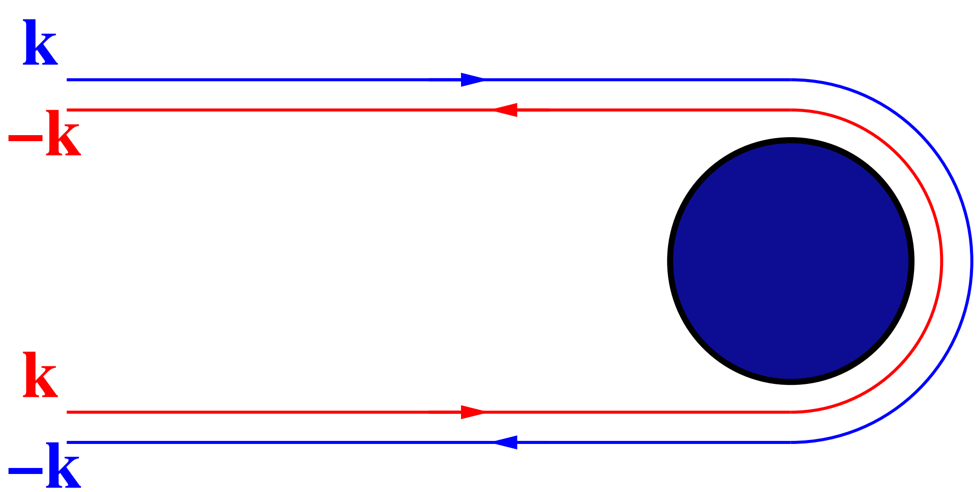
$$\sigma_{xy} = \frac{e^2}{2h} \qquad \alpha_{xx} = 0$$

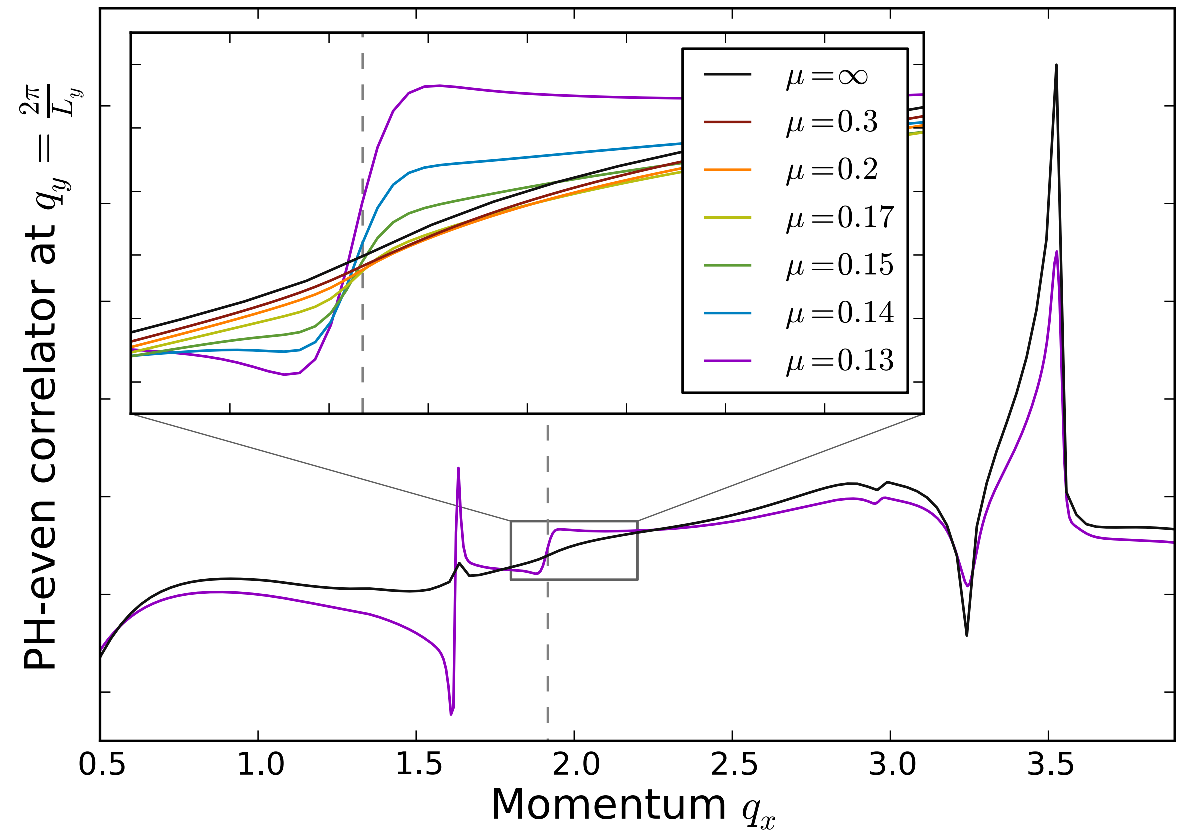
HLR $\rho_{xy} = \frac{2h}{e^2}$

Potter, Serbyn, Vishwanath 2015

Consequences of Dirac CF

Suppression of Friedel oscillations in correlations of particle-hole symmetric observables $\hat{O} = (\rho - \rho_0)\nabla^2\rho$





Geraedts, Zaletel, Mong, Metlitsky, Vishwanath, Montrunich, 2015

Direct proof of Berry phase π of the composite fermion