HOW TO FIND THE QCD CRITICAL POINT AT RHIC, IF IT IS AT $\mu_B \lesssim 400$ MeV

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Critical Point and the Onset of Deconfinement

June 2009, Brookhaven

HOW TO FIND THE QCD CRITICAL POINT AT RHIC,

IF IT IS AT $\mu_B \lesssim 400$ MeV

KRISHNA RAJAGOPAL (MIT)

Talks and discussions at:

INT program, August 2008

CPOD conference, June 2009

and reviews:

Koch 0810.2520 ; Lombardo 0808.3101 ;

Philipsen 0808.0672 ; Karsch 0711.0658 ;

Stephanov hep-lat/0701002

have all been very helpful as I prepared this talk.

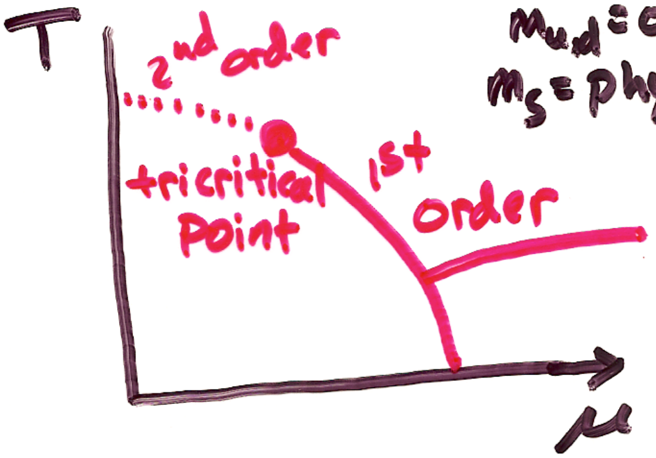
This talk is **NOT** a summary of CPOD.

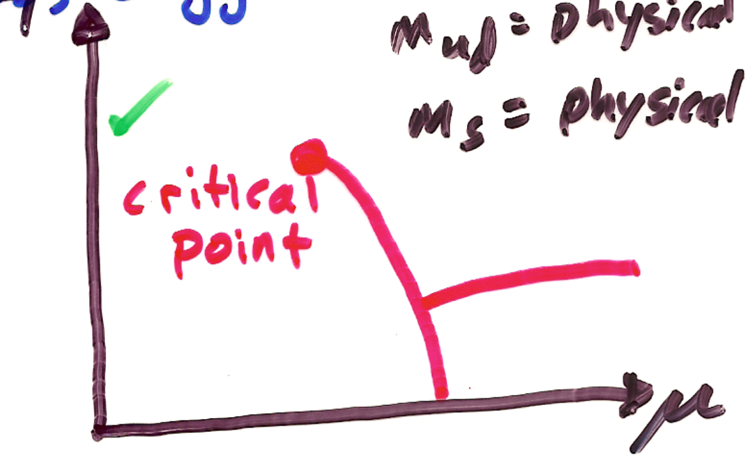
It is my attempt to describe a suite of measurements that RHIC can do that will either find experimental evidence for the QCD critical point or demonstrate convincingly that it is not at $\mu \leq 400$ MeV.

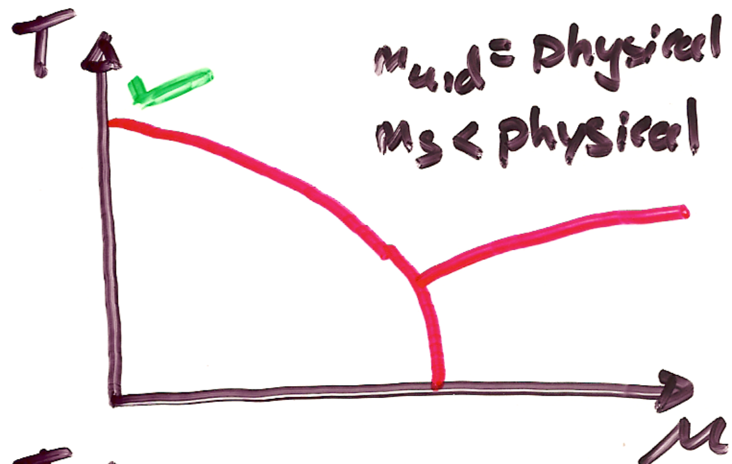
This talk is my attempt to describe a suite of measurements that RHIC can perform that will either find experimental evidence for the QCD critical point or demonstrate convincingly that it is not at $\mu \lesssim 400$ MeV.

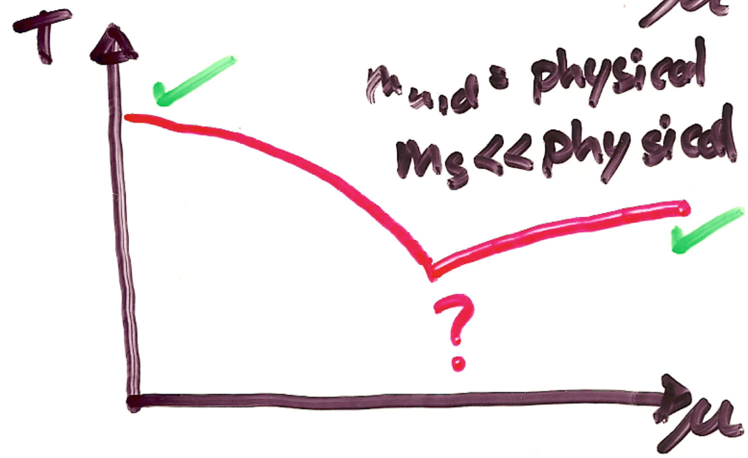
WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at $\mu = 0$ with varying quark masses, suggest:









✓ : known

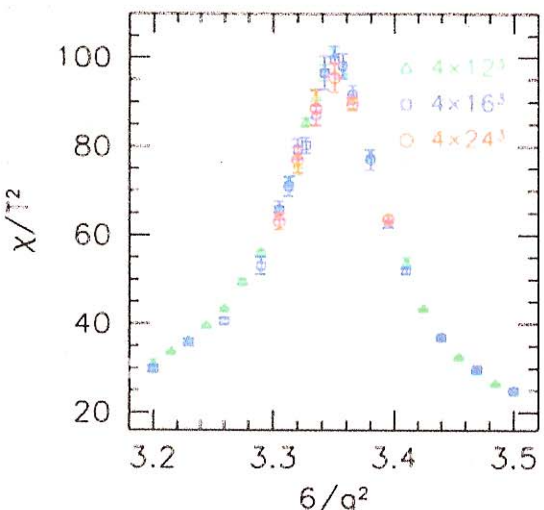
- Universality class of the QCD critical point is known. (ISING)
- Experiments, and lattice calculations with $T \neq 0, \mu \neq 0$, needed to locate it.

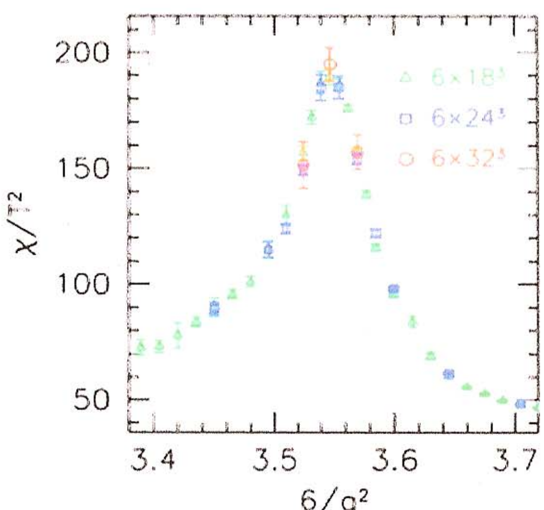
Order of Phase Transition for $\mu_B \sim 0$

Physical quark masses
Continuum limit
Simulations along Lines of Constant Physics
$m_K/m_\pi = 3.689$; $f_K/m_\pi = 1.185$
Staggered fermionic action

$$\chi(N_s, N_t) = \partial^2/(\partial m_{ud}^2)(T/V) \cdot \log Z$$

Y. Aoki et al., Nature443:675-678,2006





1st order :
Peak height ~ V
Peak width ~ 1/V
Cross over :
Peak height ~ const.
Peak width ~ const.
2nd order :
Peak height ~ V^α

T grows with $6/g^2$, g : gauge coupling

No significant volume dependence (8 times difference in volumes)

Phase transition at high T and $\mu_B = 0$ is a cross over

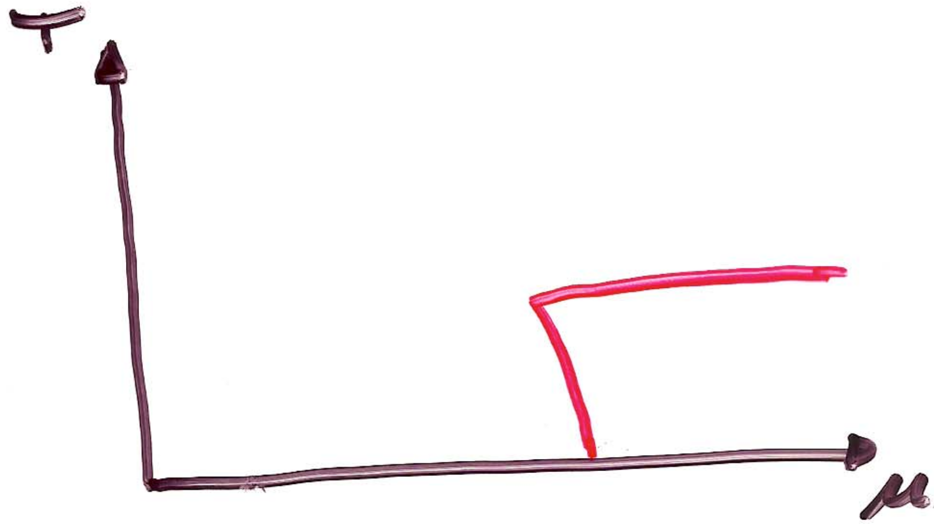
Relevant to LHC and current RHIC regimes

Lattice results on electroweak transition in standard model is an analytic cross-over for large Higgs mass

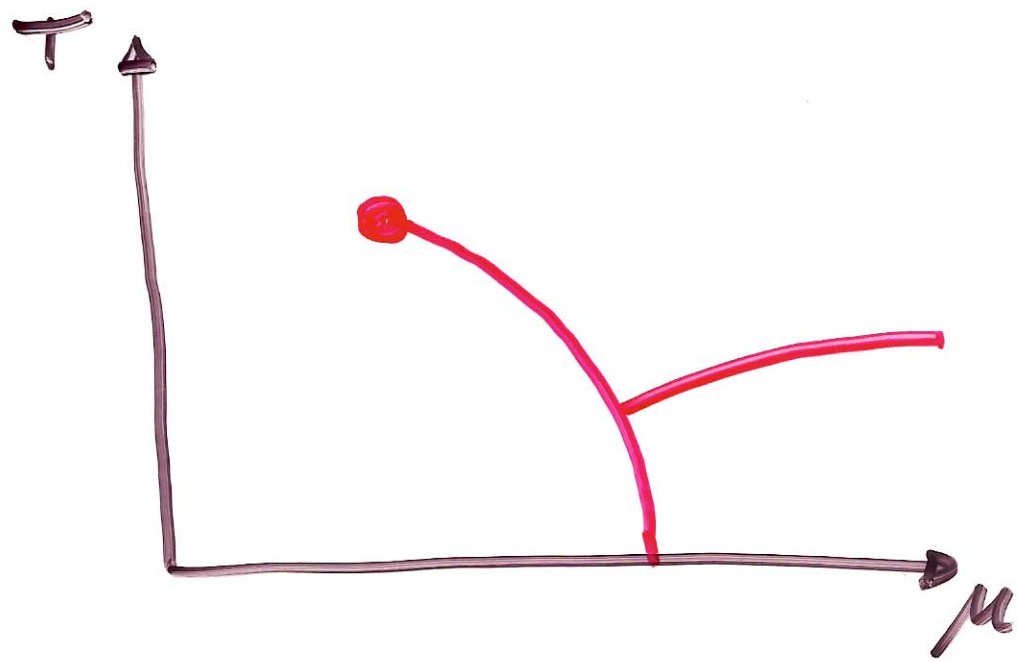
K. Kajantie et al., PRL 77, 2887-2890,2006

WARNING

Nothing we know precludes pushing the critical point so far to the right that:



although models and some lattice calculations favor



LOCATING THE CRITICAL POINT...

- either via lattice calculations
- or via experimental detection of its signature

would add a point and a line to the known QCD phase diagram.

An opportunity to write a new chapter in any future textbook on QCD.

LATTICE QCD WITH $T \neq 0$, $\mu \neq 0$, μ/T NOT LARGE

- $\mu \neq 0 \rightarrow$ complex Euclidean action
  - $\rightarrow$ sign problem
  - $\rightarrow$ difficulty of standard monte carlo $\sim \exp V$
- Several lattice methods now in use rely on smallness of $\mu_q/T = \mu_B/3T$ to control the sign problem:
  - reweighting (Fodor + Katz)
  - continue from imaginary μ (de Forcrand & Philipsen; D'Elia & Lombardo)
  - Taylor expansion of P; radius of convergence (RBC-Bielefeld; Gavai + Gupta)

  Uncertainties still dominated by systematics; (different systematics for different methods, but in all cases includes coarseness of lattice spacing)

  Steady progress; "crawling towards the continuum limit".
- Several groups exploring calculations at fixed n_B, instead of μ. (deForcrand + Kratochvila; Li, Alexandru & Liu; ...)

LATTICE RESULTS

- via reweighting (Fodor & Katz)

$$\mu_0 = 360 \pm 40^* \text{ MeV}$$

- via continuation from imaginary μ (de Forcrand & Philipsen)

$$\frac{\mu_0}{T_0} > 3$$

- via radius of convergence of Taylor expansion

$$\frac{\mu_0}{T_c(\mu=0)} = 1.7 \pm .1^* \quad \text{(Gavai + Gupta)}$$

$$> 2 \quad \text{(RBC - Bielefeld)}$$

*: statistical errors only.

⇒ clearly still systematics - dominated

BUT, STILL, ONE LESSON:

Lattice calculations provide strong evidence that $\boxed{\mu_0 > 200 \text{ MeV}}$

HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

① At large $\sqrt{s}$, ie small μ, collisions equilibrate well above the crossover....

② Decrease $\sqrt{s}$ in steps, moving freezeout point to larger and larger μ.

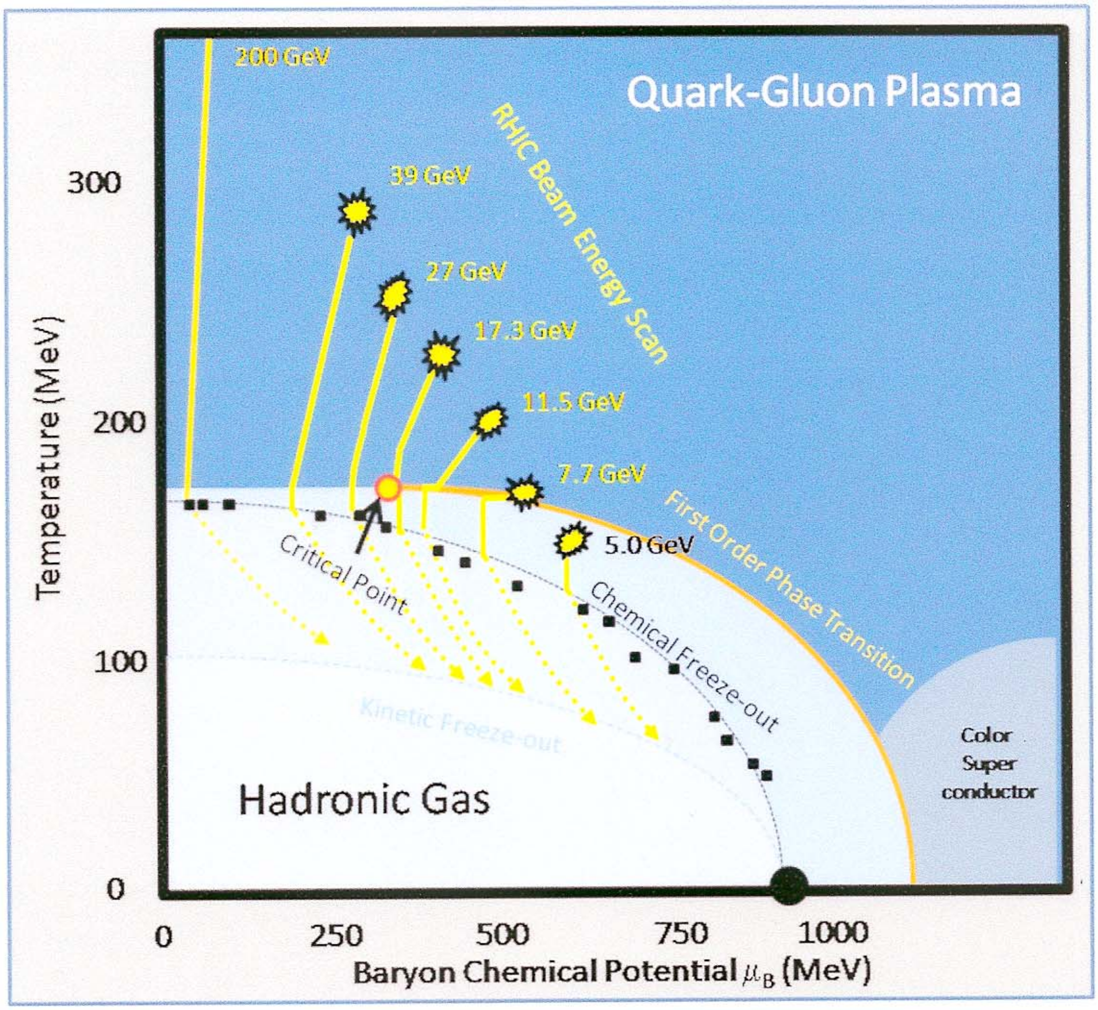
③ Look for signatures:

a) Of the critical point itself. I.e. signatures of the long wavelength fluctuations occurring only near ●. Rise and then fall as μ increases.

b) Onset of signatures of non-equilibrium "lumpy" final state expected after cooling through a first order transition. (Mishustin; Dumitru Paech Stöcker; Randrup; Koch Majumder Randrup; ....)

Eg: enhanced event-by-event fluctuations of v_2.

SEARCHING FOR THE CRITICAL POINT



Decreasing $\sqrt{s}$: decreases T and increases μ at which collision equilibrates, "landing on the phase diagram".
$\Rightarrow$ increases μ at which the trajectory followed by the cooling plasma crosses the transition or crossover.

NB: location of ● in fig. is merely illustrative – we don't know where ● is!

ISENTROPIC TRAJECTORIES

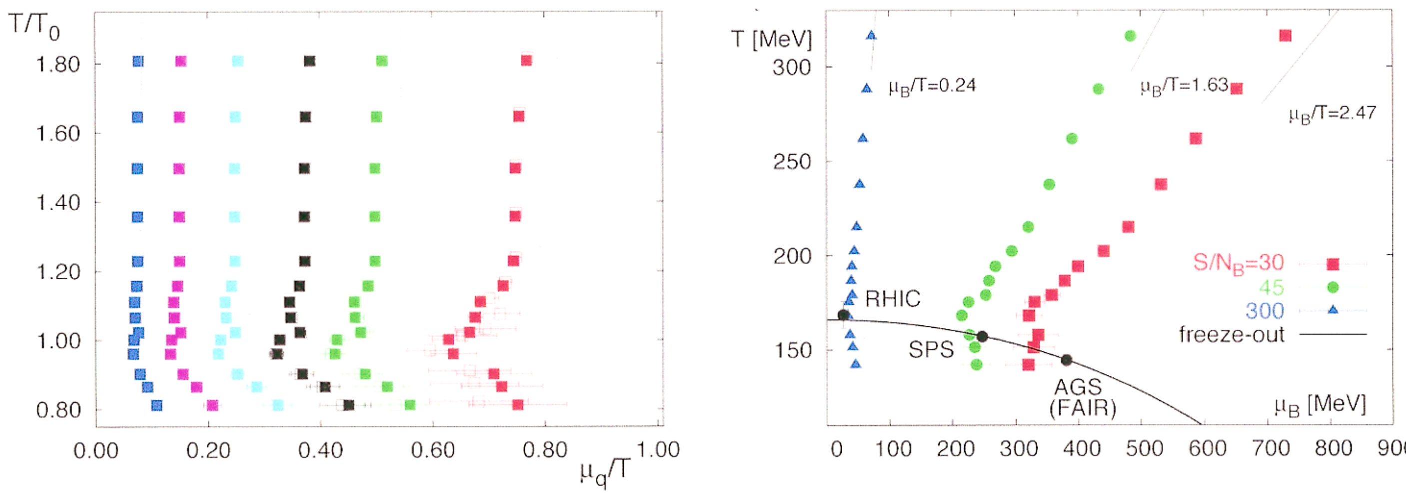


FIG. 3: Lines of constant entropy per quark number versus μ_q/T (left) and in physical units using $T_0 = 175$ MeV to set the scales (right). In the left hand figure we show results obtained using a 4^{th} (full symbols) and 6^{th} (open symbols) order Taylor expansion of the pressure, respectively. Data points correspond to $S/N_B = 300, 150, 90, 60, 45, 30$ (from left to right). The vertical lines indicate the corresponding ideal gas results, $\mu_q/T = 0.08, 0.16, 0.27, 0.41, 0.54$ and 0.82 in decreasing order of values for S/N_B. For a detailed description of the right hand figure see the discussion given in the text.

Ejiri Karsch Laermann Schmidt

- shape of isentropic trajectories in QGP phase and in crossover region is known from lattice calculations
- isentropic trajectories zigzag as they cross first order line

HOW LOW TO GO?

Down to what $\sqrt{s}$ should we look?

$\Rightarrow$ Up to what μ can we look?

This question should be answered experimentally.

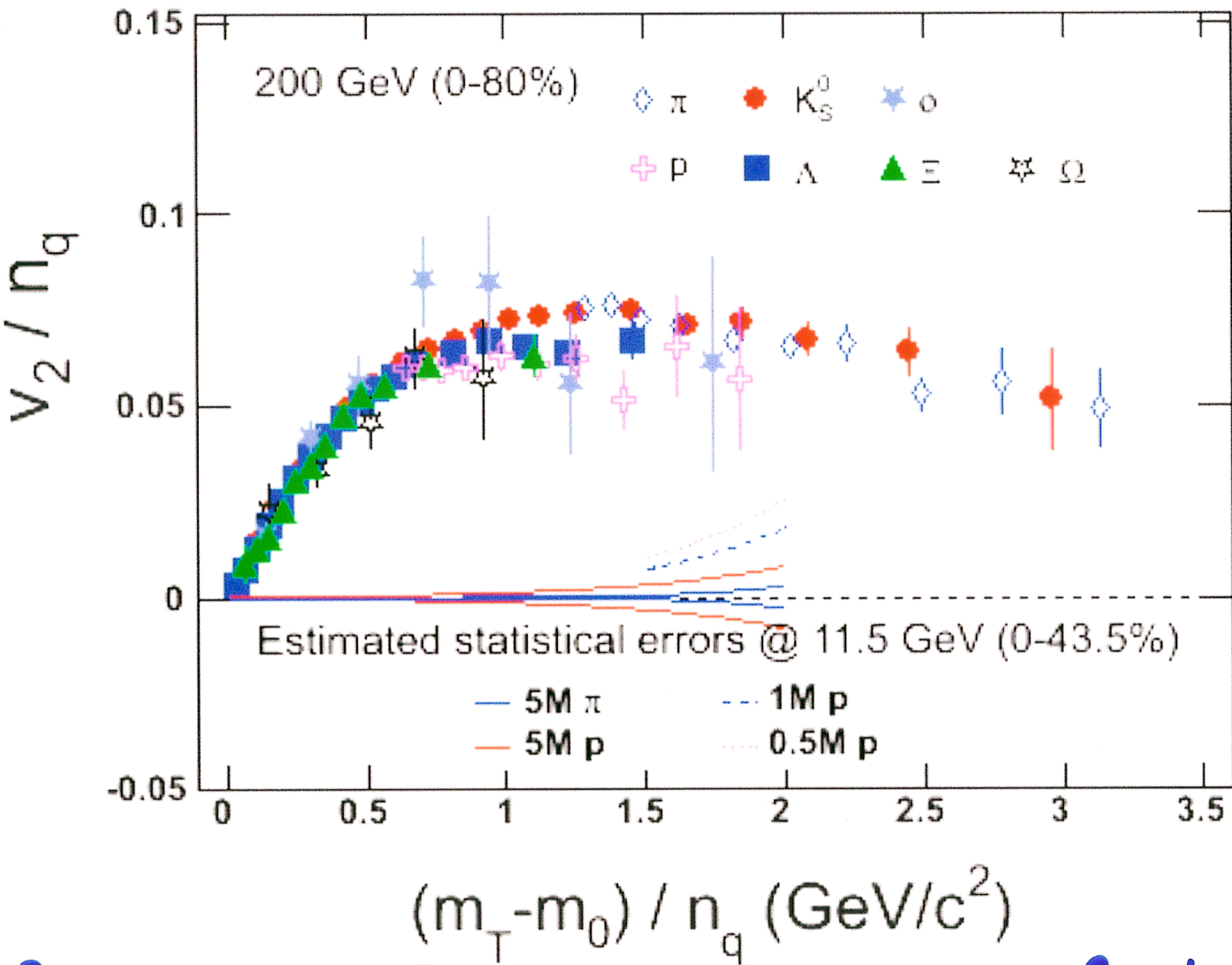
Need an effect that is:

- well-measured at $\sqrt{s} = 200$ GeV
- expected only in collisions that do begin above the crossover/transition
- expected at lower $\sqrt{s}$, as long as collisions do begin above crossover.

  [i.e. jet quenching won't do since that can turn off due to absence of jets]

Here are two suggestions....

n_q - SCALING OF V_2

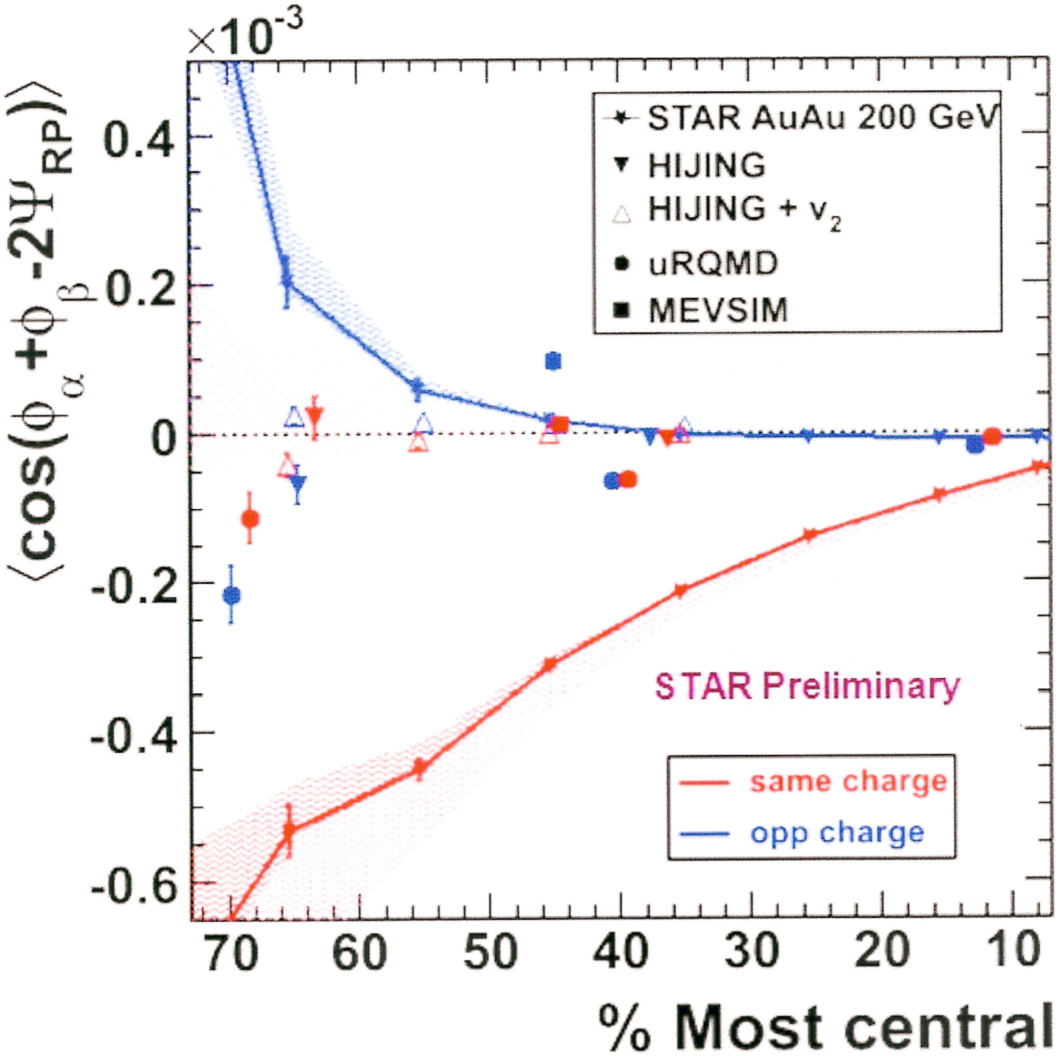


STAR B.U.R.

- V_2 same for mesons of varying mass; same for baryons of varying mass ⇒ V_2 developed before hadrons formed
- Measurement can be done for π/K/P/Λ with 5M min. bias events @ $\sqrt{s}$ = 11.5 GeV with errors shown

PARITY VIOLATING FLUCTUATIONS

Need 5M min. bias events per $\sqrt{s}$ to measure. (STAR B.U.R.)



- Data ⇒ charge separation !
  ⇒ an electric field (⊥ to reaction plane, parallel or antiparallel to $\vec{L}$ & $\vec{B}$) that is coherent over a volume corresponding to many charged particles
- No hadronic explanation of data
- Kharzeev's explanation requires deconfinement

So.....

1. Decrease $\sqrt{s}$ in steps
2. Measure the $\sqrt{s}$ at which
   - n_q scaling of v_2
   - parity violating fluctuations (charge separation)

   turn off.
3. You can only look for signatures of down to, or perhaps slightly below, that $\sqrt{s}$.
4. In this way, learn up to what μ heavy ion collision experiments can find .

SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of σ (ie fluctuations couple to $\pi\pi$ and $p\bar{p}$) and baryon number.

Fujii Ohtani; Son Stephanov

The longer the correlation length ξ gets, the bigger the signatures.

Signatures are event-by-event fluctuations of specific observables, calculable in magnitude in terms of ξ. Stephanov KR Shuryak

- Vary μ by varying $\sqrt{s}$
- Search for enhancement of these fluctuations in a window in $\sqrt{s}$, ie μ
- Analogue of critical opalescence
- Long wavelength fluctuations $\Rightarrow$ effects greatest at low $p_\perp$.

Examples ..... But, first :

SIGNATURES OF CRITICAL POINT

- Decreasing $\sqrt{s} \rightarrow$ Increasing μ
- Vary $\sqrt{s}$, and hence μ, and look for nonmonotonic enhancement (rise and then fall) of:

  i) Event-by-event fluctuations of mean p_T of low p_T pions

  ii) Event-by-event fluctuations of net proton number $(N_p - N_{\bar{p}})$

  iii) Event-by-event fluctuations of particle ratios involving pions and/or protons

  iv) Kurtosis of the N_p or $(N_p - N_{\bar{p}})$ event-by-event distribution

  ⋮

In all cases, the enhancement should be greater at lower p_T.

MEAN P_T OF LOW P_T PIONS

Stephanov KR Shuryak (1999)

First example of a quantititative connection between long wavelength fluctuations of the chiral order parameter with correlation length ξ and magnitude of event-by-event fluctuations of an experimental observable.

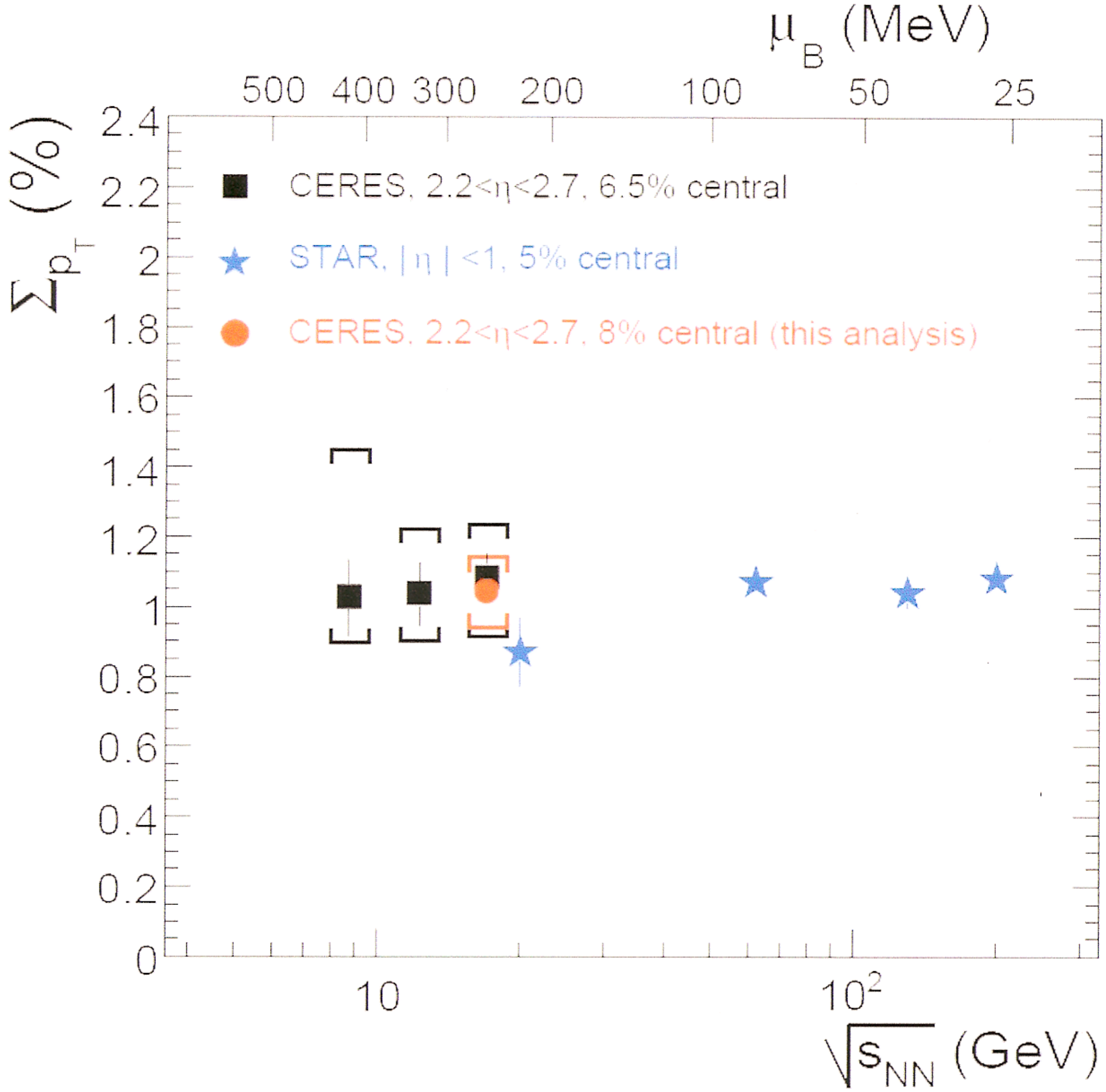
DISADVANTAGES:

- Effect predicted is not large for $\xi = 3$ fm
- Will fluctuations in P_T survive the late time hadronic gas? Will they get washed out between chemical and kinetic freezeout?

RESULT:

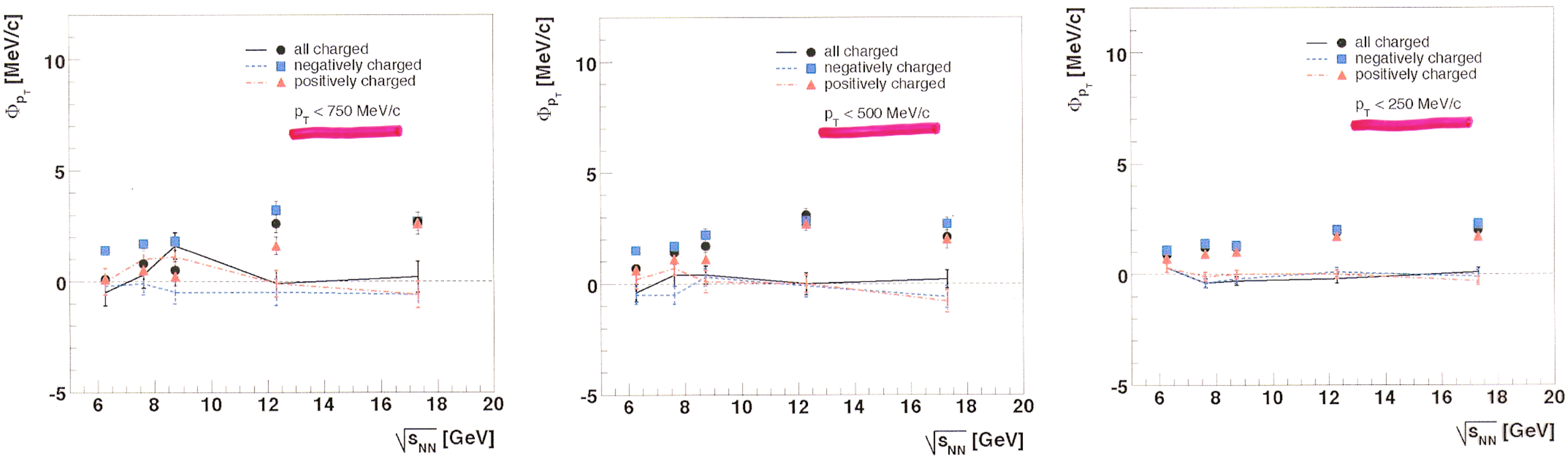
NA49 has done a beautiful analysis and sees no $\sqrt{s}$ dependence....
CERES has done a beautiful analysis and sees no $\sqrt{s}$ dependence....

EVENT-BY-EVENT FLUCTUATIONS OF MEAN p_T



CERES, Nucl Phys A, 2008

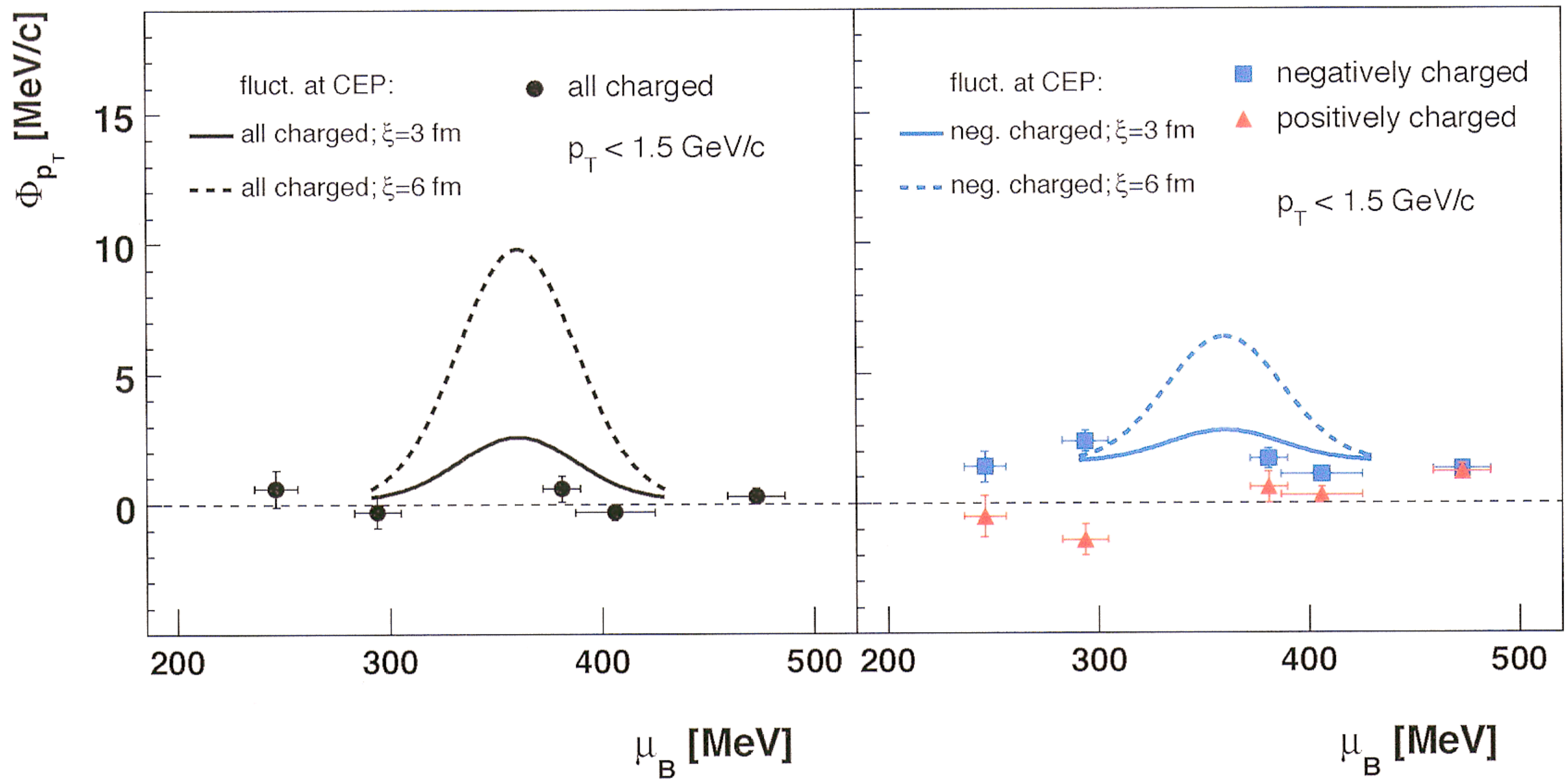
EVENT-BY-EVENT FLUCTUATIONS OF MEAN P_T OF PARTICLES WITH P_T BELOW A CUT



NA49, 2008

No enhancement at low P_T.

MEAN P_T FLUCTUATIONS, vs. μ



NA49, 2008

Height of solid (dashed) curve is magnitude of effect predicted for $\xi = 3$ fm ($\xi = 6$ fm).

POSSIBLE CONCLUSIONS

- $\mu_{\bullet} > 470$ MeV ?
- p_T fluctuations washed out.
  Predicted effect was not large, and was susceptible to being erased after chemical freezeout.

$\Rightarrow$ Look for event-by-event fluctuations of other observables

a) for which the predicted effect of proximity to the critical point is larger

b) which cannot be washed out after chemical freezeout

EVENT-BY-EVENT FLUCTUATIONS OF K/π AND p/π

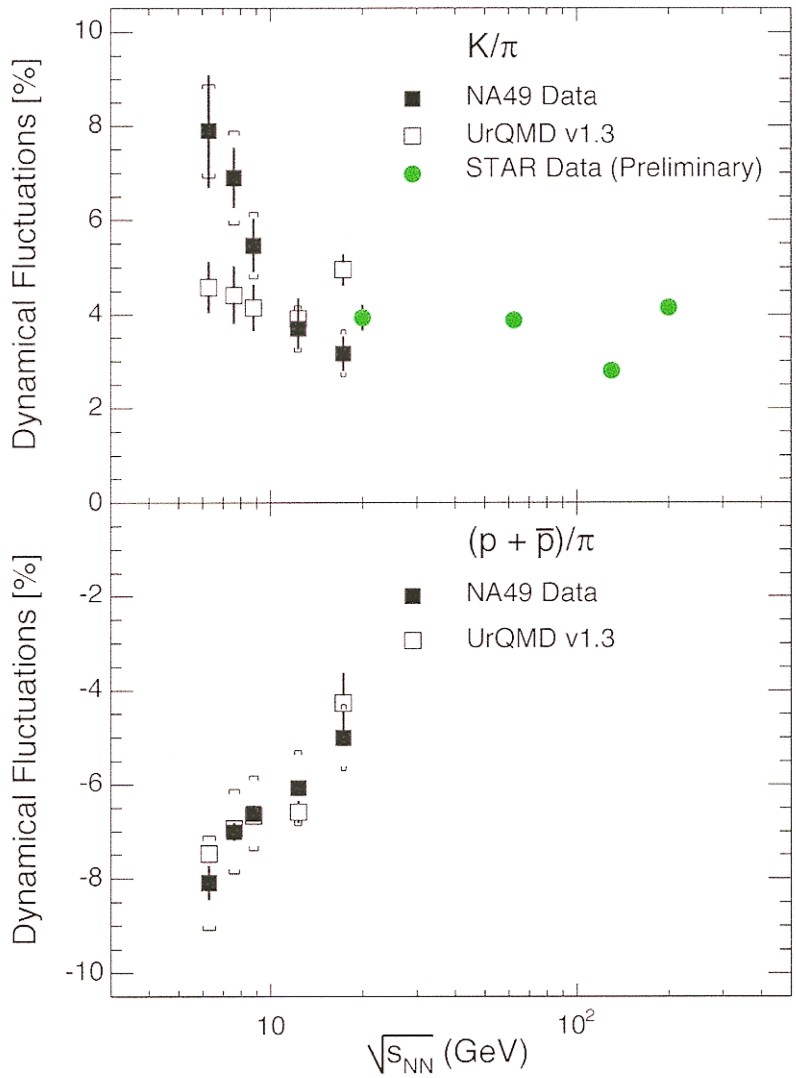


FIG. 8: Energy dependence of the event-by-event non-statistical fluctuations of the K/π ratio (top panel) and the $(p+\bar{p})/\pi$ ratio (bottom panel). Filled symbols show data, open symbols show calculations with the UrQMD transport code, using NA49 acceptance tables. Systematic uncertainties are shown as brackets.

NA49, 2008

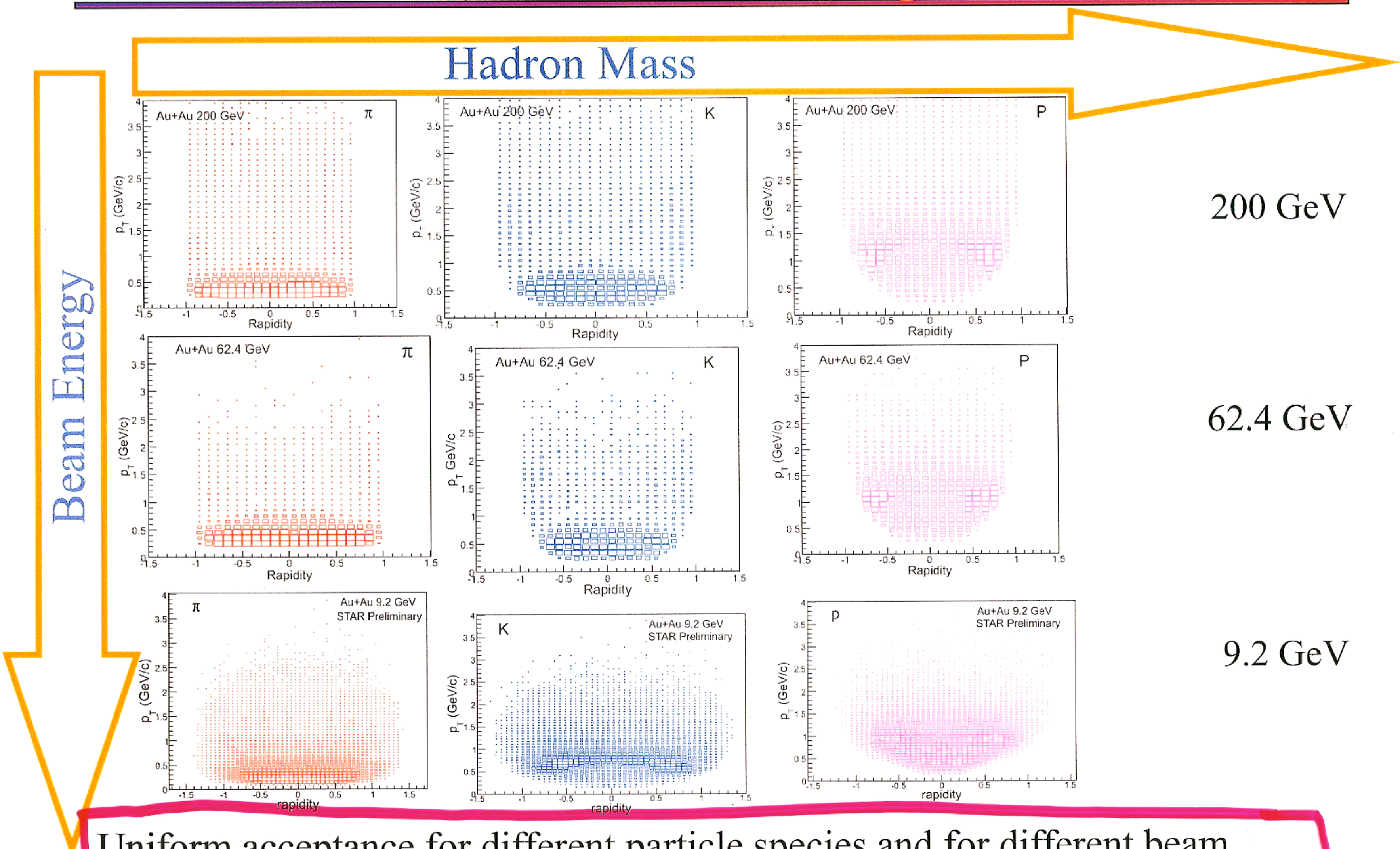
Intriguing....

- Large event-by-event fluctuations in K/π ratio at $\mu \sim 350-450$ MeV
- Why only "intriguing"?
  - error bars
  - why no p/π fluctuations?
    - → K/π fluctuations apparently not dominated by low p_T pions

STAR has used this observable as a case study to see how much they would be able to improve on this measurement with a beam energy scan at RHIC.

A collider has advantages....

RHIC Critical Point Search Program - Advantage



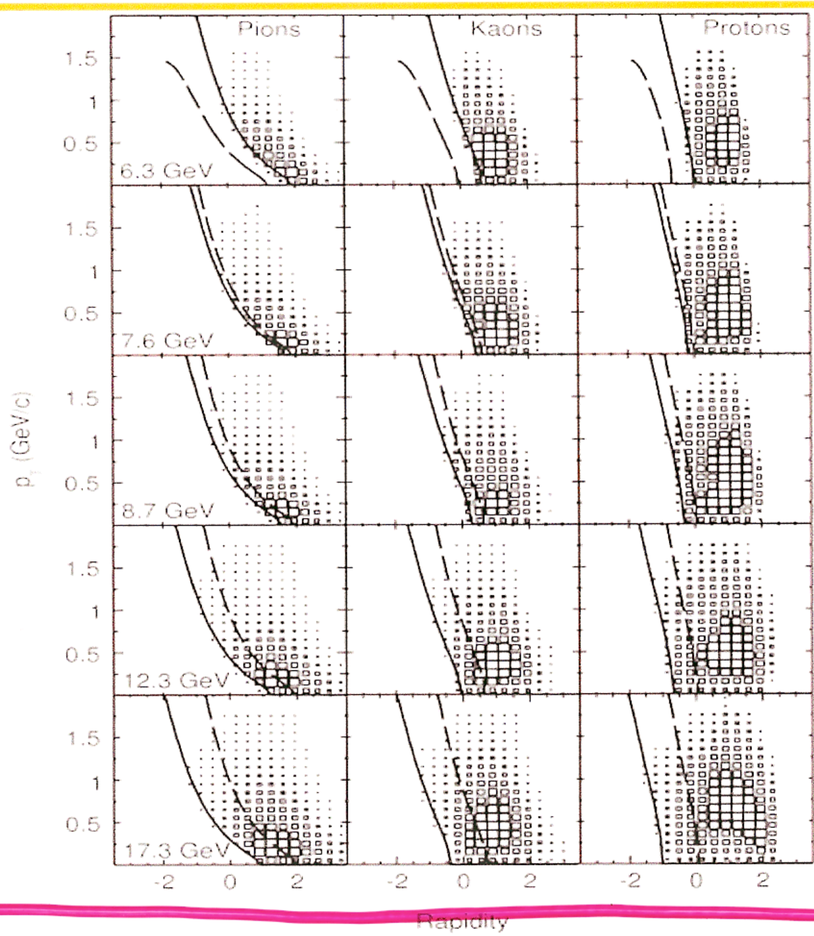
Uniform acceptance for different particle species and for different beam energies in the same experimental setup (advantage over fixed traget expt.)

Mohanty, QM 09

Old Program : Fixed Target

Hadron Mass

Beam Energy



NA49 : arXiv : 0808.1237

6.3 GeV

7.6 GeV

8.7 GeV

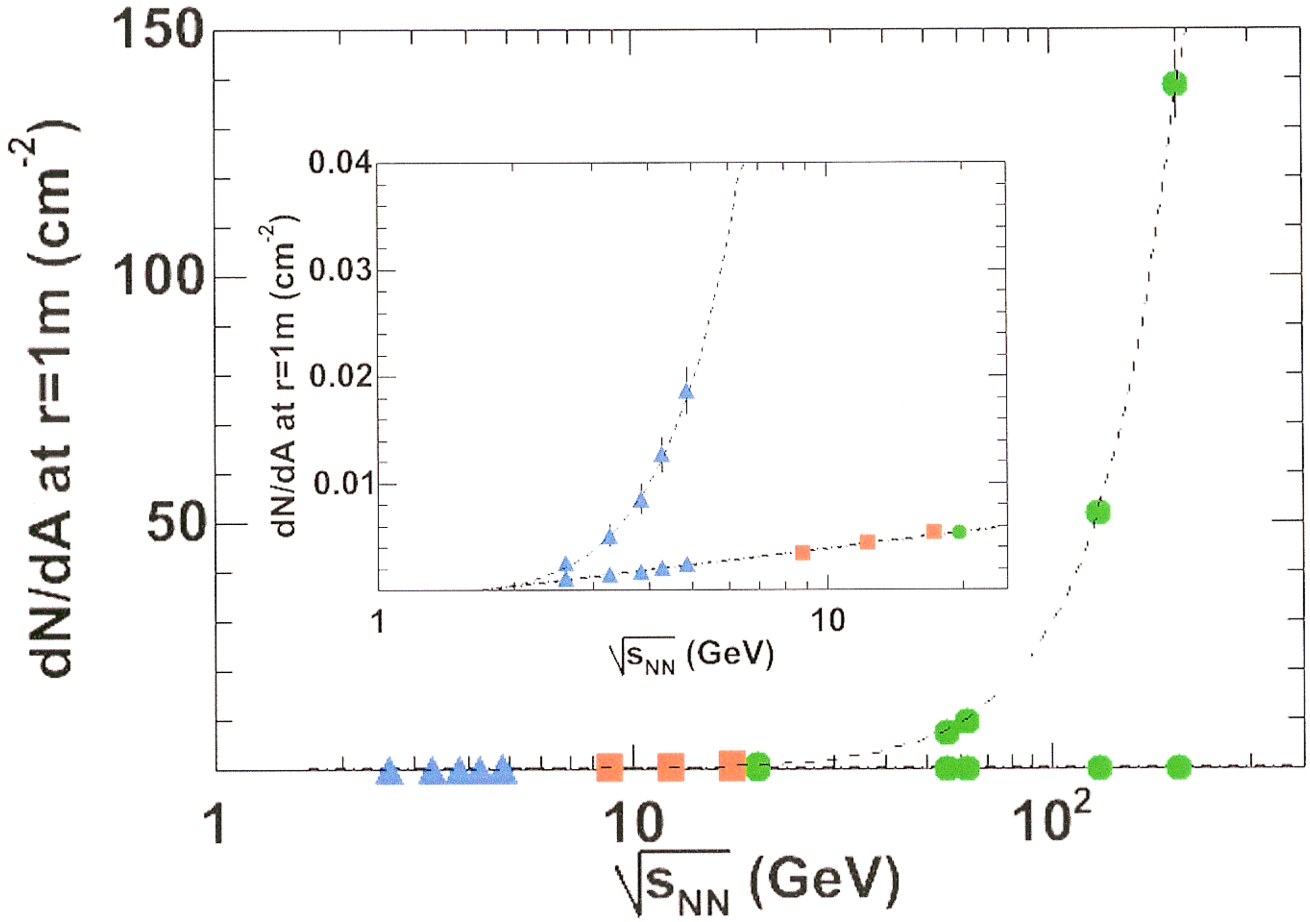
12.3 GeV

17.3 GeV

Non-Uniform acceptance for different particle species
and for different beam energies in the same experimental setup

Mohanty, QM09

RHIC Critical Point Search Program - Advantage



G. Roland

Collider experiment : Variation of particle density with beam energy slower. Occupancy in detectors reasonable compared to fixed target experiments at similar collision energy

Mohanty, QM09

K/π FLUCTUATIONS WITH A RHIC BEAM ENERGY SCAN

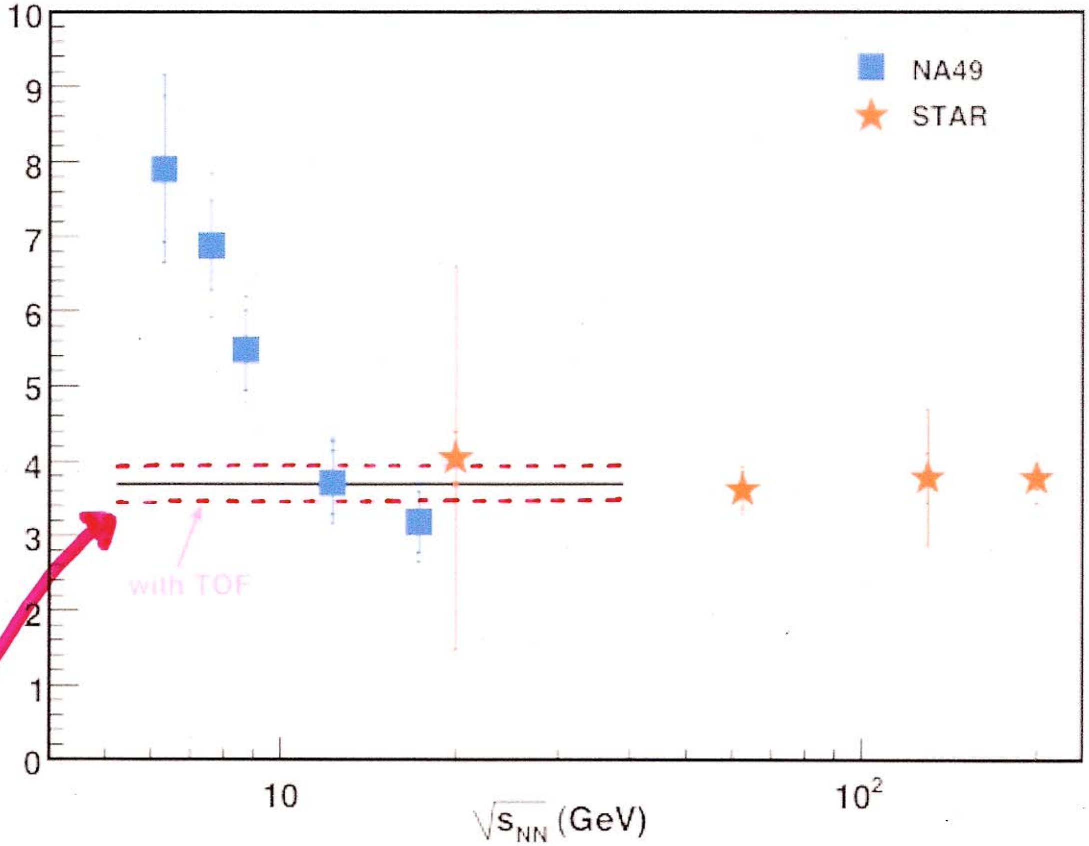


Fig. 3-7: Estimate of the error in σ_{dyn} for charge-integrated K/π fluctuations, based on 100K central events analyzed in the STAR detector (with the newly completed ToF). Shown for comparison are the current measurements from NA49 and STAR.

STAR B.U.R.

Error bars with 1M min bias events per energy

KURTOSIS OF EVENT-BY-EVENT DISTRIBUTION OF (NET) PROTON NUMBER

Stephanov; "a direct consequence of discussions" at Aug 2008 INT workshop

Critical fluctuations couple to $\pi\pi$, pp

→ event-by-event fluctuations in their multiplicities, multiplicity ratios, p_T, that are $\propto \xi^2$

Higher moments of the event-by-event distributions receive effects that are more sensitive to ξ.

Skewness $\propto \xi^{4.5}$

Kurtosis $\propto \xi^7$!!!

The prefactors work out particularly nicely for kurtosis of proton distribution, but Stephanov makes predictions for π & p, skewness & kurtosis.

DEFINITIONS:

N = # of protons in an event

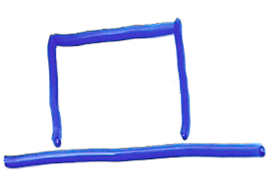
$\bar{N} = \langle N \rangle$ = mean

$\delta N \equiv N - \bar{N}$ in an event

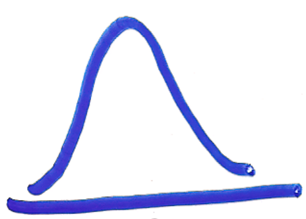
$\langle (\delta N)^2 \rangle$ = variance

$$K^{eff} \equiv \underbrace{K}_{\text{Kurtosis} \sim \frac{1}{\bar{N}}} \underbrace{\langle (\delta N)^2 \rangle}_{\text{variance} \sim \bar{N}} \equiv \frac{\langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2}{\langle (\delta N)^2 \rangle}$$

⌈In lattice QCD literature, K^{eff} is called κ_4 / κ_2. It is calculated for N = baryon number.⌋

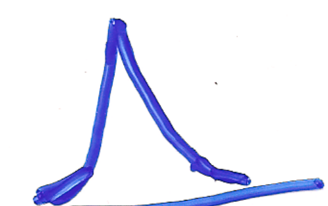


$K^{eff} < 0$



$K^{eff} = 0$

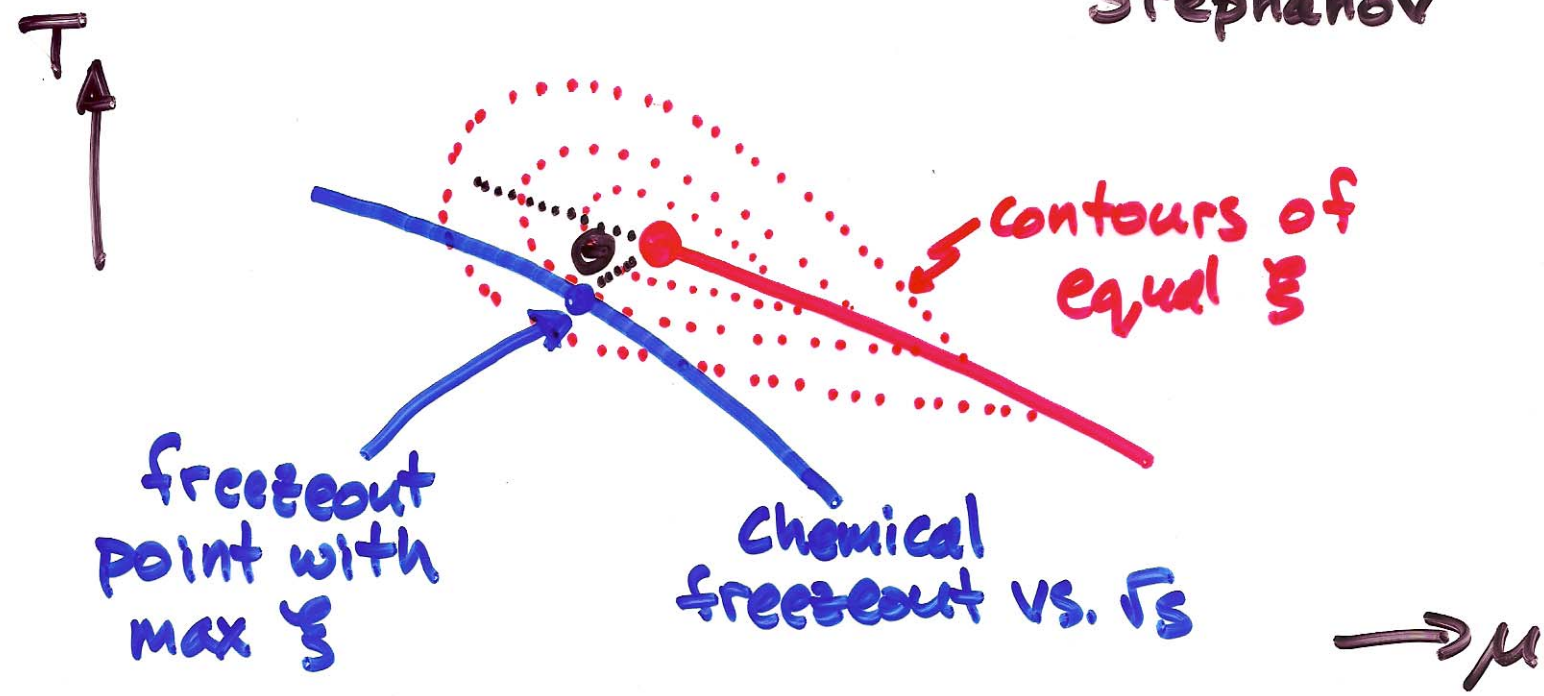
for Gaussian



$K^{eff} > 0$

EFFECT OF CRITICAL FLUCTUATIONS

Stephanov



$$K^{eff} \quad \underbrace{\frac{\bar{N}}{\langle(\delta N)^2\rangle}}_{\cong 1} = 46 \underbrace{\left(\frac{2\tilde{\lambda}_3^2 - \tilde{\lambda}_4}{50}\right)\left(\frac{g}{10}\right)^4}_{\mathcal{O}(1)} \underbrace{\left(\frac{\xi}{1\,\mathrm{fm}}\right)^7}_{!!}$$

!!

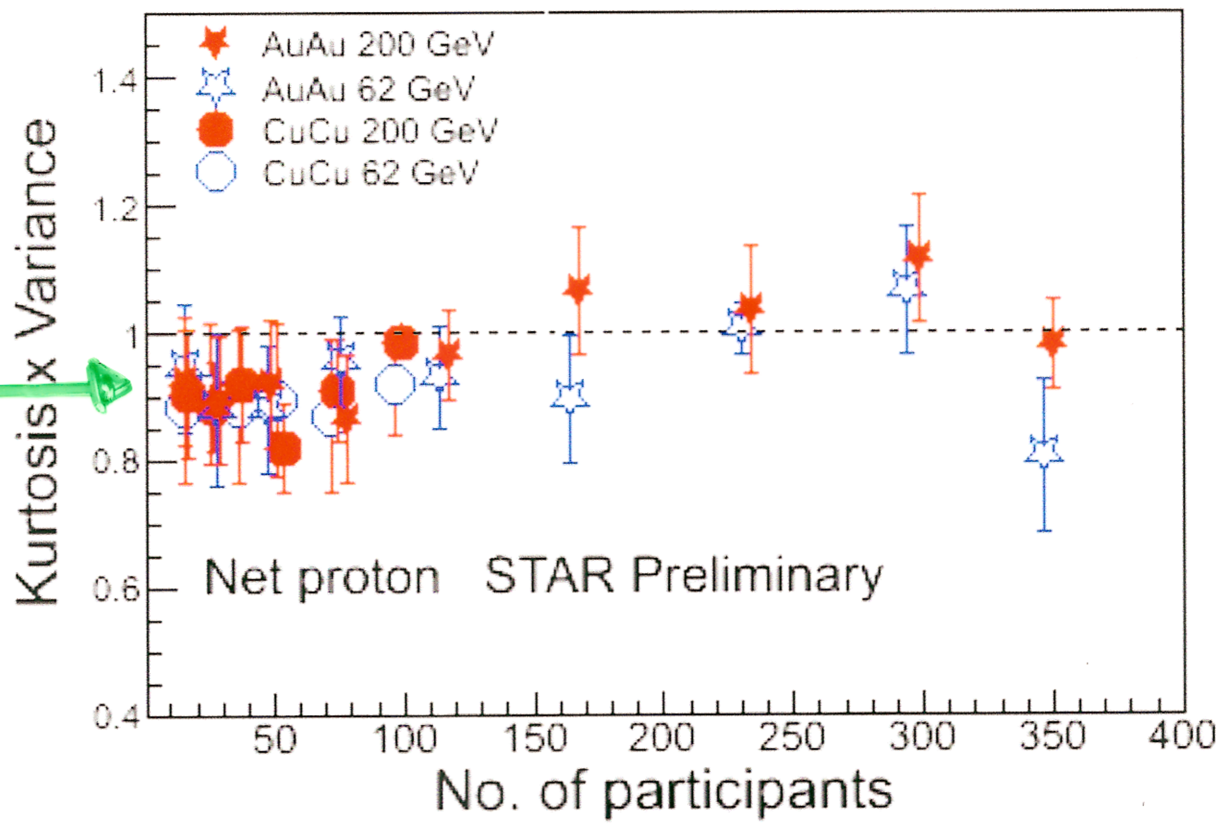
$\tilde{\lambda}_3, \tilde{\lambda}_4$: Obscure, but universal, constants that depend on θ. Known for Ising model.

g: σpp coupling. $\sim M_p/f_\pi$.

K^{eff} AT $\sqrt{s} = 200$ GeV AND 62 GEV

ie at very low μ, far from ●

K^{eff}



!

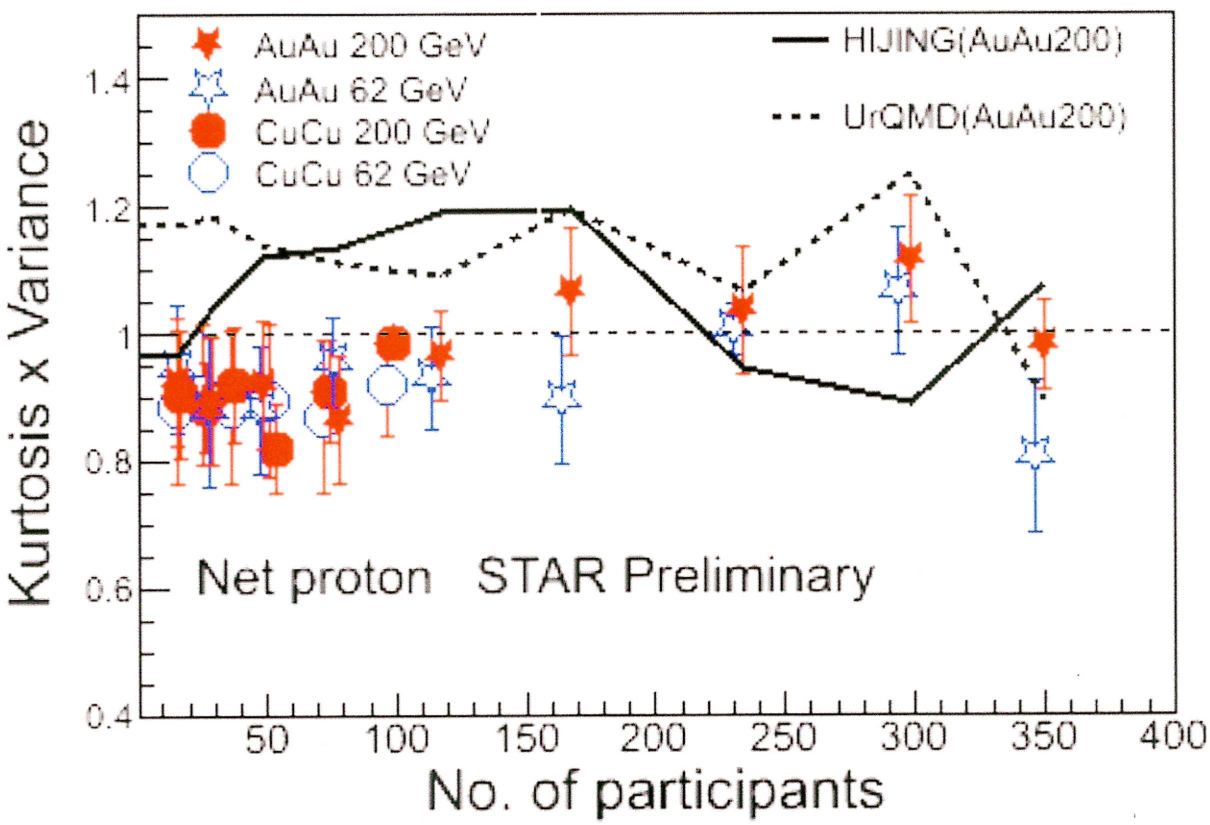
Nayak, QM09

Background K^{eff} measured, ~ 1.

With largest uncertainty coming from value of g^4, signal is predicted to be $\sim \frac{1}{2}$ for $\xi = 0.5$ fm

~ 50 for $\xi = 1$ fm !

K^{eff}



Nayak, QM09

PROTON NUMBER KURTOSIS

- Predicted effect of proximity to is large
- Measured background is not.
- Cannot be washed out between chemical and kinetic freezeout
- STAR can measure to ±0.1 with 5M min bias events per energy

STAR B.U.R.

- Calculations done for $\bar{N} \to \infty$. Finite $\bar{N}$ corrections and acceptance corrections still need to be assessed. The latter should be minimal for STAR.
- Proton number kurtosis is most fully analyzed example so far, but analogous analyses can be done for pions also, and for other moments. Many signatures will be in play if/when found.

A BEAM ENERGY SCAN TO SEARCH FOR

$\sqrt{s}/A$ (GeV)	μ (MeV)*	8 hr days† per 5 M events
5	550	
6.2	485	
7.7	420	56
9.8	355	30
12.7	290	13
17.3	230	5
27	155	2
39	110	1
200	25	
		107 days

*: from Cleymans et al's 2005 empirical fit to compilation of data

†: from STAR B.U.R.

WHAT NEED BE MEASURED AT EACH $\sqrt{s}$

- Enough <particle ratios> to evaluate μ.
  So you know where on phase diagram you are freezing out.
- n_q scaling of V_2 for $\pi/K/p/\Lambda$
  parity violating fluctuations
  So you know whether collision got above the crossover/transition
- Event-by-event fluctuations of K/π and p/π ratios with significantly smaller error bars than in NA49 data
- Variance, skewness & kurtosis of event-by-event distributions of
  N_p, $N_p - N_{\bar{p}}$, N_π.
- All can be done with 5M min bias events per $\sqrt{s}$. STAR B.U.R.

	Collision Energies (GeV)	**5**	**7.7**	**11.5**	**17.3**	**27**	**39**
Section	Observables	Millions of Events Needed					
A1	v_2 (up to ~1.5 GeV/c)	**0.3**	**0.2**	**0.1**	**0.1**	**0.1**	**0.1**
A1	v_1	0.5	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**
A2	Azimuthally sensitive HBT	4	**4**	**3.5**	**3.5**	**3**	**3**
A3	PID fluctuations (K/π)	1	**1**	**1**	**1**	**1**	**1**
A3	net-proton kurtosis	5	**5**	**5**	**5**	**5**	**5**
A3	differential corr & fluct vs. centrality	4	**5**	**5**	**5**	**5**	**5**
A3	integrated p_T fluct (T fluct)						
B1	n_q scaling π/K/p/Λ $(m_T-m_0)/n$<2GeV		**6**	**5**	**5**	**4.5**	**4.5**
B1	ϕ/Ω up to p_T/n_q=2 GeV/c		56	25	**18**	**13**	**12**
B2	R_{CP} up to p_T ~4.5 GeV/c (at 17.3) 5.5 (at 27) & 6 GeV/c (at 39)				**15**	**33**	**24**
B3	untriggered ridge correlations		27	13	**8**	**6**	**6**
B4	parity violation		**5**	**5**	**5**	**5**	**5**
See[1]:	charge-photon fluctuations (DCC)	1	**1**	**1**	**1**	**1**	**1**
	kink/step/horn	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
	v_2 fluctuations	0.5	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**
	HBT (R_l, R_o/R_s)	0.8	**0.8**	**0.5**	**0.5**	**0.5**	**0.5**
	Jet/ridge 2<trig<4, 1<assoc<trig				30	**8.8**	**4.5**
	Jet/ridge 3<trig<6, 1.5<assoc<trig					53	**24**
	Baryon-Strangeness cor (hypernuc)						50
	Forward π^- yield (rapidity scaling)						
	Forw. $\gamma(\pi^0)$ yield (rapidity scaling)						
	Long-range forward-backward corr.						
	Other PID fluctuations (esp. K/p)						
	Particle ratios (many examples)						
	p_T spectra						
	Prod. of light nuclei & antinuclei						
	Yields of species & stat model fits						

Table 3-2: Observables and statistics needed for the first BES run. The observables in the yellow-shaded area relate to the search for a phase transition or critical point (see section A), while observables in the blue-shaded area search for turn-off of new phenomena already established at higher RHIC energies (see section B). The numbers listed in boldface above are all within reach (nominally require no more than 1.5 times the proposed statistics) in the first BES run plan as set out in Table 3-1. The remaining numbers (not boldface) will need to wait for higher

CAN WE DISCOVER THE QCD CRITICAL POINT IN RUN 10 AT RHIC?

YES, IF:

Nature is kind, and puts $\mu_{\bullet} \lesssim 420\,\text{MeV}$

IF YES:

- The landmark discovered. Our map of the QCD phase diagram then anchored by experiment.
- Assuming reasonable progress in lattice QCD, quantitative comparison between theory & experiment for $\mu_{\bullet}$ will come.
- RHIC (and FAIR) can study the first order phase transition. RHIC can further probe the region around $\bullet$.

IF NO:

- We learn that $\mu_{\bullet} > 420\,\text{MeV}$
- We will be able to make a data-driven decision about whether to run at $\sqrt{s} = 6.2$ and 5 in a future year.