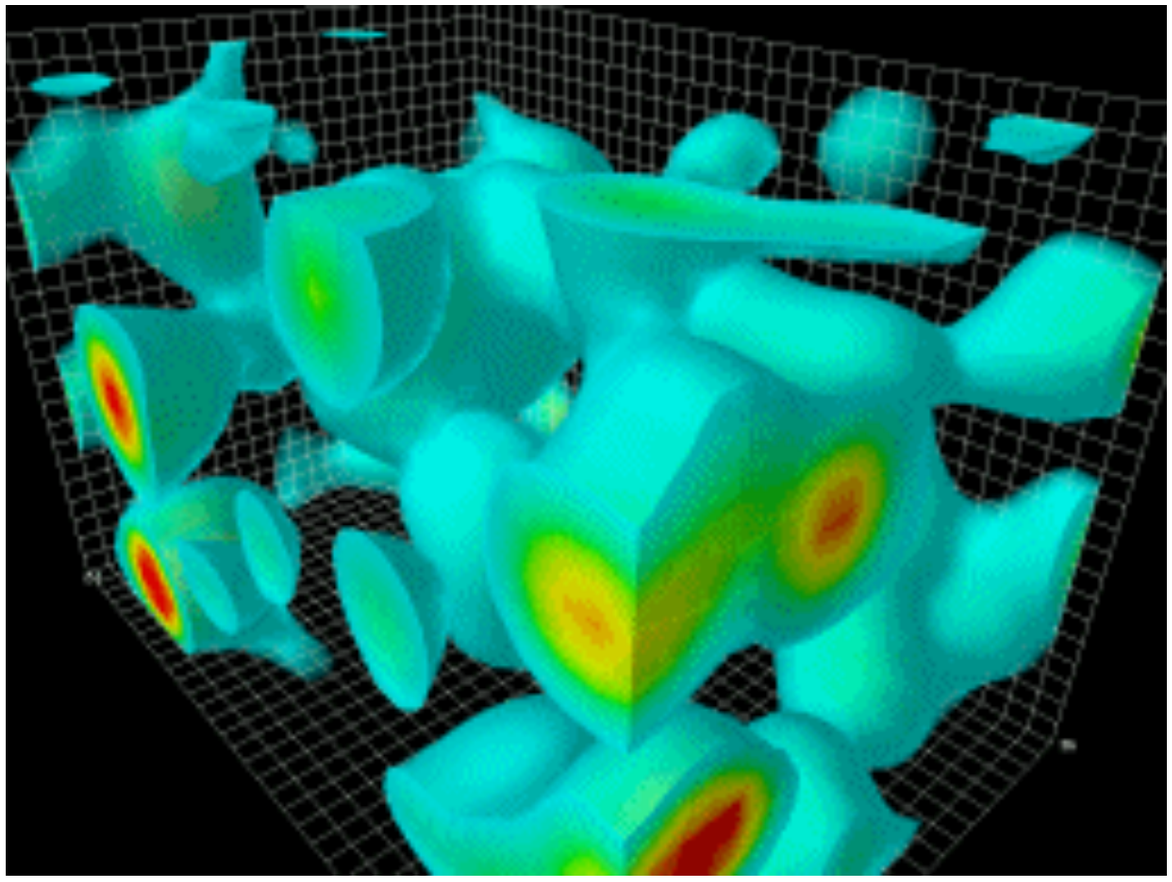
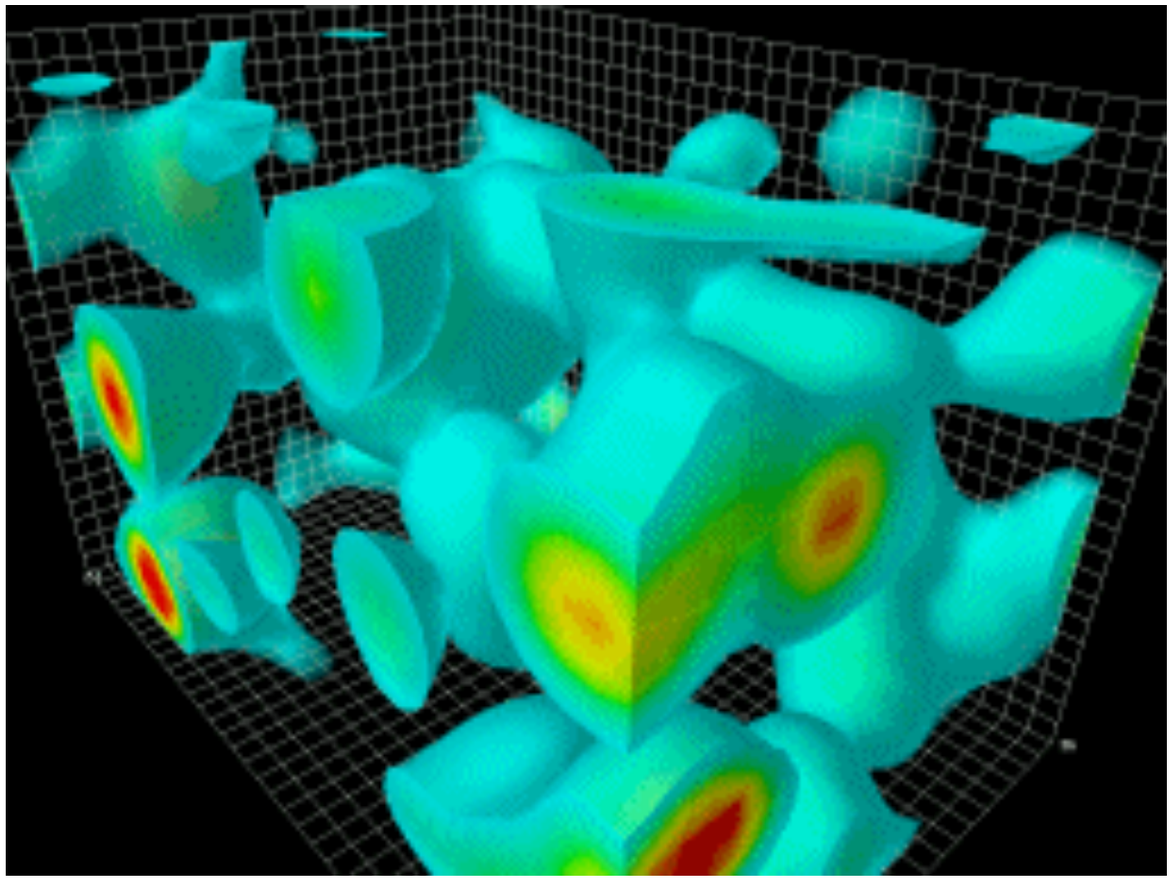
Dualities, Old and New

David Tong: MIT Pappalardo Fellow, 2001-2004

Quantum Field Theory



Quantum Field Theory



... is hard

How to Proceed?

1. Numerics:

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2. Toy Models (e.g. supersymmetry)

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1. Numerics:

2. Toy Models (e.g. supersymmetry)

3. Guess

A particularly useful method is to invoke *dualities*

Particle-Vortex Duality

Peskin 1978
Dasgupta and Halperin 1981
Fisher and Lee, 1989

Abelian-Higgs Model:

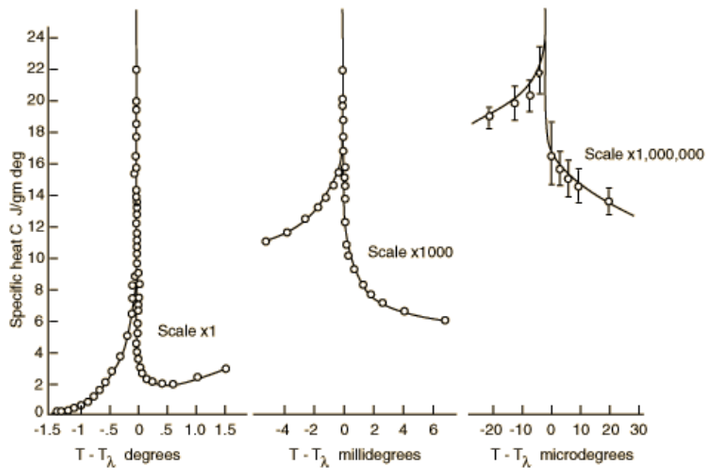
$$S_A = \int d^3x \; -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + |\mathcal{D}_\mu \Phi|^2 - a|\Phi|^2 - b|\Phi|^4$$

XY Model:

$$S_B = \int d^3x \; |\partial_\mu \phi|^2 - a'|\phi|^2 - b|\phi|^4$$

Duality = Universality

Superfluids and superconductors have the same second-order phase transition



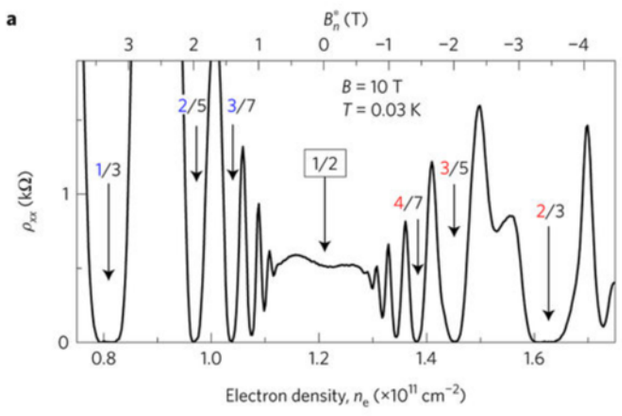
Fermionic Particle-Vortex Duality

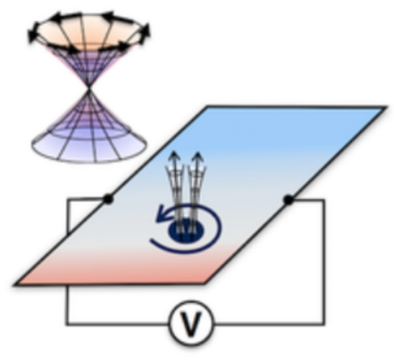
“3d QED” $$S_A = \int d^3x \ -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + i\bar{\Psi}\, \not{D} \Psi$$

Free Fermion $$S_B = \int d^3x \ i\bar{\psi}\, \not{\partial} \psi$$

Son, 2015

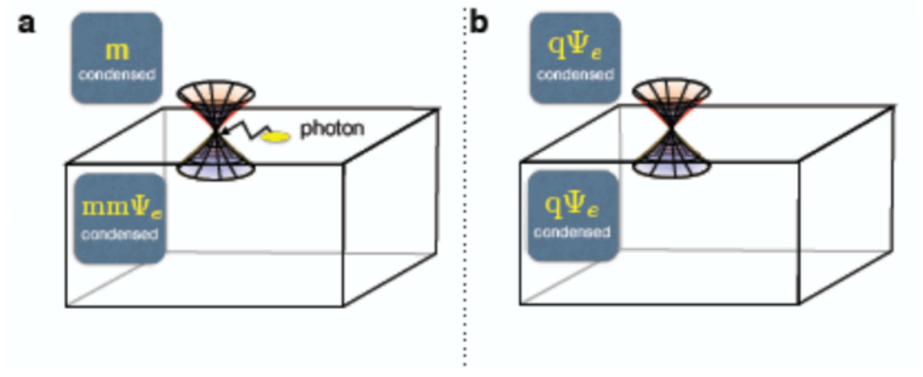
Half-filled Landau level





Wang and Senthil; Metlitski and Vishwanath

Topological insulators



Supersymmetric Particle-Vortex Duality

a.k.a mirror symmetry

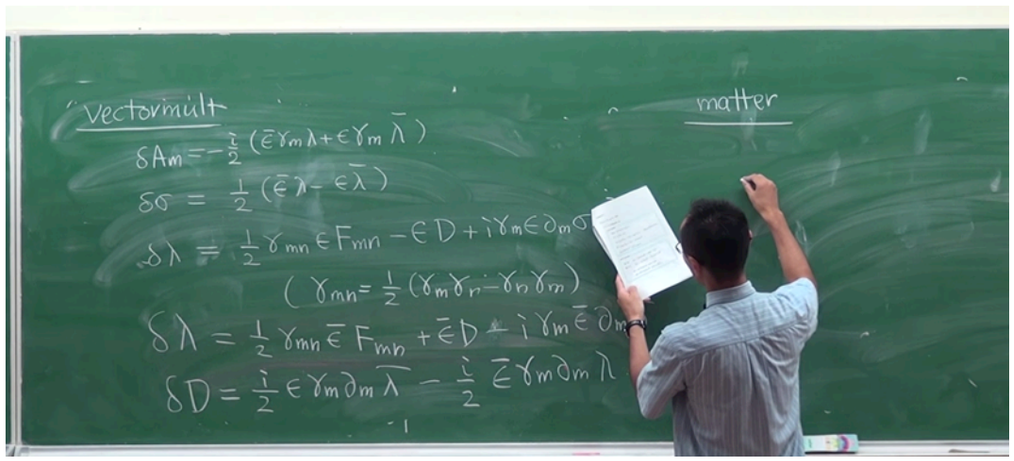
Intriligator and Seiberg 1996
Aharony, Hanany, Intriligator, Seiberg, Strassler 1997

Deser, Jackiw, Templeton 1982

<u>Supersymmetric QED:</u> *N=2*, U(1) gauge theory + chiral multiplet + Chern-Simons = ½

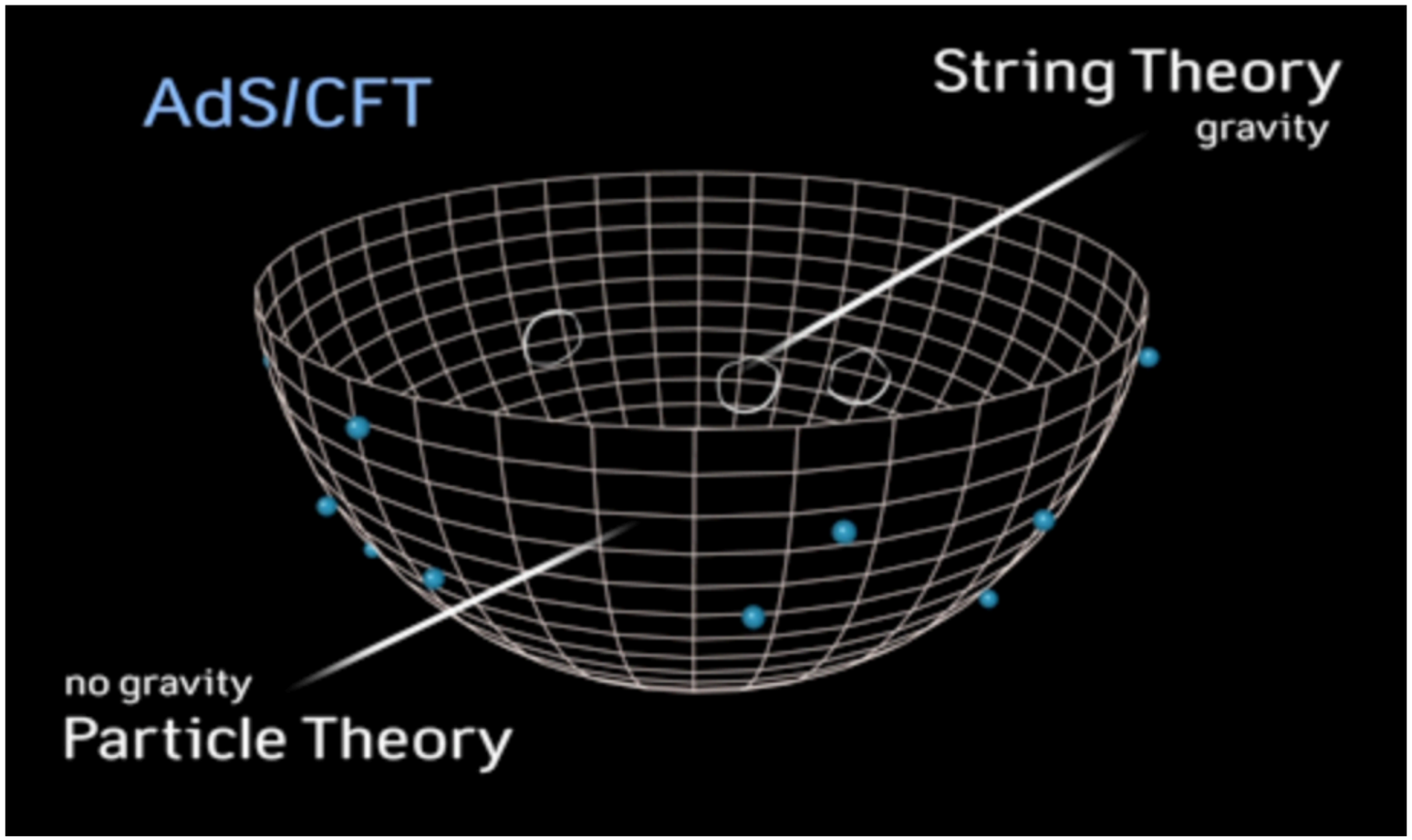
<u>Free chiral multiplet:</u> free Dirac fermion + complex scalar

Dorey and Tong, 1999



How is this related to the other dualities?!

Help from a Surprising Direction



Higher spin theories in the bulk = Simple theories on the boundary

Vasiliev, 1980s and 1990s

Klebanov and Polyakov 2002
Sezgin and Sundell 2002
Giombi and Yin 2009

The Dual of Higher Spin Theories

Minwalla et al. 2011-2015
Aharony et al. 2011-2015

Theory A: U(N) Yang-Mills + WF boson + CS = k

Theory B: SU(k) Yang-Mills + fermion + CS = -N+1/2

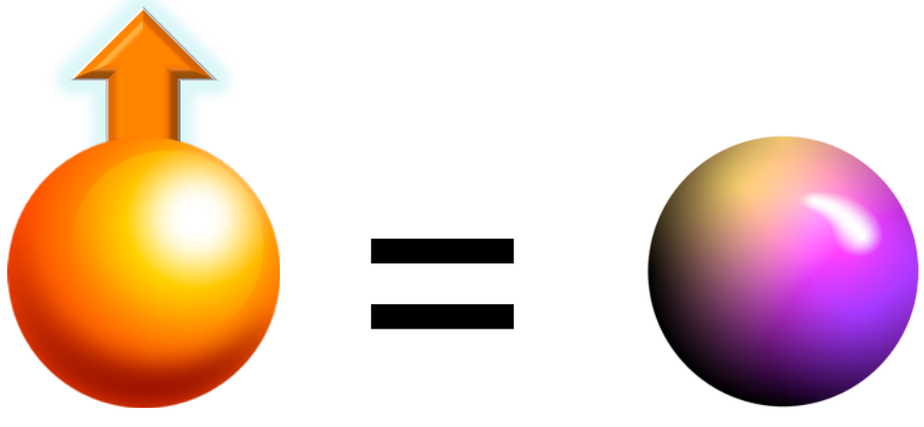
Tested beyond all reasonable doubt at large N and k

The Simplest Bosonization Duality

Scalar + CS: $$S_A = \int d^3x \; |\mathcal{D}_\mu \phi|^2 - |\phi|^4 + \frac{1}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

Free Fermion $$S_B = \int d^3x \; i\bar{\psi} \not{\partial} \psi$$

The equation of motion for *a* attaches flux to particles: $$\rho_{\text{scalar}} + \frac{f}{2\pi} = 0$$



Boson + 2π flux Fermion

Long known in non-relativistic theories: Wilczek 1982

More complicated in relativistic theories: Polyakov 1988

A Web of Dualities

Karch and Tong 2016
Seiberg, Senthil, Wang and Witten 2016

Scalar + CS:

$$S_A = \int d^3x \ |\mathcal{D}_\mu \phi|^2 - |\phi|^4 + \frac{1}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

Free Fermion

$$S_B = \int d^3x \ i\bar{\psi}\, \partial\!\!\!/ \psi$$

This is the seed duality. From this we can derive all others!

A Web of Dualities

Karch and Tong 2016
Seiberg, Senthil, Wang and Witten 2016

Seed Duality: $U(1)_1$ + complex WF boson = free fermion

Key idea:
- Identify currents on both sides
- Gauge currents to get new dualities
- Repeat

What we get:
- $U(1)_{-1/2}$ + fermion = WF boson
- Bosonic particle vortex duality
- Fermionic particle vortex duality
- An infinite number of new dualities...
  - e.g. U(1) + 2 fermions is self-dual with emergent SU(2) x SU(2) global symmetry.

See also Barkeshli and McGreevy, 2012

What Happened Next?

<u>"Derivations" of bosonization</u>

Break supersymmetry = Bosonization

Lattice derivation

Kachru et al; Chen, Son, Wang, Raghu

<u>Proliferation of dualities</u>

Almost anything = Something else

Dozens of papers, including Benini, Cordova, Jensen, Karch, Komargodski, Hsin, Radicevic, Robinson, Seiberg, Tong

<u>Numerical Tests</u>

Critical points in QED_3

Karthik and Narayanan

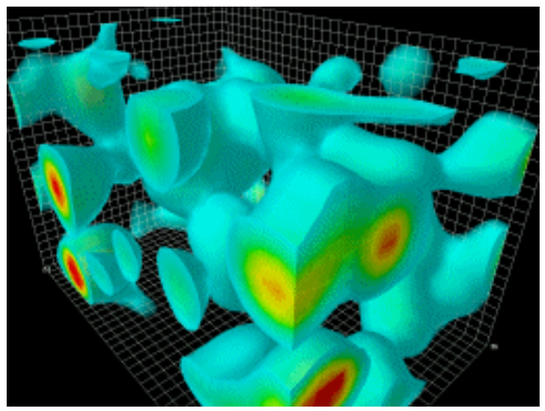
<u>Condensed Matter Physics</u>

Bewildering number of applications

See last week's Aspen conference

Summary: we're making progress!

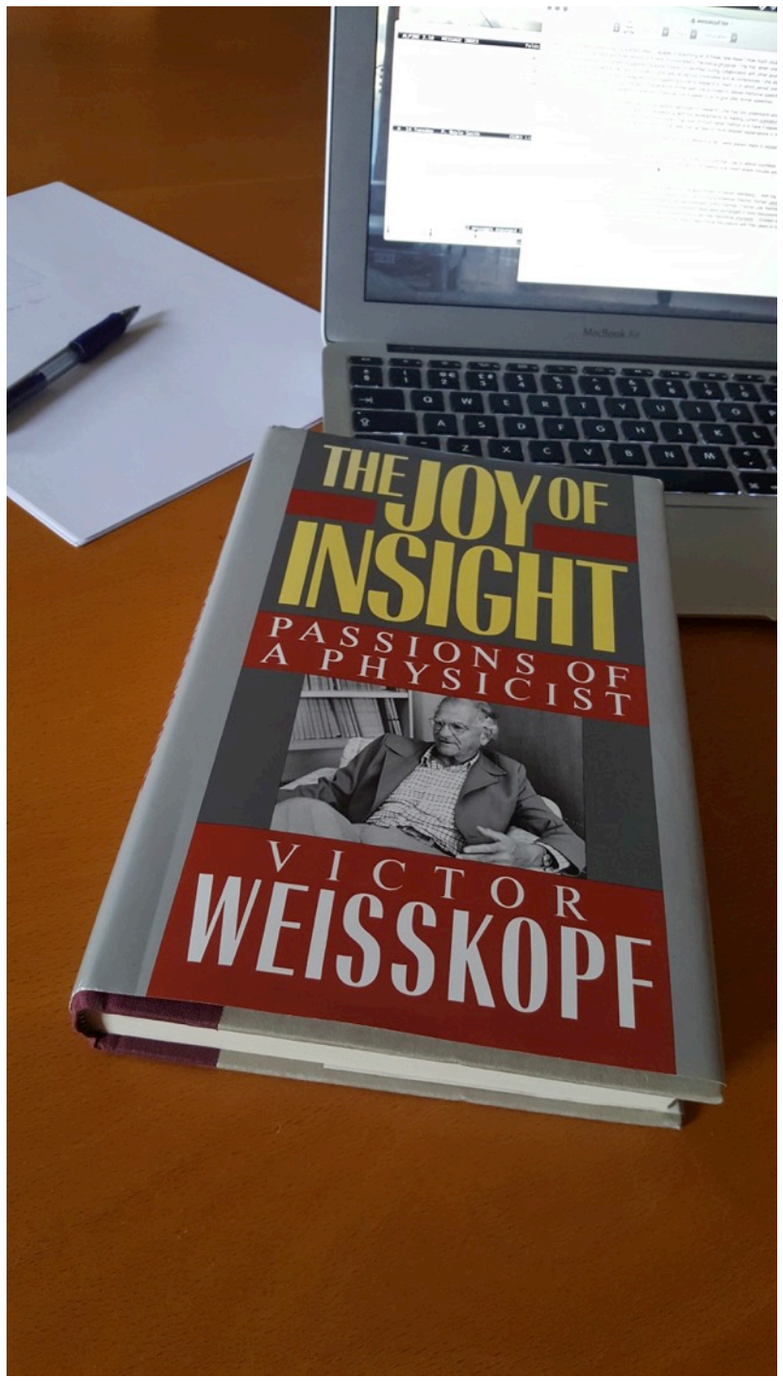
Condensed matter



Supersymmetry

Holography

Viki Weisskopf and CTP



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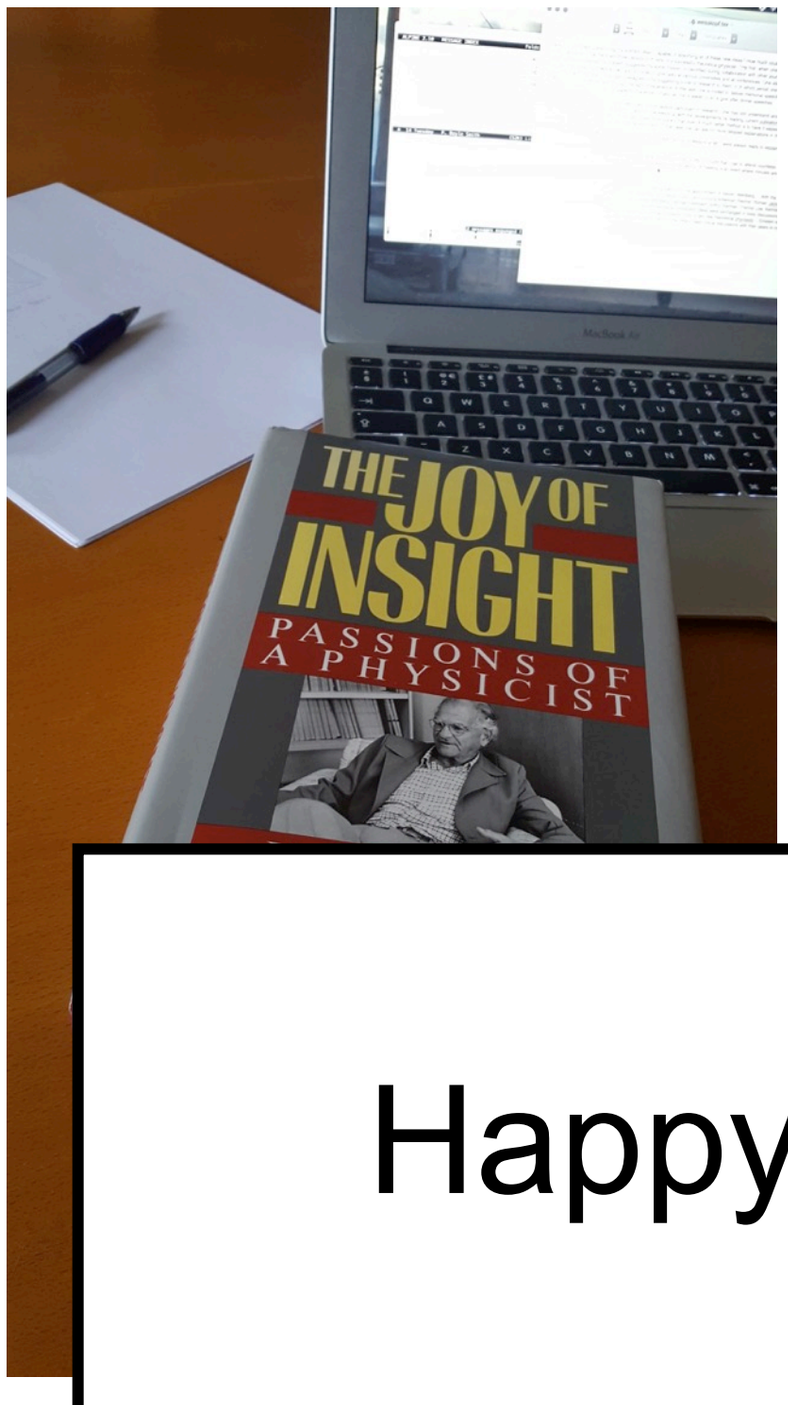
Our center vibrated with activity.””

Viki Weisskopf and CTP



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Happy Birthday CTP!