MATCHING HEAVY-TO-LIGHT CURRENTS AT 2-LOOP

[GUIDO BELL]

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G. Bell, M. Beneke, T. Huber, X. Q. Li, in preparation

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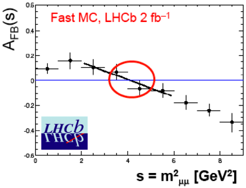
Precision B physics?

Expect precision measurements from current + future B physics experiments

examples:
- $\delta\mathrm{Br}(B \to X_s\gamma) \sim 5-10\%$
- $\delta V_{ub}\big|_{\mathrm{inclusive}} \sim 5-10\%$
- zero of $A_{FB}(B \to K^*\ell^+\ell^-) \sim 10\%$



Theory is challenged to match the experimental precision

- develop strategies where hadronic uncertainties largely cancel
- need progress from non-perturbative methods
- work out subleading corrections in SCET: NNLO and $1/m_b$

B physics at the NNLO frontier

NNLO programme complete for $\mathcal{H}_{\text{eff}} = \sum_i C_i\, Q_i$

- 2-loop / 3-loop matching corrections [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]
- 3-loop / 4-loop anomalous dimensions [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

$\rightarrow$ need hadronic matrix elements to same level of precision

Some (incomplete) NNLO analysis based on SCET

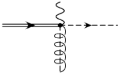
- $B \rightarrow X_s\gamma$ [Becher, Neubert 06]
- $B \rightarrow X_u\ell\nu$ [$\rightarrow$ talk by Ben Pecjak]
- $B \rightarrow V\gamma$ [Ali, Greub, Pecjak 07]
- $B \rightarrow MM$ [Beneke, Jäger 05,06; Kivel 06; GB 07,09; Pilipp 07; Jain, Rothstein, Stewart 07]

Heavy-to-light currents in SCET$_I$

2-body operators

3-body operators

$$\bar{q}\,\Gamma b \;=\; \sum_i \int ds\; \tilde{C}_i^{(A)}(s)\; O_i^{(A)}(s) \;+\; \sum_i \int ds_1 ds_2\; \tilde{C}_i^{(B)}(s_1,s_2)\; O_i^{(B)}(s_1,s_2) \;+\; \dots$$



Heavy-to-light currents in SCET$_I$

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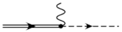

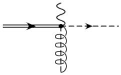

Inclusive decays	leading contribution $\langle O_i^{(A)} \rangle \;\to\; J \otimes S$	power-correction $\langle O_i^{(B)} \rangle \;\to\; \sum_i j_i \otimes s_i$
Exclusive decays	non-factorizable in SCET$_{II}$ $\langle O_i^{(A)} \rangle \;\to\; \xi_P$ soft-overlap contribution	factorizable in SCET$_{II}$ $\langle O_i^{(B)} \rangle \;\to\; \phi_B \otimes J_{\|\|} \otimes \phi_M$ hard spectator scattering

Heavy-to-light currents in SCET$_I$

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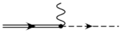

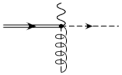

Hard coefficients
QCD $\to$ SCET$_I$

$\mathcal{O}(\alpha_s)$: 1-loop
[Bauer, Fleming, Pirjol, Stewart 00]

$\mathcal{O}(\alpha_s^2)$: 2-loop

$\mathcal{O}(\alpha_s^2)$: 1-loop
[Beneke, Kiyo, Yang 04; Becher, Hill 04]

$\to$ 2-loop matching required for NNLO analysis of inclusive and exclusive B decays

Heavy-to-light currents in SCET_I

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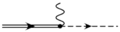
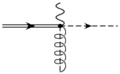




Status of 2-loop calculation

- $\Gamma = \gamma^\mu$ $\quad B \to X_u \ell\nu,\ B \to \pi\ell\nu$ [Bonciani, Ferroglia 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08; GB 08]
- $\Gamma = 1, \sigma^{\mu\nu}$ $\quad B \to X_s \ell^+\ell^-,\ B \to K^*\ell^+\ell^-$ [GB, Beneke, Huber, Li in preparation]

  ($q^2 = 0$ known from $B \to X_s\gamma,\ B \to K^*\gamma$) [Melnikov, Mitov 05; Asatrian, Ewerth, Ferroglia, Gambino, Greub 06; Ali, Greub, Pecjak 07]

NDR-scheme relates currents with and without γ_5

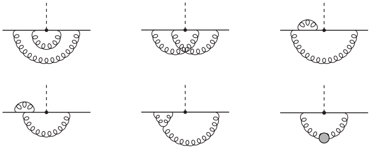
Set-up for 2-loop matching

Heavy-to-light currents in leading power

$$\bar{q}\,\Gamma\, b \;\simeq\; \sum_i \int ds\; \tilde{C}_i^{(A)}(s)\; [\xi W_{hc}](sn_+)\, \Gamma_i'\, h_v$$

Strategy: compute $\langle q|\ldots|b\rangle$ in QCD and SCET$_I$
matching simplifies in Dimensional Regularization with on-shell quarks
$\rightarrow$ SCET diagrams are scaleless and vanish

2-loop QCD calculation



similar to 2-loop calc. in $B \rightarrow \pi\pi$ [GB 07,09]
here: less diagrams / integrals
no evanescent operators

Multi-loop techniques

Automatized reduction algorithm

- integration-by-parts identities [Chetyrkin, Tkachov 81]
- solve large system of equations efficiently [Laporta 00]

$\rightarrow$ reduce $\mathcal{O}(1.000)$ scalar 2-loop integrals to 14 Master Integrals

Calculation of Master Integrals

- method of differential equations [Kotikov 91; Remiddi 97]
- harmonic polylogarithms [Remiddi, Vermaseren 00]
- Mellin-Barnes techniques [Smirnov 99; Tausk 99]
- method of sector decomposition [Binoth, Heinrich 04]

$\rightarrow$ analytical results known from 2-loop analysis in $B \rightarrow \pi\pi$ [GB 07]

confirmed by several independent calculations [Bonciani, Ferroglia 08; Asatrian, Greub, Pecjak 08; Beneke, Huber, Li 08; Huber 09]

Renormalization

Standard QCD counterterms, e.g.

- wave-function renormalization (on-shell scheme) [Gray, Broadhurst, Grafe, Schilcher 90]

$$Z_{2,b}^{(2)} = \frac{74}{3\varepsilon^2} + \left(-\frac{83}{9} + \frac{56}{3} \ln \frac{\mu^2}{m_b^2} \right) \frac{1}{\varepsilon}$$
$$- \frac{5543}{54} - \frac{14\pi^2}{9} - \frac{32\pi^2}{9} \ln(2) + \frac{16}{3}\zeta_3 - \frac{178}{3} \ln \frac{\mu^2}{m_b^2} + \frac{10}{3} \ln^2 \frac{\mu^2}{m_b^2} + \mathcal{O}(\varepsilon)$$

- current renormalization ($\overline{\text{MS}}$-scheme) [Tarrach 81; Broadhurst, Grozin 94]

$$Z_S^{(2)} = C_F \left\{ \left[\frac{9}{2} C_F + \frac{11}{2} C_A - 2 n_f T_F \right] \frac{1}{\varepsilon^2} + \left[-\frac{3}{4} C_F - \frac{97}{12} C_A + \frac{5}{3} n_f T_F \right] \frac{1}{\varepsilon} \right\}$$

$$Z_V^{(2)} = 0$$

$$Z_T^{(2)} = C_F \left\{ \left[\frac{1}{2} C_F - \frac{11}{6} C_A + \frac{2}{3} n_f T_F \right] \frac{1}{\varepsilon^2} + \left[-\frac{19}{4} C_F + \frac{257}{36} C_A - \frac{13}{9} n_f T_F \right] \frac{1}{\varepsilon} \right\}$$

IR-subtractions

Matching: do not simply drop remaining poles!

instead use $\overline{\text{MS}}$-counterterm of SCET$_I$ currents

$$Z_J^{(1)} = C_F \left\{ -\frac{2}{\varepsilon^2} - \frac{4}{\varepsilon} \ln \frac{\mu}{n_+ p} - \frac{5}{\varepsilon} \right\}$$

[Bauer, Fleming, Pirjol, Stewart 00]

$Z_J^{(2)}$ from 2-loop calculation of jet and shape function [Becher, Neubert 05,06]

$\rightarrow$ IR-subtractions involve finite terms from $C_i^{(1)} Z_J^{(1)}$

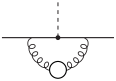
Subtlety: QCD-side contains *b*-quark loops, SCET-side does not

account for additional renormalization from $\alpha_s^{(5)} \rightarrow \alpha_s^{(4)}$

$$\delta\alpha_s^{(1)} = T_F \left[\frac{4}{3} \ln \frac{\mu^2}{m_b^2} + \left(\frac{2}{3} \ln^2 \frac{\mu^2}{m_b^2} + \frac{\pi^2}{9} \right) \varepsilon + \mathcal{O}(\varepsilon^2) \right]$$

Charm mass effects

charm quark enters at 2-loop through fermion bubble



$\rightarrow$ not tremendously important, but naively $\frac{m_c^2}{m_b^2} \ln \frac{m_b^2}{m_c^2} \sim 0.20 \gtrsim \frac{\alpha_s(m_b)}{\pi}$

Choose power-counting

- $m_c \sim \mu_{hc} \sim (\Lambda_{QCD} m_b)^{1/2}$
  IR-scale in hard matching ($m_c = 0$)
  m_c-dependence in jet-function

- $m_c \rightarrow \infty$, $m_b \rightarrow \infty$, m_c/m_b fixed
  m_c-dependence in hard matching
  jet function with 3 massless quarks

Adopt second scenario

$\rightarrow$ 4 new Master Integrals, modified UV- and IR-subtractions, numerical results

Wilson coefficients in NNLO

$\Gamma = \gamma^\mu \quad \longrightarrow \quad C_{V,1}, C_{V,2}, C_{V,3}$

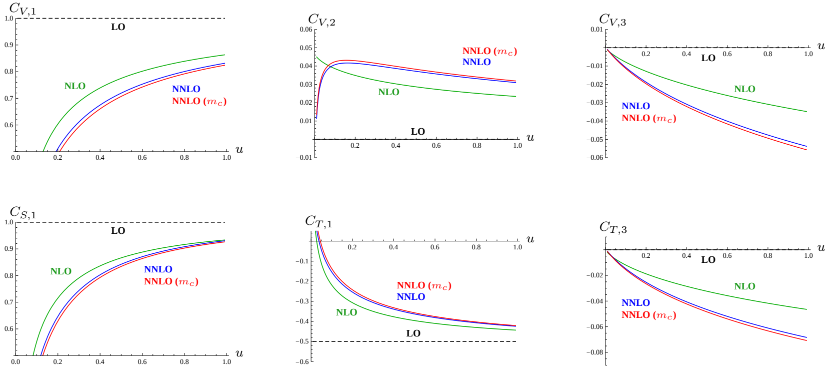
$\Gamma = 1 \quad \longrightarrow \quad C_{S,1}$

$\Gamma = i\sigma^{\mu\nu} \quad \longrightarrow \quad C_{T,1}, C_{T,3} \quad (C_{T,2} = C_{T,4} = 0)$

momentum transfer $q^2 = (1-u)m_b^2$

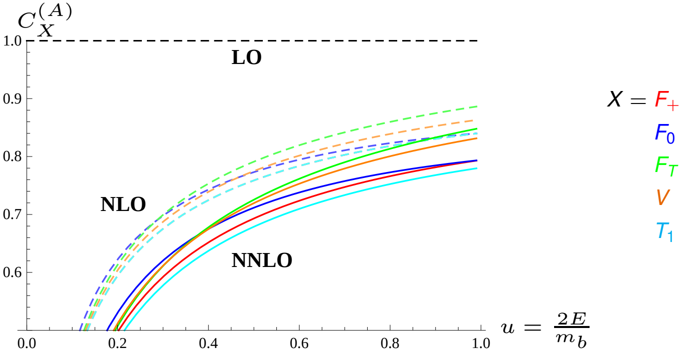
$u = \frac{n_+ p}{m_b} = \frac{2E}{m_b}$

$\mu = m_b$



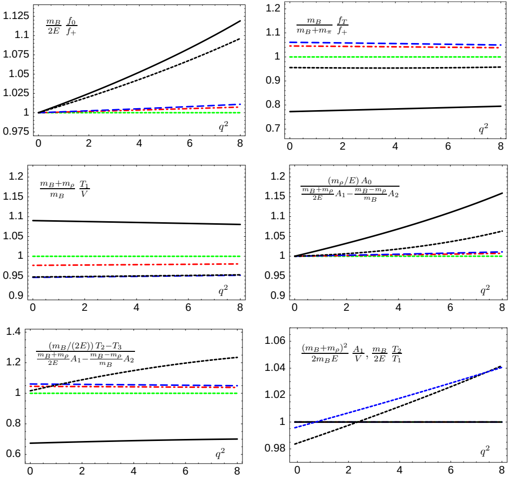
Heavy-to-light form factors

$$F_X^{B\to M}(E) \simeq C_X^{(A)}(E)\,\xi_M(E) + \int d\omega \int du\, \phi_B(\omega)\, T_X(E,\omega,u)\, \phi_M(u)$$



→ "universal" corrections to heavy-to-light form factors

Symmetry-breaking corrections to form factor ratios



- symmetry limit
- with 1-loop $C_X^{(A)}$
- with 2-loop $C_X^{(A)}$
- with spectator scattering
- QCD sum rules [Ball, Zwicky 04]

2-loop corrections "largest" for

$$\mathcal{R}_2 = \frac{m_B+m_V}{m_B}\,\frac{T_1}{V}$$

which enters

- $|V_{td}/V_{ub}|$-determination from

$$\frac{\Gamma(B\to\rho\gamma)}{d\Gamma(B\to\rho\ell\nu)/dq^2\,d\cos\theta}$$

- zero of $\frac{dA_{\rm FB}(B\to V\ell^+\ell^-)}{dq^2}$

[GB, Beneke, Huber, Li in preparation]

Summary

- Computed 2-loop hard matching corrections for heavy-to-light currents in SCET_I
- Find $\sim$ 10% corrections to individual heavy-to-light form factors,

  but very small corrections to form factor ratios
- 2-loop corrections presumably more important in $B \to X_u \ell \nu$ and $B \to X_s \ell^+ \ell^-$,

  where they represent only part of the NNLO corrections

  (to be combined with 2-loop jet function and SCET resummation)

  [$\to$ talk by Ben Pecjak]

Backup slides