Different formulations of SCET and their quantization

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LNF and INFN

based on work done in collaboration with

C. W. Bauer and G. Ovanesyan

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Outline

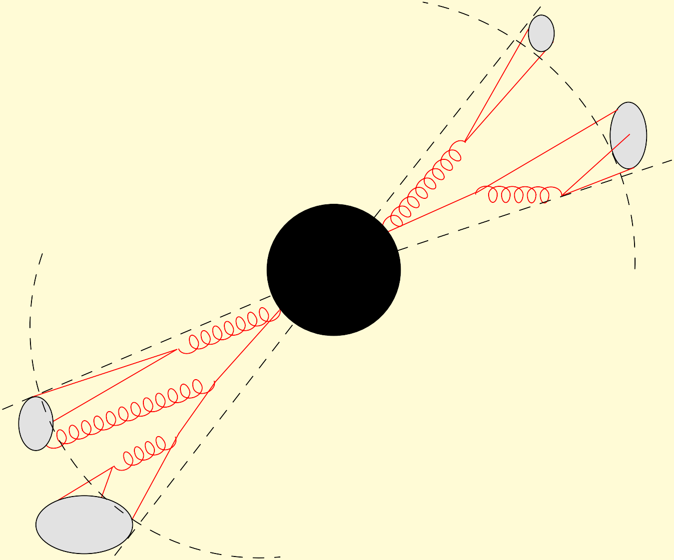
Motivation

Different formulations of SCET

Yet another one

Summary and outlook

From quarks to hadrons

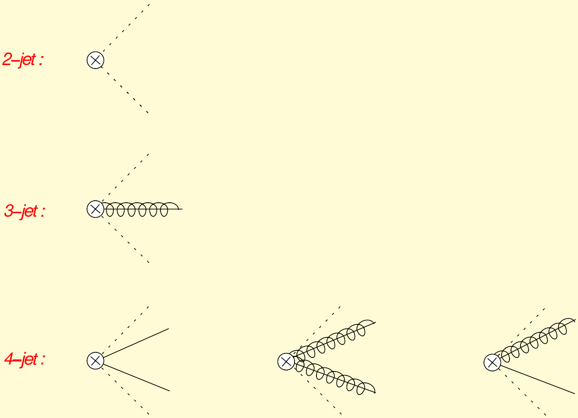


Hard kernel: QCD-SCET matching

[Bauer, Schwartz'06]

- Consider the QCD current $\mathcal{J} = \bar{\psi}\Gamma\psi$. In SCET:

$$\mathcal{J} = c_2\mathcal{O}_2 + c_3\mathcal{O}_3 + c_4\mathcal{O}_4 + \cdots$$



- Matching equation (between operators):

$$\mathcal{J} = c_2\,\mathcal{O}_2 + c_3\,\mathcal{O}_3 + c_4\,\mathcal{O}_4 + \cdots$$

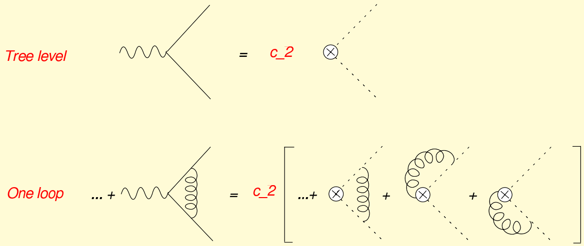
- Matching can be done iteratively:

$$\langle 2|\ \mathcal{J}\ |\gamma\rangle = c_2\ \langle 2|\ \mathcal{O}_2\ |\gamma\rangle$$

$$\langle 3|\ \mathcal{J}\ |\gamma\rangle = c_2\ \langle 3|\ \mathcal{O}_2\ |\gamma\rangle + c_3\ \langle 3|\ \mathcal{O}_3\ |\gamma\rangle$$

$$\langle 4|\ \mathcal{J}\ |\gamma\rangle = c_2\ \langle 4|\ \mathcal{O}_2\ |\gamma\rangle + c_3\ \langle 4|\ \mathcal{O}_3\ |\gamma\rangle + c_4\ \langle 4|\ \mathcal{O}_4\ |\gamma\rangle$$

$e^+e^- \to$ *Hadrons: 2-jet matching*



$$c_2(\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left[8 - \frac{\pi^2}{6} + \log^2\left(-Q^2/\mu^2\right) - 3\log\left(-Q^2/\mu^2\right)\right]$$

$$\mathcal{O}_2 = \bar{\chi}_1 \Gamma \chi_2$$

- Operators specified by tree level matching, demanding $c_i = 1$.

$e^+e^- \to$ *Hadrons to* $\mathcal{O}(\alpha_s^2)$

- Up to 4-jet events are needed.

$$\begin{aligned}\sigma(e^+e^- \to H)[\alpha_s^2] \quad &\sim \quad \left|c_2(\mu)\mathcal{O}_2^{(2)}(\mu)\right|^2 && (2-loop)\\ &+ \quad \left|c_2(\mu)\mathcal{O}_2^{(3)}(\mu) + c_3(\mu)\mathcal{O}_3^{(3)}(\mu)\right|^2 && (1-loop)\\ &+ \quad \left|c_2\mathcal{O}_2^{(4)} + c_3\mathcal{O}_3^{(4)} + c_4\mathcal{O}_4\right|^2 && (tree\ level)\end{aligned}$$

- Declaration of intent:
  - Determine $c_3(\mu)$ at 1 loop analytically ($c_2(\mu)$ to 2 loops known);
  - We want to show explicitly IR cancellations (need for IR regulators) in order to understand the 2-jet to 3-jet transition in SCET.

SCET operator basis

- In QCD,

$$\bar{\psi}\Gamma\psi, \qquad \Gamma = \mathbf{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$$

- In SCET, the absence of the soft spinor reduces the basis to

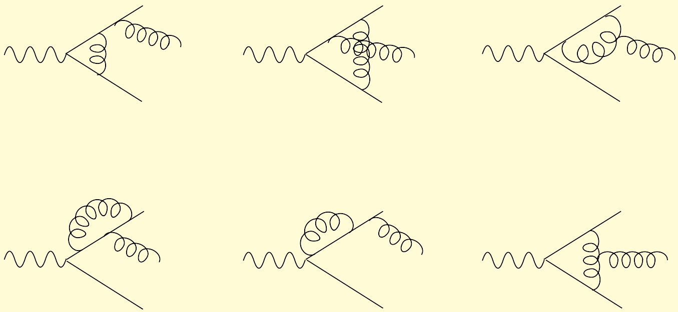
$$\bar{\chi}_n \mathbf{1} \chi_{\bar{n}} \qquad \bar{\chi}_n \gamma_5 \chi_{\bar{n}} \qquad \bar{\chi}_n \gamma_\mu^\perp \chi_{\bar{n}}$$

- Operators for 3 and 4 collinear directions more complicated, involving gauge-invariant operators $\chi_n, \mathcal{B}_n^\mu$.

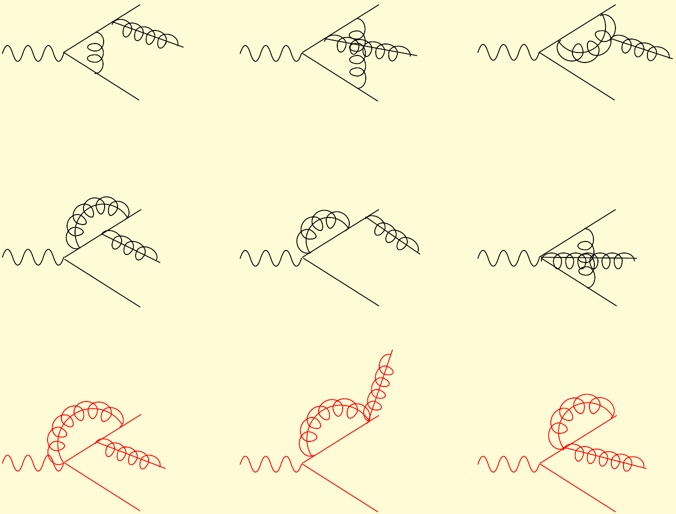
[Marcantonini, Stewart'07]

- It would be useful to compute the diagrams directly in terms of gauge-invariant fields.

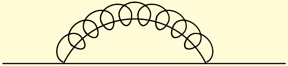
QCD diagrammatics



SCET diagrammatics



A one-loop computational example



Collinear diagrams not computationally-friendly...

$$\sim \int \frac{d^dk}{(2\pi)^d}\left[\frac{\not{p}_\perp \not{k}_\perp}{\bar{n}\cdot p(p+k)^2k^2}+\cdots\right]$$

- Modified Feynman parameters (inhomogeneous denominators):

$$\frac{1}{a^nb^m}=\int_0^\infty d\lambda 2^m \frac{\Gamma[n+m]}{\Gamma[n]\Gamma[m]}\frac{\lambda^{m-1}}{(a+2b\lambda)^{m+n}}$$

- SCET (light-cone) diracology:

$$\gamma_\mu^\perp\gamma_\perp^\mu = d-2$$

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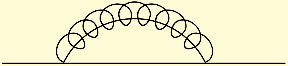

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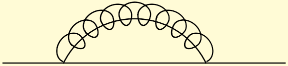

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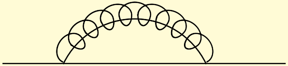

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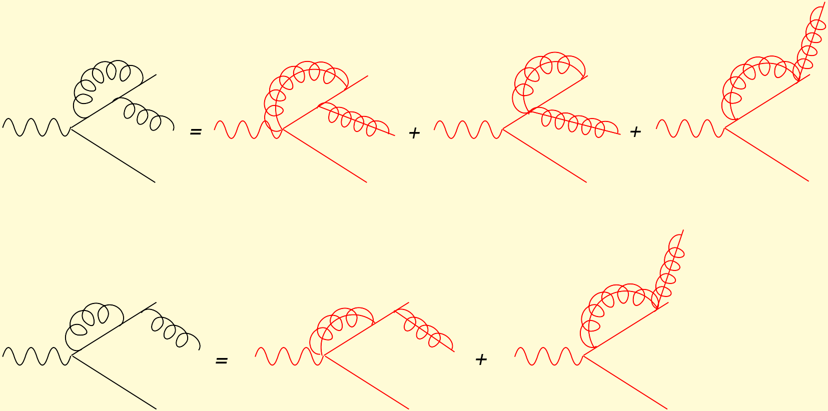
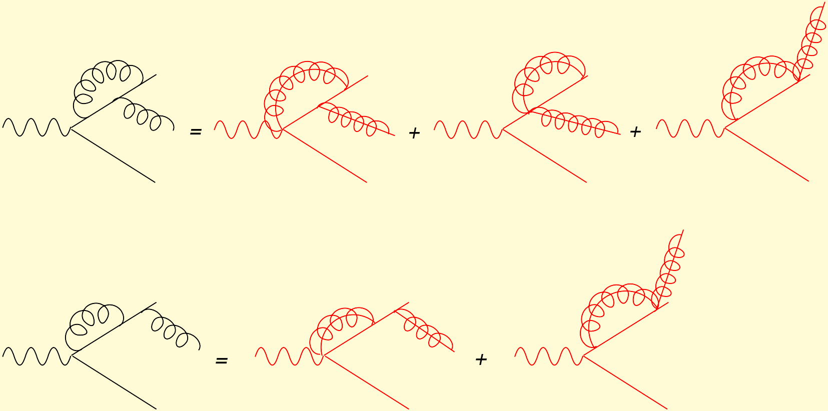
diagram matching



- Is there a way to compute directly in terms of $\chi_n, \mathcal{B}_n^\mu$?
- Can the diagrammatics be simplified?

diagram matching



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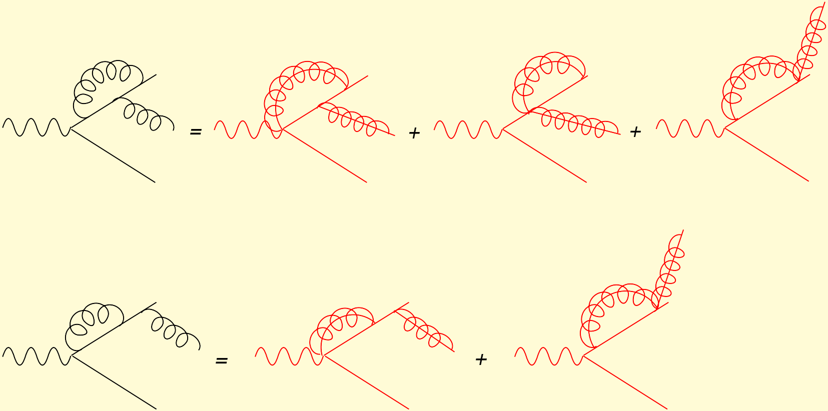

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Quantization of SCET

[Bauer, O.C., Ovanesyan'08]

- Original formulation of the theory:

$$\mathcal{L}_I^n(\xi_n, A_n) = \bar{\xi}_n \left[in\cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp \right] \frac{\not{\bar{n}}}{2}\xi_n - \frac{1}{2}\mathrm{Tr}\, F^n_{\mu\nu}F_n^{\mu\nu}$$

  where the soft spinor can be either eliminated through EOM, or integrated out.

- However, in order to build gauge-invariant operators, the following collinear quark operators are preferable:

$$\chi_n = W_n^\dagger \xi_n\,, \qquad W_n = \mathrm{P}\exp\left[-ig_s\int_0^\infty \mathrm{d}s\ \bar{n}\cdot A_n(\bar{n}s + x)\right]\,.$$

- The gluon field can also be made gauge-invariant through:

$$\begin{aligned} \mathcal{D}_n^\mu &= W_n^\dagger D_n^\mu W_n \\ &= i\partial_n^\mu + g_s \mathcal{B}_n^\mu \end{aligned}$$

  where

$$\mathcal{B}_n^\mu = \left[\frac{1}{\bar{n}\cdot\partial}\left[i\bar{n}\cdot\mathcal{D}_n, i\mathcal{D}_n^\mu\right]\right] = \frac{1}{g_s}\left[W_n^\dagger iD_n^\mu W_n\right]\,,$$

Quantization of SCET

- Therefore, in principle 4 possible (naive) formulations, only 3 used:

$$\mathcal{L}_I^n(\xi_n, A_n) = \bar{\xi}_n \left[in\cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp \right] \frac{\not{\bar{n}}}{2}\xi_n - \frac{1}{2}\mathrm{Tr}\, F^n_{\mu\nu}F_n^{\mu\nu}$$

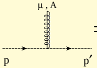
$$\mathcal{L}_{II}^n(\chi_n, A_n) = \bar{\chi}_n W_n^\dagger \left[in\cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp \right] \frac{\not{\bar{n}}}{2} W_n\chi_n - \frac{1}{2}\mathrm{Tr}\, F^n_{\mu\nu}F_n^{\mu\nu}$$

$$\mathcal{L}_{III}^n(\chi_n, \mathcal{B}_n) = \bar{\chi}_n \left[in\cdot \mathcal{D}_n + i\not{\mathcal{D}}_n^\perp \frac{1}{i\bar{n}\cdot \partial} i\not{\mathcal{D}}_n^\perp \right] \frac{\not{\bar{n}}}{2}\chi_n - \frac{1}{2}\mathrm{Tr}\, \mathcal{F}^n_{\mu\nu}\mathcal{F}_n^{\mu\nu}$$

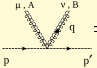
- We want to proof that indeed

$$\mathcal{L}_I^n(\xi_n, A_n) \equiv \mathcal{L}_{II}^n(\chi_n, A_n) \equiv \mathcal{L}_{III}^n(\chi_n, \mathcal{B}_n)$$

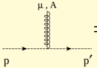
Quantization of SCET

$V_{\mathcal{L}_I}^{(1)} =$ 

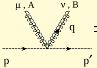
$= igT^A \left[n^\mu + \frac{\gamma_\perp^\mu \not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}'_\perp \gamma_\perp^\mu}{\bar{n}\cdot p'} - \frac{\not{p}'_\perp \not{p}_\perp}{\bar{n}\cdot p'\,\bar{n}\cdot p} \bar{n}^\mu \right] \frac{\not{\bar{n}}}{2}$

$V_{\mathcal{L}_I}^{(2)} =$ 

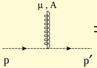
$= ig^2 \left[\frac{T^A T^B}{\bar{n}\cdot(p-q)} \gamma_\perp^\mu \gamma_\perp^\nu + \frac{T^B T^A}{\bar{n}\cdot(q+p')} \gamma_\perp^\nu \gamma_\perp^\mu \right] \frac{\not{\bar{n}}}{2} + (\dots)$

$V_{\mathcal{L}_{II}}^{(1)} =$ 

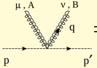
$= V_{\mathcal{L}_I}^{(1)} + igT^A \left[\frac{1}{\bar{n}\cdot(p-p')} \left(\frac{p^2}{\bar{n}\cdot p} - \frac{p'^2}{\bar{n}\cdot p'} \right) \bar{n}^\mu \right] \frac{\not{\bar{n}}}{2}$

$V_{\mathcal{L}_{II}}^{(2)} =$ 

$= V_{\mathcal{L}_I}^{(2)} + (\dots)$

$V_{\mathcal{L}_{III}}^{(1)} =$ 

$= igT^A \left[n^\mu + \frac{\gamma_\perp^\mu \not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}'_\perp \gamma_\perp^\mu}{\bar{n}\cdot p'} \right] \frac{\not{\bar{n}}}{2}$

$V_{\mathcal{L}_{III}}^{(2)} =$ 

$= ig^2 \left[\frac{T^A T^B}{\bar{n}\cdot(p-q)} \gamma_\perp^\mu \gamma_\perp^\nu + \frac{T^B T^A}{\bar{n}\cdot(q+p')} \gamma_\perp^\nu \gamma_\perp^\mu \right] \frac{\not{\bar{n}}}{2}$

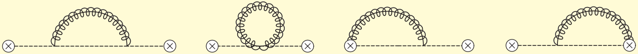
$\Delta_{\mathcal{L}_{III}} =$ 

$= -i \frac{\delta^{AB}}{k^2 + i\epsilon} \left(g_{\mu\nu} - \frac{\bar{n}_\mu k_\nu + \bar{n}_\nu k_\mu}{\bar{n}\cdot k} \right)$

Quantization of SCET

- Equivalence not obvious. Consider for instance,

$$\langle 0|T\chi_n(x)\bar{\chi}_n(y)|0\rangle = \langle 0|TW_n^\dagger(x)\xi_n(x)\bar{\xi}_n(y)W_n(y)|0\rangle$$



- To actually check that

$$\mathcal{L}_I^n(\xi_n, A_n) \equiv \mathcal{L}_{II}^n(\chi_n, A_n)$$

is relatively easy. One can show that

$$D_{I,a} + D_{I,b} + D_{I,c} + D_{I,d} = D_{II,a} + D_{II,b} = D_I$$

$$D_I = i\frac{\alpha_s C_F}{4\pi}\frac{\not{n}}{2}\frac{\bar{n}\cdot p}{p^2}\left(\frac{\mu^2}{-p^2}\right)^\epsilon\left[\frac{4}{\epsilon^2}+\frac{3}{\epsilon}+7-\frac{\pi^2}{3}\right].$$

- However, a naïve computation with $\mathcal{L}_{III}^n(\chi_n, \mathcal{B}_n)$ seems to bring a puzzle.

Quantization of SCET

- In order to proof the equivalence rigorously, one should go to the path integrals for the actions.

$$Z[J] = \int \mathcal{D}\bar{\xi}_n \mathcal{D}\xi_n \mathcal{D}A_n^\mu \exp\left[i \int d^4x \, \mathcal{S}_I(\xi_n, A_n^\mu, J_n) \right]$$

$$\begin{aligned} \mathcal{S}_I \quad = \quad & \sum_n \left[\mathcal{L}_I^n + \bar{J}_n^\xi \xi_n + \bar{\xi}_n J_n^\xi + \bar{J}_n^\chi W_n^\dagger \xi_n + \bar{\xi}_n W_n J_n^\chi + J_{n\mu}^A A_n^\mu + J_{n\mu}^\mathcal{B} \mathcal{B}_n^\mu(A_n) \right. \\ & + \sum_k J_k \mathcal{O}_k \left(W_n^\dagger \xi_n, \mathcal{B}_n^\mu(A_n) \right) \end{aligned}$$

where we enlarge the action with general external sources and

$$\mathcal{L}_I^n(\xi_n, A_n) = \bar{\xi}_n \left[in\cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp \right] \frac{\not{\bar{n}}}{2} \xi_n - \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu}^n F_n^{\mu\nu}$$

FlaviA net

Quantization of SCET

- $\mathcal{S}_{II}$ can be trivially worked out. Since the Wilson lines are unitary $W_n^\dagger W_n = 1$, the Jacobian is trivial $\mathcal{D}\xi_n \mathcal{D}\bar{\xi}_n \mathcal{D}A_n^\mu = \mathcal{D}\chi_n \mathcal{D}\bar{\chi}_n \mathcal{D}A_n^\mu$ and

$$Z[J] = \int \mathcal{D}\bar{\chi}_n \mathcal{D}\chi_n \mathcal{D}A_n^\mu \exp\left[i \int d^4x\, \mathcal{S}_{II}(\chi_n, A_n^\mu, J_n)\right]$$

$$\mathcal{S}_{II} = \sum_n \left[\mathcal{L}_{II}^n + \bar{J}_n^\xi W_n \chi_n + \bar{\chi}_n W_n^\dagger J_n^\xi + \bar{J}_n^\chi \chi_n + \bar{\chi}_n J_n^\chi + J_{n\mu}^A A_n^\mu + J_{n\mu}^{\mathcal{B}} \mathcal{B}_n^\mu(A_n)\right] + \sum_k J_k \mathcal{O}_k(\chi_n, \mathcal{B}_n(A_n))$$

where

$$\mathcal{L}_{II}^n(\chi_n, A_n) = \bar{\chi}_n W_n^\dagger \left[in\cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp\right] \frac{\not{\bar{n}}}{2} W_n \chi_n - \frac{1}{2} \mathrm{Tr}\, F_{\mu\nu}^n F_n^{\mu\nu}$$

Quantization of SCET

- Subtleties arise however when moving from $\mathcal{S}_{II}(\xi, A_n^\mu) \to \mathcal{S}_{III}(\chi, \mathcal{B}_n^\mu)$. A_μ has initially 4 degrees of freedom, so gauge-fixing is necessary. However, $\mathcal{B}_n^\mu$ is gauge-invariant. The right degrees of freedom are ensured by $\bar{n}\cdot\mathcal{B}_n = 0$, which follows from

$$\bar{n}\cdot\mathcal{B}_n = \frac{1}{g_s}\left[W_n^\dagger \underbrace{i\bar{n}\cdot D_n W_n}_{0}\right] = 0$$

- We can reduce the gauge redundancy of the A_n^μ field through the Faddeev-Popov procedure:

$$Z_{\text{YM}} = \int \mathcal{D}\alpha \int \mathcal{D}A_n^\mu \, \delta[G(A_n)] \, E_G[A_n] \, ,$$

where

$$E_G[A_n] = \det\left(\frac{\delta G(A_n^\alpha)}{\delta\alpha}\right)[A_n] \, e^{iS_{\text{YM}}[A_n]} \, .$$

Quantization of SCET

- Then formally,

$$\mathcal{D}A_n^\mu \, \delta[G(A_n)] = J_G[\mathcal{B}_n] \, \mathcal{D}\mathcal{B}_n^\mu \, \delta[\bar{n}\cdot\mathcal{B}_n]$$

$$Z_{\rm YM} = \int \mathcal{D}\mathcal{B}_n^\mu \; \delta[\bar{n}\cdot\mathcal{B}_n] \, J_G[\mathcal{B}_n] \, E_G[A_n(\mathcal{B}_n)]$$

- In a general gauge the result is highly non-trivial. However, the generating functional is gauge-invariant. A smart choice is Light-Cone gauge, $G(A_n) = \bar{n}\cdot A_n$, which implies

$$\mathcal{B}_n^\mu = A_n^\mu \,, \qquad J_{G_{\rm LC}}[\mathcal{B}_n] = 1 \,, \qquad E_{LC}[\mathcal{B}_n] = \det(\bar{n}\cdot\partial) e^{iS_{\rm YM}[\mathcal{B}_n]}$$

- The gauge-invariant gluon propagator is therefore

$$(\Delta_{\mathcal{B}})^{ab}_{\mu\nu}(k) = \frac{-i\delta^{ab}}{k^2 + i\epsilon} \left(g_{\mu\nu} - \frac{\bar{n}_\mu k_\nu + \bar{n}_\nu k_\mu}{\bar{n}\cdot k} \right)$$

- Note: LC gauge is unitary and therefore ghost-free.

SCET in a QCD disguise

- The goal is to describe SCET in terms of QCD interactions. Start with

$$Z[J] = \int \mathcal{D}\bar{\psi}_n \mathcal{D}\psi_n \mathcal{D}A_n^\mu \exp\left[i\int d^4x\, S_{\rm QCD}(\psi_n, A_n, J)\right]$$

  with

$$\begin{aligned} S_{\rm QCD} &= \sum_n \left[\mathcal{L}_n^{\rm QCD} + \bar{J}_n^\xi \mathcal{M}_n^\xi \psi_n + \bar{\psi}_n \bar{\mathcal{M}}_n^\xi J_n^\xi + \bar{J}_n^\chi \mathcal{M}_n^\chi \psi_n \right.\\ &\left. + \bar{\psi}_n \bar{\mathcal{M}}_n^\chi J_n^\chi + J_{n\mu}^A A_n^\mu + J_{n\mu}^{\mathcal{B}} \mathcal{B}_n^\mu(A_n)\right] \\ &+ \sum_k J_k \mathcal{Q}_k(\psi_n, A_n) \end{aligned}$$

  and

$$\mathcal{L}_n^{\rm QCD} = \bar{\psi}_n\, i\not{D}\psi_n$$

- We need to determine $\mathcal{M}_n$ and $\mathcal{Q}_k$ such that the previous action is the SCET action. We project the spinor field into ξ_n and ϕ_n and integrate out the soft components using:

$$\int \mathcal{D}\phi\mathcal{D}\bar{\phi}\, \exp\left[i\int d^4x(\bar{\phi}M\phi + \bar{J}\phi + \bar{\phi}J)\right] = \det(-iM)\exp\left[-i\int d^4x \bar{J}\frac{1}{M}J\right]$$

SCET in a QCD disguise

- The determinant is trivial:

$$\det\left(\frac{\not{n}}{2}\bar{n}\cdot D\right) = \int \mathcal{D}\eta_n \mathcal{D}\bar{\eta}_n \exp\left[-\int d^4x\, \bar{\eta}_n \left(\frac{\not{n}}{2}\bar{n}\cdot D\right)\eta_n\right]$$

$$= \int \mathcal{D}\eta_n' \mathcal{D}\bar{\eta}_n' \exp\left[-\int d^4x\, \bar{\eta}_n' \left(\frac{\not{n}}{2}W_n^\dagger \bar{n}\cdot D W_n\right)\eta_n'\right]$$

$$= \det\left(\frac{\not{n}}{2}\bar{n}\cdot\partial\right),$$

- Alternatively, before integrating out the soft field,

$$\mathcal{L} = \sum_n \sum_{p,p'} e^{-i(p-p')x}\Big[\bar{\chi}_{n,p'}\; iW^\dagger n\cdot DW\,\frac{\bar{\not{n}}}{2}\,\chi_{n,p} + \bar{\phi}_{n,p'}\,(iW^\dagger \not{D}_\perp W)\,\chi_{n,p}$$

$$+\; \bar{\phi}_{n,p'}\,(i\bar{n}\cdot\partial)\,\frac{\not{n}}{2}\,\phi_{n,p} + \bar{\chi}_{n,p'}\,(iW^\dagger \not{D}_\perp W)\,\phi_{n,p}\Big]$$

SCET in a QCD disguise

- The result is

$$Z = \int \mathcal{D}\bar{\xi}_n \mathcal{D}\xi_n \mathcal{D}A_n^\mu \exp\left[i \int d^4x\, S^{\text{SCET}}\right]$$

with

$$\begin{aligned} S^{\text{SCET}}(J_k = 0) \quad = \quad & \sum_n \mathcal{L}_l^n + \bar{J}_n^\xi \mathcal{M}_n^\xi \mathcal{R}_n \xi_n + \bar{\xi}_n \bar{\mathcal{R}}_n \bar{\mathcal{M}}_n^\xi J_n^\xi + \bar{J}_n^\chi \mathcal{M}_n^\chi \mathcal{R}_n \xi_n \\ & + \bar{\xi}_n \bar{\mathcal{R}}_n \bar{\mathcal{M}}_n^\chi J_n^\chi - \left(\bar{J}_n^\xi \mathcal{M}_n^\xi + \bar{J}_n^\chi \mathcal{M}_n^\chi\right) \frac{1}{i\bar{n}\cdot D} \frac{\bar{\not{n}}}{2} \left(\bar{\mathcal{M}}_n^\xi J_n^\xi + \bar{\mathcal{M}}_n^\chi J_n^\chi\right) \end{aligned}$$

with

$$\mathcal{R}_n = \left[1 + \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \frac{\bar{\not{n}}}{2}\right]$$

SCET in a QCD disguise

- Fulfilled if

$$\mathcal{M}_n^\xi \mathcal{R}_n \xi_n \equiv \xi_n \,, \qquad \mathcal{M}_n^\chi \mathcal{R}_n \chi_n \equiv W_n^\dagger \xi_n$$

Two solutions:

$$\begin{aligned} \mathcal{M}_n^\xi &= \mathcal{R}_n^{-1} & \text{or} \quad \mathcal{M}_n^\xi &= P_n \,, \\ \mathcal{M}_n^\chi &= W_n^\dagger \mathcal{R}_n^{-1} & \text{or} \quad \mathcal{M}_n^\chi &= W_n^\dagger P_n \end{aligned}$$

- We want to avoid proliferation of contact terms $\Longrightarrow$ Projector solution $\mathcal{M}_n^\xi = P_n$.

SCET in a QCD disguise

- For the external currents, use

$$\mathcal{Q}_k(\psi_n, A_n) = \mathcal{O}_k(W_n^\dagger P_n \psi_n, \mathcal{B}_n(A_n))$$

- The final answer then reads

$$\begin{aligned} S_{\rm QCD} &= \sum_n \big[\mathcal{L}_n^{\rm QCD} + \bar{J}_n^\xi P_n \psi_n + \bar{\psi}_n P_{\bar{n}} J_n^\xi + \bar{J}_n^\chi W_n^\dagger P_n \psi_n \\ &\quad + \bar{\psi}_n P_{\bar{n}} W_n J_n^\chi + J_{n\mu}^A A_n^\mu + J_{n\mu}^{\mathcal{B}} \mathcal{B}_n^\mu(A_n)\big] \\ &\quad + \sum_k J_k \mathcal{O}_k(W_n^\dagger P_n \psi_n, \mathcal{B}_n(A_n)) \end{aligned}$$

- The collinear sector of SCET is a sum of multiple copies of QCD-like Lagrangians, whose interactions are mediated by $\mathcal{O}_k$.
- usoft degrees of freedom treated conventionally.

Summary

- We provided the rules for directly computing SCET correlators with gauge-invariant fields $(\chi_n, \mathcal{B}_n^\mu)$ from a path integral approach to SCET. It turns out that the full theory in terms of gauge-invariant fields is equivalent to a light-cone gauge-fixed SCET in the Landau gauge ($\xi = 0$).
- Matching can in principle be greatly simplified if one computes in the QCD-like interaction representation of SCET.

Summary

- We provided the rules for directly computing SCET correlators with gauge-invariant fields $(\chi_n, \mathcal{B}_n^\mu)$ from a path integral approach to SCET. It turns out that the full theory in terms of gauge-invariant fields is equivalent to a light-cone gauge-fixed SCET in the Landau gauge $(\xi = 0)$.
- Matching can in principle be greatly simplified if one computes in the QCD-like interaction representation of SCET.

Things (I) still do not understand

- Prescriptions for the spurious poles in LC gauge troublesome... One possibility is the Mandelstam-Leibbrandt prescription,

$$(\Delta_{\mathcal{B}})^{ab}_{\mu\nu}(k) = \frac{-i\delta^{ab}}{k^2 + i\epsilon}\left(g_{\mu\nu} - n\cdot k\,\frac{\bar{n}_\mu k_\nu + \bar{n}_\nu k_\mu}{\bar{n}\cdot k n\cdot k + i\varepsilon}\right)$$

- There should be a compact expression like

$$\mathcal{L}^c_{matching} \simeq QCD_n(\psi, A_\mu) - SCET(\psi, A_\mu)$$

to compute the QCD-SCET matching.

- Applications?