Why Glauber Gluons Are Relevant?

Ahmad Idilbi
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A.I and A. Majumder, arXiv:0808.1087 [hep-ph]

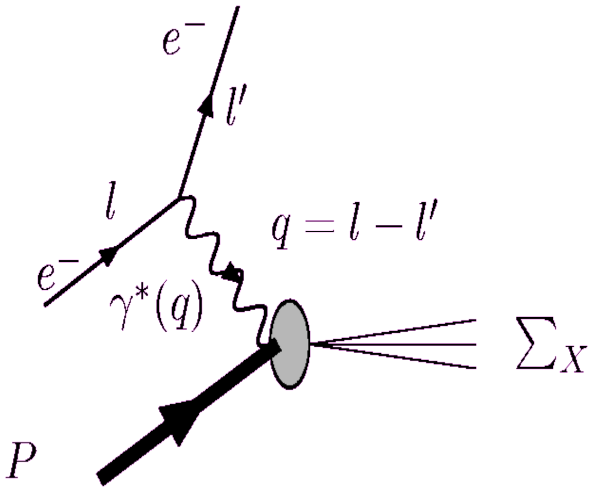
Semi-Inclusive Deep-Inelastic Scattering (SIDIS) and
Transverse Momentum Parton Distribution (TMDPDF)

Jet Broadening In The Transverse Direction: DIS
On Extended Nuclei

Review Of DIS In QCD

In The Threshold Region: $x \to 1$

$$F_2^{\rm ns}(x,Q^2) = H(Q^2,\mu)\, Q^2 \int_x^1 \frac{dz}{z}\, J\Big(Q^2\,\frac{1-z}{z},\mu\Big)\, \frac{x}{z}\, \phi_q^{\rm ns}\Big(\frac{x}{z},\mu\Big)$$



J :Jet Function Representing Final State Jet

[Sterman 87]

ϕ :Standard Parton Distribution Function (PDF). In Full QCD

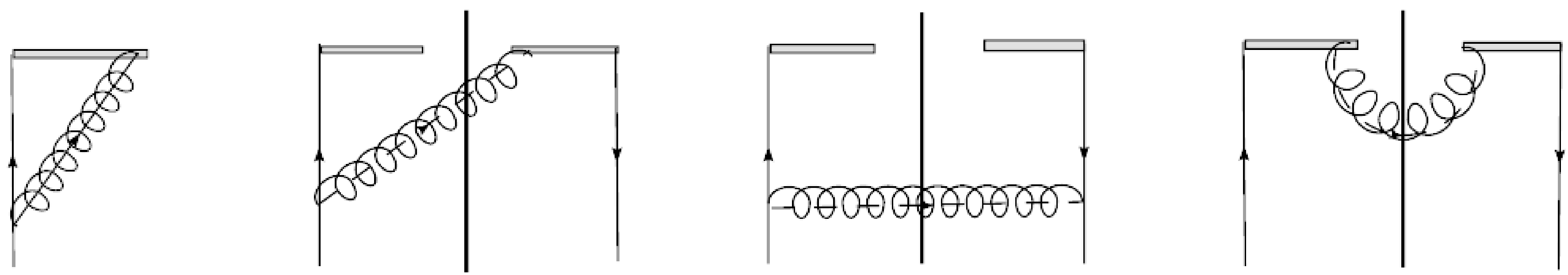
$$\phi_q^{\rm ns}(\xi,\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi t n\cdot p}\, \langle N(p)|\, \bar\psi(tn)\, [tn,0]\, \frac{\not n}{2}\, \psi(0)\, |N(p)\rangle$$

Ligh-Cone Gauge Link Insures Gauge Invariance

PDF IN SCET

$$\phi(x,\mu_I^2) = \left\langle P \left| \bar{\xi}_{\bar{n}} W_{\bar{n}} \delta\left(x - \frac{n\cdot\mathcal{P}_+}{n\cdot p}\right) \frac{\not{n}}{\sqrt{2}} W_{\bar{n}}^\dagger \xi_{\bar{n}} \right| P \right\rangle$$

$$W_{\bar{n}}(z) = Pexp\left[-ig_s \int_{-\infty}^{0} dt\ n\cdot A_{\bar{n}}(nt+z)\right]$$



$$\hat{\phi}_N^R = 1 + \frac{\alpha_s}{4\pi} C_F \left(-\frac{1}{\varepsilon_{\mathrm{IR}}}\right)\left[3 - 4\ln\overline{N}\right]$$

[Manohar:03]

- Equivalent To The Full QCD Result In The Threshold Limit

$$F_2^{\rm ns}(x,Q^2) = H(Q^2,\mu)\, Q^2 \int_x^1 \frac{dz}{z}\, J\Big(Q^2\,\frac{1-z}{z},\mu\Big)\, \frac{x}{z}\, \phi_q^{\rm ns}\Big(\frac{x}{z},\mu\Big)$$

- Can Be Successfully Reproduced In Soft-Collinear ET

[Chay and Kim:05
Becher,Neubert and Pecjak:06]

- Not Surprising: The Soft And Collinear Modes Generate The Infra-Red Divergences IN pQCD

[Sterman:95]

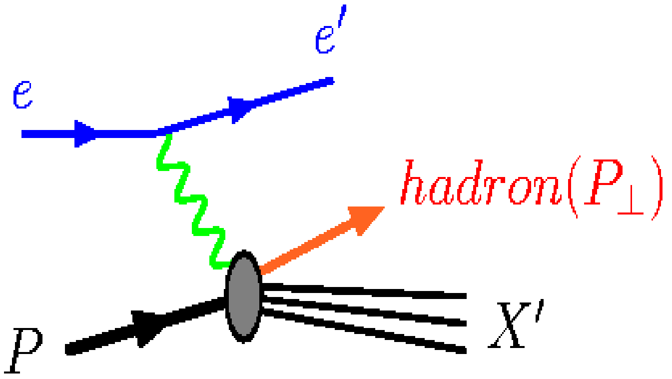
IN SCET:

$$F_{2,N} = 1 + \frac{\alpha_s}{4\pi} C_F \left\{ -\frac{1}{\varepsilon_{\rm IR}}[3 - 4\ln \overline{N}] - 2\ln^2\left(\frac{Q^2}{\mu^2}\right) + 6\ln\left(\frac{Q^2}{\mu^2}\right) + 2\ln^2\left(\frac{Q^2}{\overline{N}\mu^2}\right) - 3\ln\left(\frac{Q^2}{\overline{N}\mu^2}\right) - 9 - \frac{\pi^2}{3} \right\} ,$$

- Equivalent To The Full QCD Result In The Threshold Limit

SIDIS: Factorization Theorem

For very small p_T



$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\dots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp \times q\left(x_B, k_\perp, \mu^2, \rho,\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho) \times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) ,$$

$\hat{q}_T$: Fragmentation Function Of Quark Into Hadron

q : Transverse Momentum PDF

[Ji, Ma and Yuan (2004)]

TMDPDF

$$f(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\vec{\xi}_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \times \langle P|\bar{\psi}(\xi^-,\vec{\xi}_\perp) L^\dagger_{\vec{\xi}_\perp}(\infty,\xi^-) \times \gamma^+ L_0(\infty,0)\psi(0,\vec{0}_\perp)|P\rangle$$

Wilson line (Analogous to W)

$$L_{\vec{\xi}_\perp}(\infty,\xi^-) = P\exp\left(ig_s \int_{\xi^-}^{\infty} d\xi^- A^+(\xi^-,\vec{\xi}_\perp)\right)$$

Gauge Invariance Requires Additional Gauge Link:

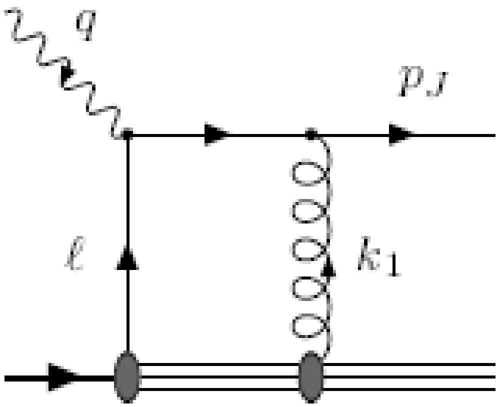
$$L_{\xi^-=\infty}(\vec{\xi}_\perp,\vec{0}_\perp) = P\exp\left(ig_s \int_0^\infty d\vec{\xi}_\perp \cdot \vec{A}_\perp(\xi^- = \infty,\vec{\xi}_\perp)\right)$$

[Ji and Yuan 02]

Where It Comes From?

•Transverse Gauge Link: Final State Interactions With Gluons Carrying Only Transverse Momentum

[Belitsky, Ji and Yuan 03]



$$\langle p_J, \bar{N}|j_\nu(0)|P\rangle_{(1)} = g\bar{u}(p_J)\int \frac{d^4k_1}{(2\pi)^4}\langle \bar{N}|\not{A}(k_1)S_0(p_J-k_1)\gamma_\nu\psi(0)|P\rangle$$

•In Light-Cone Gauge: $A^+ = 0$

$$\bar{u}(p_J)\langle \bar{N}|\not{A}(k_1)S_0(p_J-k_1)\gamma_\nu\psi(0)|P\rangle \approx \bar{u}(p_J)\frac{\gamma_\alpha \not{\boldsymbol{k}}_1\gamma_\nu}{2p_{J-}k_{1+}+\boldsymbol{k}_1^2-i0}\langle \bar{N}|\boldsymbol{A}_\alpha(k_1)\psi(0)|P\rangle$$

Use Chisholm`s Representation

$$\frac{1}{2p_{J-}k_{1+}+\boldsymbol{k}_1^2-i0} = i\int_0^\infty d\lambda\, \mathrm{e}^{-i\lambda(2p_{J-}k_{1+}+\boldsymbol{k}_1^2-i0)}$$

Integrate Over k_{1+} → $A^\mu(x^- = 2\lambda p_{J\_}, x^+ = 0, \mathbf{k_1})$

Important Step: Take The Scaling Limit: $p_{J-} \longrightarrow \infty,$

$$A^\mu(x^- = \infty, x^+ = 0, \mathbf{k_1})$$

The λ Integral Becomes Trivial:

$$\langle p_J, \bar{N}|j_\nu(0)|P\rangle_{(1)} = g\bar{u}(p_J)\int \frac{d^2\boldsymbol{k}_1}{(2\pi)^2}\gamma_\alpha \frac{\not{\boldsymbol{k}}_1}{\boldsymbol{k}_1^2 - i0}\gamma_\nu \langle \bar{N}|\boldsymbol{A}_\alpha(x^- - = \infty, x^+ = 0, \mathbf{k_1})\psi(0)|P\rangle$$

All Information On Other Momentum Components Vanished

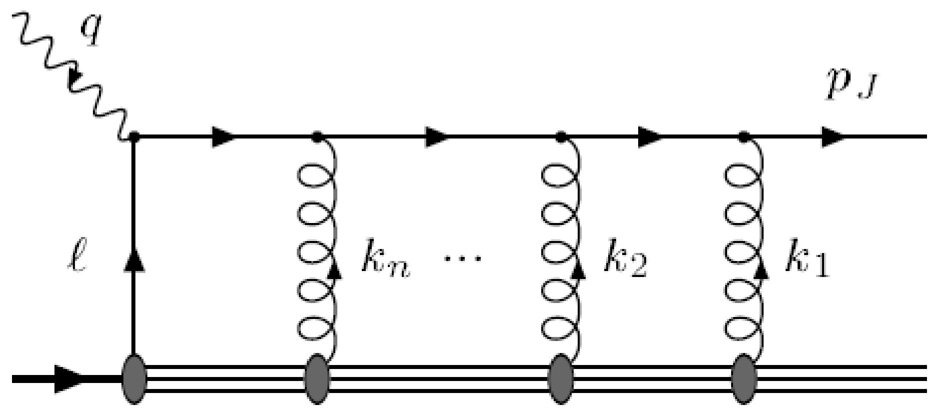
At Light-cone Infinity The Field Strength Tensor Vanishes

$A_\alpha(x^- = \infty, x^+ = 0, \mathbf{x}_\perp) = \boldsymbol{\nabla}_\alpha \, \phi(\mathbf{x}_\perp)$ ⟶ Pure Gauge

Fourier Transform Of $\phi(\mathbf{x}_\perp)$ And Integration By Part:

$\langle p_J, \bar{N}|j_\nu(0)|P\rangle_{(1)} = -ig\bar{u}(p_J)\gamma_\nu \langle p_J, \bar{N}|\phi(\mathbf{0})\psi(0)|P\rangle$ ⟶ $\phi(\mathbf{0}) = -\int_0^\infty d\boldsymbol{\xi} \cdot \boldsymbol{A}(\infty, \boldsymbol{\xi})$

Multi-Gluon Exchange



In The Scaling Limit $Q \to \infty$ And In Light-Cone Gauge:

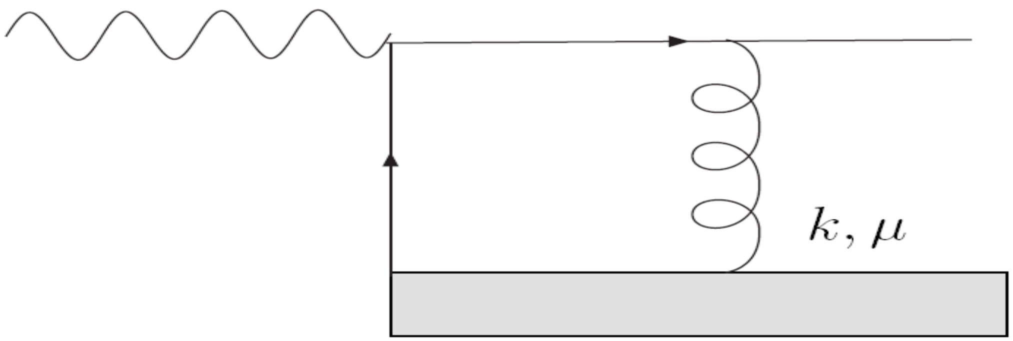
$$\langle p^-, N | j_\nu(0) | P \rangle \;=\; (-1)^n (i)^n (ig_s)^n \bar{u}(p^-) \Pi_{i=1}^n \langle N | \int \frac{d^2\vec{k}_i}{(2\pi)^2} \; A(\infty, \vec{k}_i) \frac{\sum_{j=1}^i \; \not{\vec{k}}_{j\perp}}{|\sum_{j=1}^i \vec{k}_{j\perp}|^2 - i\varepsilon} \gamma_\nu \psi(0) | P \rangle$$

The Information Is Only On Transverse Components

Are These Glauber Gluons? $k^\mu = (k^+, k^-, k_\perp) = Q(\lambda^2, \lambda^2, \lambda)$

What Are The Relevant Momentum Modes Needed To describe SIDIS Or TMDPDF?

BJY Revisited: One Gluon



$$I_1 \;=\; g_s\, \bar{\xi}_{\bar{n}} \frac{\gamma^{\mu\perp}}{2p^-} \int \frac{dk^+}{2\pi} \int \frac{d^2\vec{k}_\perp}{(2\pi)^2} \frac{\not{k}_\perp}{k^+ + \frac{|\vec{k}_\perp|^2}{2p^-} - i\varepsilon} A_{\mu\perp}(k^+, \vec{k}_\perp)$$

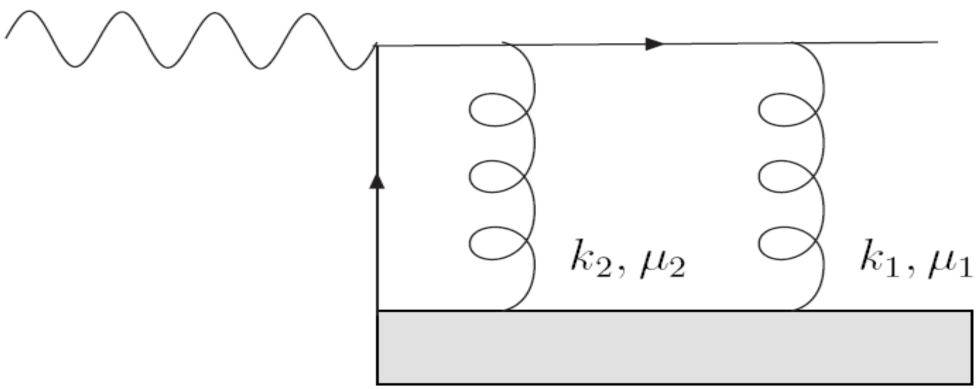
$$A_{\mu\perp}(k^+, \vec{k}_\perp) \;=\; \int dx^- \int d^2\vec{x}_\perp \tilde{A}_{\mu\perp}(x^-, \vec{x}_\perp) e^{i(k^+x^- - \vec{k}_\perp \cdot \vec{x}_\perp)}$$

We Keep The Glauber Scaling

$$I_1 \;=\; i g_s \int_0^\infty d|\vec{x}_\perp| \hat{n} \cdot A(\infty, \vec{x}_\perp)$$

The First Term In The Expansion Of The Transverse Gauge Link

Two Gluon Exchange



$$I_2 = -(ig_s)^2 \, \bar{\xi}_{\bar{n}} \frac{\gamma^{\mu_1 \perp}}{(2p^-)^2} \int \frac{dk_1^+}{2\pi} \int \frac{d^2\vec{k}_{1\perp}}{(2\pi)^2} \int \frac{dk_2^+}{2\pi} \int \frac{d^2\vec{k}_{2\perp}}{(2\pi)^2} \times$$

$$\frac{\not{k}_{1\perp} A_{\mu_1 \perp}(k_1^-, \vec{k}_{1\perp})[\gamma^{\mu_2 \perp}(\not{k}_{1\perp} + \not{k}_{2\perp}) + \not{k}_{1\perp}\gamma^{\mu_2 \perp}] A_{\mu_2 \perp}(k_2^-, \vec{k}_{2\perp})}{[k_1^+ + \frac{|\vec{k}_{1\perp}|^2}{2p^-} - i\varepsilon][(k_1^+ + k_2^+) + \frac{|\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2}{2p^-} - i\varepsilon]}$$

$$I_2 = (ig_s)^2 \int_0^\infty d|\vec{x}_{1\perp}| \; \hat{n}_1 \cdot \vec{A}_{1\perp}(\infty, |\vec{x}_{1\perp}|) \int_0^{|\vec{x}_1|} d|\vec{x}_{2\perp}| \; \hat{n}_2 \cdot \vec{A}_{2\perp}(\infty, |\vec{x}_{2\perp}|)$$

TMDPDF

There Are Two Gauge-Links:

$$L_{\vec{\xi}_\perp}(\infty,\xi^-) = P\exp\left(ig_s\int_{\xi^-}^{\infty} d\xi^- A^+(\xi^-,\vec{\xi}_\perp)\right)$$ ← Collinear Gluons

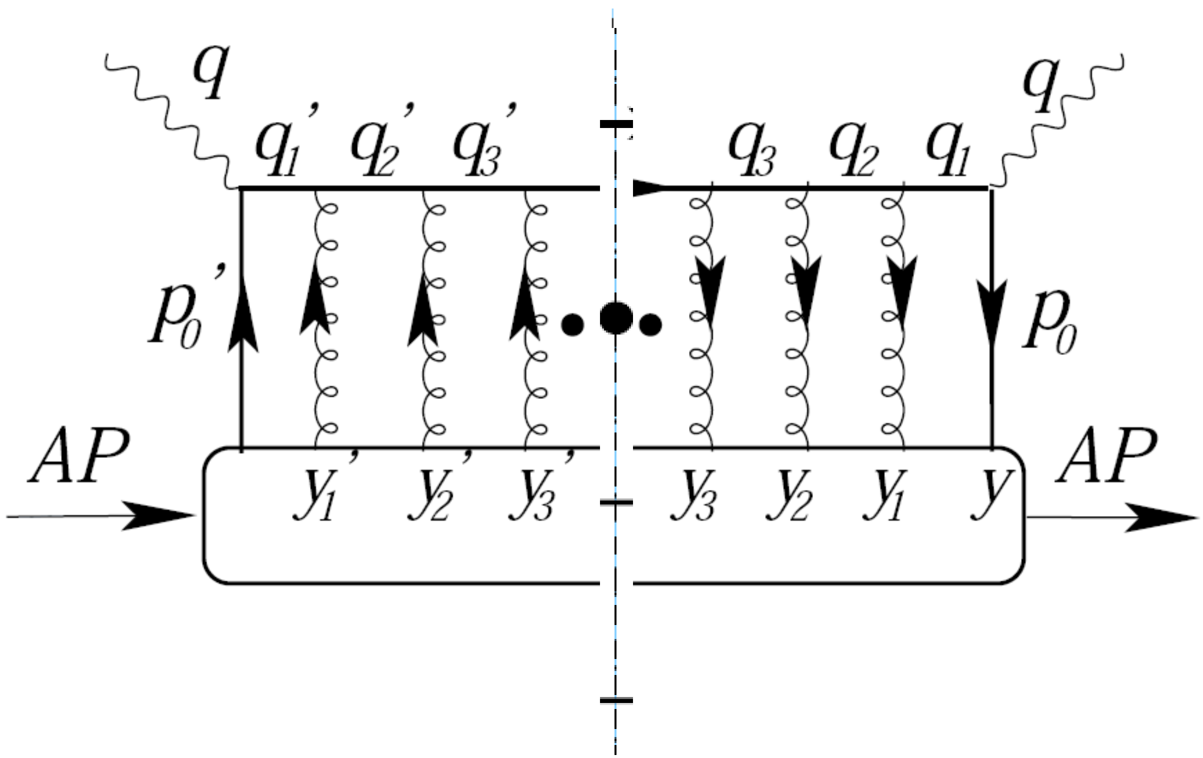
$$L_{\xi^-=\infty}(\vec{\xi}_\perp,\vec{0}_\perp) = P\exp\left(ig_s\int_0^{\infty} d\vec{\xi}_\perp\cdot\vec{A}_\perp(\xi^-=\infty,\vec{\xi}_\perp)\right)$$ ← Gluber Gluons

Transverse Broadening In Large Nuclei

- Consider The Cross Section For Semi-Inclusive Production Of Final-state Jet With Net Transverse Momentum $\vec{l}_\perp$

$$e(L_1) + A(p) \longrightarrow e(L_2) + J(\vec{l}_\perp) + X$$

Mass Number



- Focusing On Jets Broadening In The Transverse Direction:
For ``Substantial'' Effect, The Jet Should Have Very Small Off-Shellness So That It Lives Long Enough In The Medium.

The Collinear Parton Remains Collinear After Multiple Scatterings With Off-Shellness: $p^2 \sim \lambda^2 Q^2$

Consider Incoming Nucleus Collinear In The +z. The Struck Quark Is Collinear In The -z Direction With Momentum p :

$$p^+ \sim \lambda^2 Q, p^- \sim Q, \vec{p}_\perp \sim \lambda Q$$

The Quark Propagator:

$$(p+k_i)^2 = p^2 + k_i^2 + 2p^+k_i^- + 2p^-k_i^+ - 2\vec{p}_\perp \cdot \vec{k}_\perp^i \quad \sim \lambda^2 Q^2$$

→ The Gluon Momentum k Can Neither Be Soft Nor Collinear (In any Direction)

→ The First Non-Trivial Population: Glauber Gluons

$$k^\mu = (k^+, k^-, k_\perp) = Q(\lambda^2, \lambda^2, \lambda)$$

→ Recall: If The Out-going Jet Emits Soft Or Collinear Gluons It Remains With Very Small Off-Shellness.

Diffusion Equation For The Hadronic Tensor

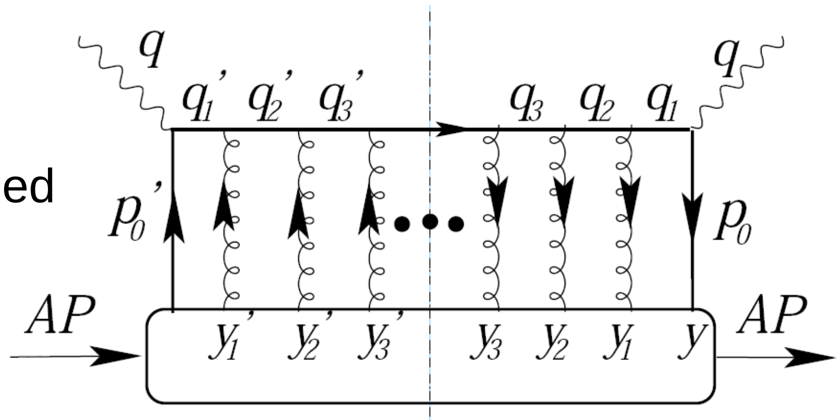
- Only Glauber Gluons

$$e(L_1) + A(p) \longrightarrow e(L_2) + J(\vec{l}_\perp) + X$$

- There Is A Confining Scale For The Medium

- The Quantum Correlations Between Different Nucleons Are Weak Enough And Are Encoded In one Parameter:

C_p^A : Probability Of Finding A Nucleon In The Nucleus



[Majumder And Muller:08, Idilbi And Majumder:08]

$$\frac{d^2W^{\mu\nu}}{d^2l_\perp} = e^{(DL^-)\nabla^2_{l_\perp}} \frac{d^2W_0^{\mu\nu}}{d^2l_\perp} \qquad \frac{d^2W_0^{\mu\nu}}{d^2l_\perp} = C_p^A W_0^{\mu\nu} \delta^2(\vec{l}_\perp)$$

L^- :Is The Length Traversed Inside The Nucleus

$$D = \frac{\pi^2\alpha_s}{2N_c}\rho \int \frac{d^3y d^2p_\perp}{(2\pi)^3 2p^+} \langle p|F^{a\,+\alpha}(y)F^a{}_{\alpha,}{}^{+}(0)|p\rangle.$$
$$\times \exp\left\{-i\left(\frac{p_\perp^2}{2q^-}y^- - p_\perp\cdot y_\perp\right)\right\}$$

ρ :Nucleon Density Inside The Nucleus

$$\frac{d^2W^{\mu\nu}}{d^2l_\perp} = e^{(DL^-)\nabla^2_{l_\perp}} \frac{d^2W_0^{\mu\nu}}{d^2l_\perp}$$

$$\frac{d^2W_0^{\mu\nu}}{d^2l_\perp} = C_p^A W_0^{\mu\nu} \delta^2(\vec{l}_\perp)$$

$$\frac{d^2W^{\mu\nu}}{d^2l_\perp} = C_p^A W_0^{\mu\nu} \phi(L^-, \vec{l}_\perp)$$

Two-dimensional Diffusion Eq.

$$\frac{\partial \phi(L^-, \vec{l}_\perp)}{\partial L^-} = D \nabla^2_{l_\perp} \phi(L^-, \vec{l}_\perp)$$

Initial Condition

$$\phi(L^- = 0, \vec{l}_\perp) = \delta^2(\vec{l}_\perp)$$

Gaussian Solution

$$\phi(L^-, \vec{l}_\perp) = \frac{1}{4\pi DL^-} \exp\left\{-\frac{l_\perp^2}{4DL^-}\right\}$$

How The Above Result Gets Modified When We Include Radiations Of Soft And Collinear Gluons?

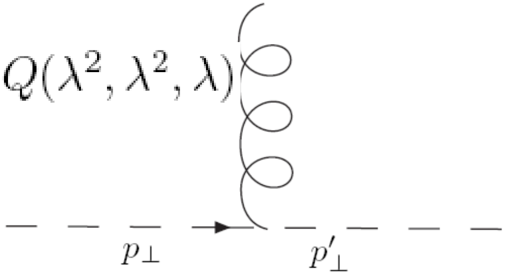
Effective Lagrangian For Glaubers

$$\bar{n} = \frac{1}{\sqrt{2}}(1,0,0,-1)$$

•Consider a Collinear Quark Along $-z$

•As In SCET

$$k^\mu = (k^+, k^-, k_\perp) = Q(\lambda^2, \lambda^2, \lambda)$$



$$\psi(x) = e^{-ip^- x^+} \sum_{\vec{p}_\perp} e^{i\vec{p}_\perp \cdot \vec{x}_\perp} \psi_{\bar{n},\vec{p}_\perp}(x)$$

•The true ``Labels'' Are The Transverse Components Only

•Power Counting of The Glauber Gluon Field: All Components Have To Scale As $Q\lambda^2$
[This Is obtained By Analyzing The Gluon Propagator In The Gleuber Region]

$$\mathcal{L}_g = \sum_{\vec{p}_\perp, \vec{p'}_\perp} e^{i(\vec{p}_\perp - \vec{p}'_\perp)\cdot \vec{x}_\perp} \bar{\xi}_{\bar{n},\vec{p'}_\perp}(x) \left[\bar{n}\cdot iD + (\not{p}_\perp + i\not{D}_\perp) \frac{1}{2n\cdot p} \left(1 - \frac{in\cdot D}{n\cdot p}\right) (\not{p}_\perp + i\not{D}_\perp) \right] \not{n} \xi_{\bar{n},\vec{p}_\perp}(x)$$

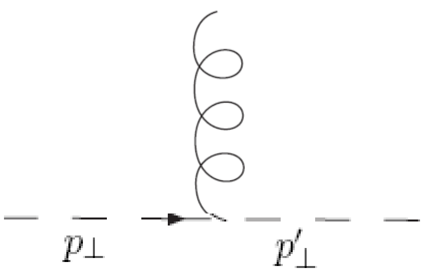
$n\cdot p = p^- \sim Q$

There Are Three Different ``Scaling'' Contributions

$$\mathcal{L}_g^{(0)} = \bar{\xi}_{\bar{n},\vec{p}_\perp} \left[i\bar{n}\cdot D + \frac{p_\perp^2}{2n\cdot p} \right] \not{n}\, \xi_{\bar{n},\vec{p}_\perp} \longrightarrow \lambda^4$$

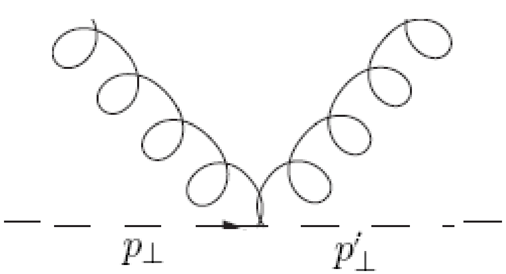
$$\mathcal{L}_g^{(1)} = \bar{\xi}_{\bar{n},\vec{p'}_\perp} \left[\frac{\not{p}'_\perp\, i\not{D}_\perp + i\not{D}_\perp\, \not{p}_\perp}{2n\cdot p} \right] \not{n}\, \xi_{\bar{n},\vec{p}_\perp} \longrightarrow \lambda^5$$

$$\mathcal{L}_g^{(2)} = \bar{\xi}_{\bar{n},\vec{p'}_\perp} \left[\frac{(i\not{D}_\perp)^2}{2n\cdot p} - \frac{\not{p}'_\perp\, in\cdot D\, \not{p}_\perp}{2(n\cdot p)^2} \right] \not{n}\, \xi_{\bar{n},\vec{p}_\perp} \longrightarrow \lambda^6$$



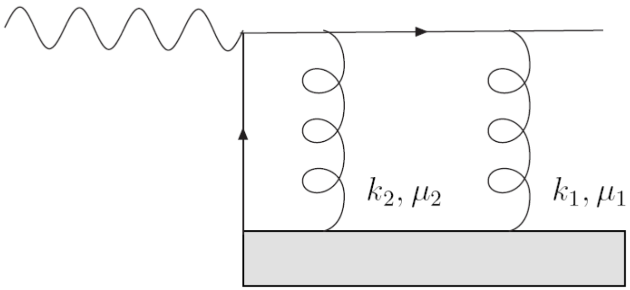
$$= ig_s t^a \left(\bar{n}_\mu + \frac{\gamma_\mu^\perp \not{p}'_\perp + \not{p}'_\perp \gamma_\mu^\perp}{2n\cdot p} \right) \not{n}$$

Difference With The SCET Lagrangian : $n\cdot A$ Scales As $Q\lambda^2$ For Glauber Gluons Instead Of Q For Collinear One.



$$= \frac{ig_s^2}{2n\cdot p}[t^a t^b \gamma_\perp^\mu \gamma_\perp^\nu + t^b t^a \gamma_\perp^\nu \gamma_\perp^\mu]\, \not{n}$$

Why The Sub-Leading Terms?



In Full QCD Abd IN Light-Cone Gauge: $\bar{n} \cdot A = 0$

$$\mathcal{J} \; = \; -(ig_s)^2 \int \frac{d^4k}{(2\pi)^4} \bar{u}(p) \frac{\not{A}_\perp(k_1)[2k_1^+ p^- \; \not{A}_\perp(k_2) + \not{k}_{1\perp} \; \not{A}_\perp(k_2)(\not{k}_{1\perp} + \not{k}_{2\perp})]}{[(p-k_1)^2 + i\varepsilon][(p-k_1-k_2)^2 + i\varepsilon]}$$

Two Contributions From The Effective Lagrangian With The Same Scaling:

Two Gluons Attached At Different Points Coming From $\mathcal{L}_g^{(1)}$

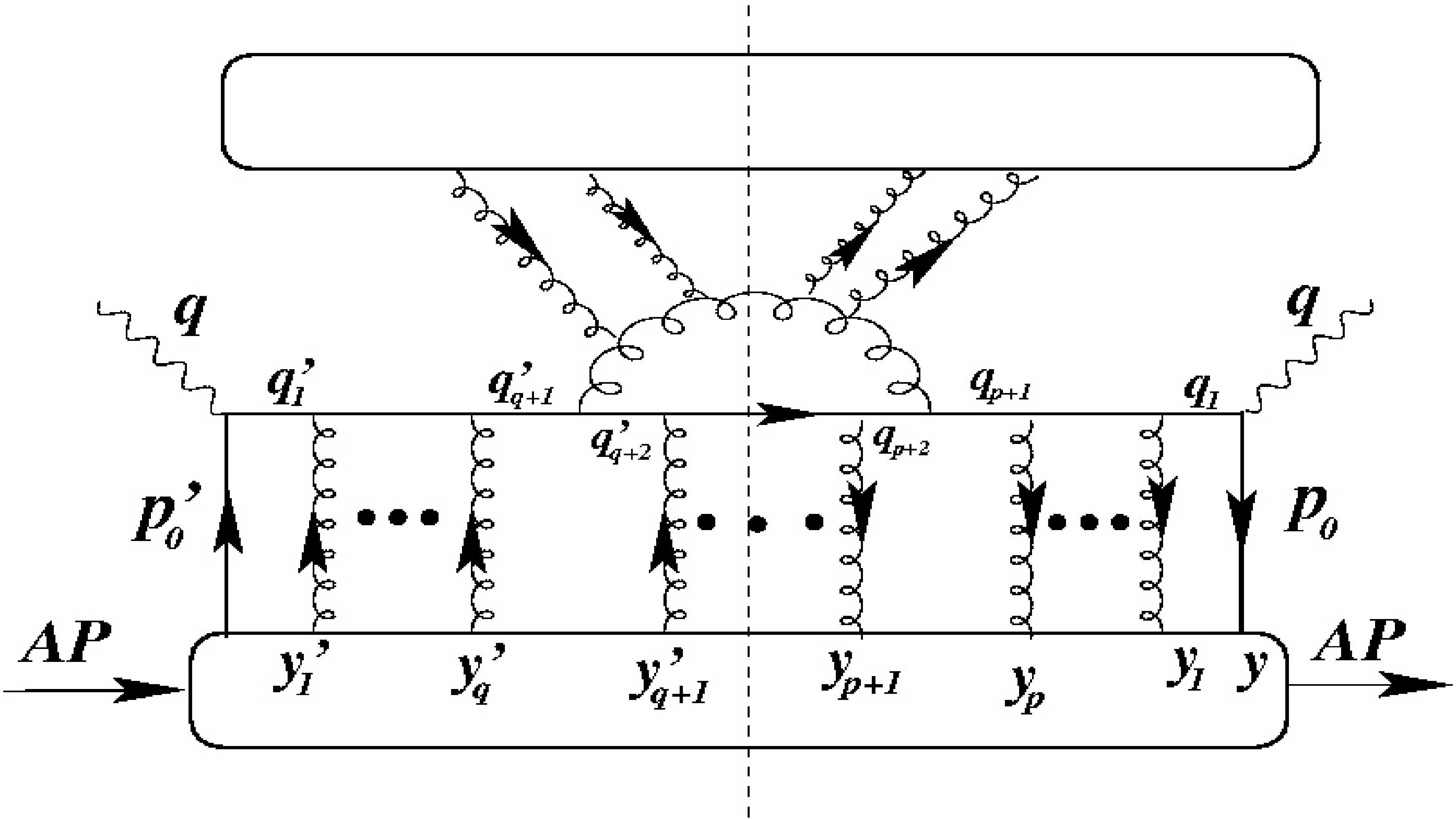
$$\mathcal{J}^{(1)} \; = \; -(ig_s)^2 \int \frac{d^4k}{(2\pi)^4} \; \bar{\xi}_{\bar{n}} \frac{\not{A}_\perp(k_1) \; \not{k}_{1\perp}[\not{A}_\perp(k_2)(\not{k}_{1\perp} + \not{k}_{2\perp}) + \not{k}_{1\perp} \; \not{A}_\perp(k_2)]}{[2p^- k_1^+ + |\vec{k}_{1\perp}|^2 - i\varepsilon][2p^-(k_1^+ + k_2^+) + |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2 - i\varepsilon]} \qquad \lambda^6/(\lambda^2 \times \lambda^2) = \lambda^2$$

Two Gluons Attached At One Poin Coming From $\mathcal{L}_g^{(2)}$

$$\mathcal{J}^{(2)} \; = \; -(ig_s)^2 \int \frac{d^4k}{(2\pi)^4} \; \bar{\xi}_{\bar{n}} \frac{\not{A}_\perp(k_1) \; \not{A}_\perp(k_2)}{[2p^-(k_1^+ + k_2^+) + |\vec{k}_{1\perp} + \vec{k}_{2\perp}|^2 - i\varepsilon]} \qquad \lambda^4/\lambda^2 = \lambda^2$$

$$\mathcal{J} = \mathcal{J}^{(1)} + \mathcal{J}^{(2)}$$

Future challenges



A combined SCET and Glauber lagrangian may allow for a factorized form for the diagrams above

Conclusions And Outlook

- There Are Many Interesting Physical Quantities And High Energy Processes Where Soft, Collinear And Glauber Gluons Are Needed To Give A Complete Description, e.g.,

  Energy loss of jets in medium

  TMDPDF

- The Path To Combine All Three Modes In One Framework (Effective Lagrangian?) Is Yet To Be Worked Out.