Electroweak Gauge Boson Production using Effective Field Theory

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Electroweak Corrections using Effective Field Theory: Applications to the LHC

Phys. Rev. D78, 073006 (2008)

Soft-Collinear Factorization and Zero-Bin Subtractions

arXiv:0901.1332 [hep-ph]

Electroweak Gauge Boson production using Effective Field theory

In preparation.

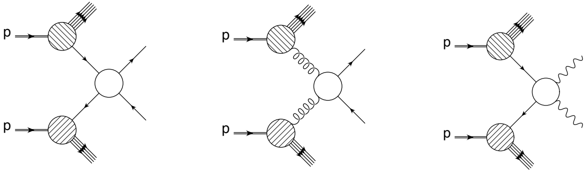
Jui-yu Chiu, Andre Hoang, Andreas Fuhrer, Randall Kelley, Aneesh Manohar

Outline

Introduction

- Typical processes:

$$q\bar{q} \to \mu^+\mu^- \qquad q\bar{q} \to q\bar{q} \qquad q\bar{q} \to t\bar{t} \qquad q\bar{q} \to WW, ZZ, \cdots$$



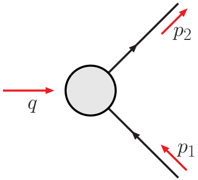
- Electroweak logarithms can be important at LHC energies.
- Last year we did external fermions and scalars.
- Present work extends work to include gauge boson production.
- Longitudinal polarization requires the GB equivalence theorem.
- Looking at fixed angle scattering with

$$\hat{s} \sim -\hat{t} \sim -\hat{u} \sim Q^2$$

Sudakov Form Factors

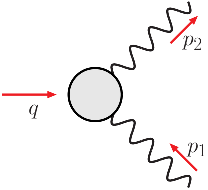
- $\hat{\mathcal{O}} = \bar{\psi}\Gamma^{\mu}\psi$ (Chiu et al. SCET 08)

$$F(Q)\left[\bar{u}(p_2)\Gamma^{\mu}u(p_1)\right] =$$



- $\hat{\mathcal{O}} = F^{A\mu\nu}F^{A}_{\mu\nu}$

$$F(Q)\left[-2q^2\left(\epsilon^{A}(p_2)\cdot\epsilon^{A}(p_1)\right)\right] =$$



SCET degrees of freedom (modes)

$$p^\mu = (p^-, p^+, p^\perp); \qquad p^2 = p^+p^- + p_\perp^2$$

- Power Counting

  $\lambda = M/Q$

- n-collinear

  $p^\mu \sim Q(1, \lambda^2, \lambda)$

- $\bar{n}$-collinear

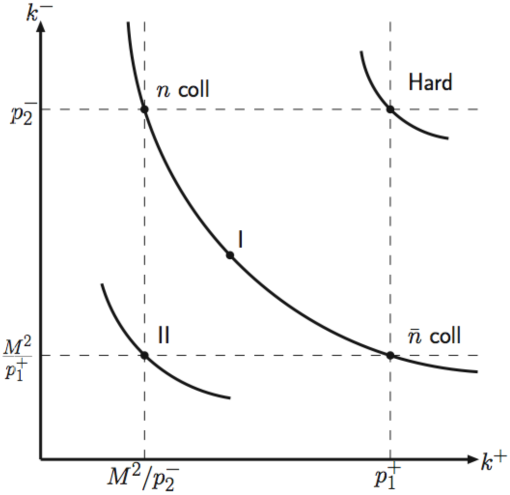
  $p^\mu \sim Q(\lambda^2, 1, \lambda)$

- "soft" Mass-Modes (I)

  $p^\mu \sim Q(\lambda, \lambda, \lambda)$

- Ultra-Soft (II)

  $p^\mu = Q(\lambda^2, \lambda^2, \lambda^2)$



SU(2) gauge theory with massive boson(M)

Full Theory

$C(Q, \mu)$ ———————————— $\mu \sim Q$

SCET ($M = 0$)

↓ γ_{SCET}

SCET ($M \neq 0$)

$D(M, \mu)$ ———————————— $\mu \sim M$

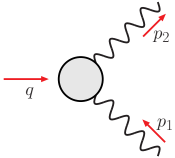
SCET (no gauge bosons)

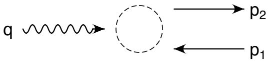
Transverse Gauge Boson Production

Simple Example

- $W_T W_T$ production (Breit Frame):

$$\mathcal{O} = F^{A\mu\nu} F^{A}_{\mu\nu}$$





- An external Gauge field requires matching onto B^μ.

(Arnesen, Kundu, Stewart and Bauer , Catà, Ovanesyan)

$$B^\mu_n = \frac{1}{g}[W^\dagger_n iD^\mu_n W_n], \qquad iD^\mu_n = i\partial_n + gA_n$$

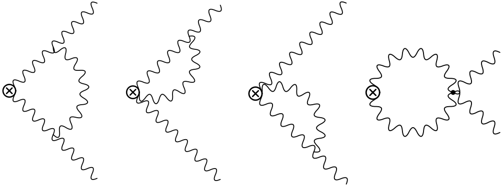
$$\bar{n} \cdot B_n = 0 \qquad \text{all orders}$$

$$B^\mu_{n\perp} = \left(A^\mu_{n\perp} - \frac{\mathcal{P}^\mu_\perp}{\bar{\mathcal{P}}} \bar{n} \cdot A_n \right) + \cdots \qquad \Delta^{\mu\nu}(k^2) = \frac{-i(g^{\mu\nu} - \frac{n^\mu \bar{n}^\nu + \bar{n}^\mu n^\nu}{2})}{k^2 - M^2 + i0^+}$$

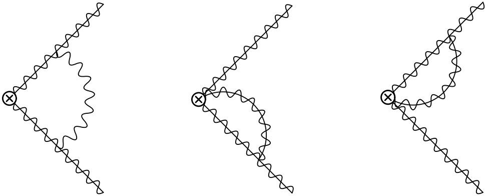
$$n \cdot B_n = \left(n \cdot A_n - \frac{n \cdot i\partial}{\bar{\mathcal{P}}} (\bar{n} \cdot A_n) \right) + \cdots \qquad \text{power suppressed}$$

High scale matching

- Full Theory: $\mathcal{O} = F^{A\mu\nu} F^{A}_{\mu\nu}$



- EFT: $\mathcal{O}^{\text{EFT}} = B_{1\perp} \cdot B_{2\perp}$ (scaleless)



High Scale matching

1 loop matching coefficient and anomalous dimension

- Matching at $\mu_1 \sim Q$ is the finite part of full theory diagram:

$$C(\mu) = \left[1 + \frac{\alpha}{4\pi} C_A \left(-\mathrm{L}_Q^2 + \frac{\pi^2}{6} \right) \right] 2Q^2, \qquad \mathrm{L}_Q = \log \frac{Q^2}{\mu^2}$$

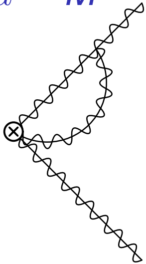
- Anomalous Dim from $\frac{1}{\epsilon_{\mathrm{IR}}}$ of the Full Theory graphs.

$$\gamma_{\mathrm{SCET}} = \frac{\alpha}{4\pi} C_A (4\mathrm{L}_Q - 4) + 2\gamma_{\mathrm{W}}$$

- Evolve from $\mu_1 \sim Q$ to $\mu_2 \sim M$ using

$$C(\mu_2) = C(\mu_1) \exp \left[\int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu} \gamma_{\mathrm{SCET}}(\mu) \right]$$

n-collinear diagram at $\mu \sim M$

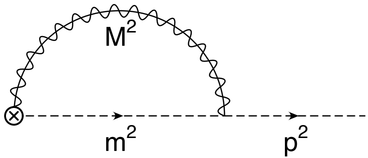


$$I_n = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\bar{n} \cdot (p_2 - k)}{[(p_2 - k)^2 - M^2][-\bar{n} \cdot k][k^2 - M^2]}$$

- Graph is IR divergent even in dimensional regularization with off-shellness.
- Use Δ- regulator:

$$I_n = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\bar{n} \cdot (p_2 - k)}{[(p_2 - k)^2 - M^2 - \Delta_2][-\bar{n} \cdot k - \delta_1][k^2 - M^2]}, \qquad \delta_i \equiv \frac{\Delta_i}{\bar{n}_i \cdot p_i}$$

Generic Collinear diagram - fermion



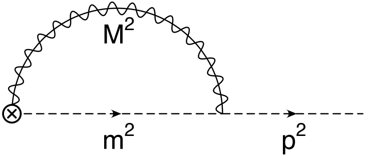
$$I_n(p^2, m^2, M^2) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\bar{n}\cdot(p-k)}{[(p-k)^2 - m^2 - \Delta_2][-\bar{n}\cdot k - \delta_1][k^2 - M^2]}$$

- Three separate scales: p^2, m^2, M^2

$$I_n(p^2, m^2, M^2) = I_n(0,0,M^2) + \underbrace{I_n(p^2, m^2, M^2) - I_n(0,0,M^2)}_{f_F\left(\frac{p^2}{M^2}, \frac{m^2}{M^2}\right)}$$

For scalar and Gauge use $f_S\left(\frac{p^2}{M^2}, \frac{m^2}{M^2}\right)$ instead.

Generic Collinear diagram - fermion



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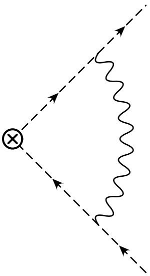
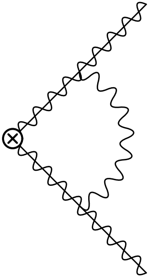
For scalar and Gauge use $f_S\left(\frac{p^2}{M^2}, \frac{m^2}{M^2}\right)$ instead.

Generic Soft diagram



or



- After using the Δ-regulator:

$$I_s^{ij} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{(n_i \cdot n_j)}{[-n_i \cdot k - \delta_i][-n_j \cdot k - \delta_j][k^2 - M^2]}$$

$$= \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \log \frac{(n_i \cdot n_j)\mu^2}{2\delta_i\delta_j} + \log \frac{M^2}{\mu^2} \log \frac{(n_i \cdot n_j)\mu^2}{2\delta_i\delta_j} - \frac{1}{2} \log \frac{M^2}{\mu^2} - \frac{\pi^2}{12}$$

Low Scale Matching

- After zero-bin subtraction

$$I_n - I_{n\varnothing} = \frac{\alpha}{4\pi} C_A \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon} \log \frac{\Delta_2}{\mu^2} - 2\left(1 - \log \frac{\Delta_2}{\mu^2}\right) \mathrm{L}_M - \mathrm{L}_M^2 + 2 - \frac{\pi^2}{2} + f_S(1,1) \right]$$

- Low Scale matching from $(I_n - I_{n\varnothing}) + (I_{\bar{n}} - I_{\bar{n}\varnothing}) + I_s + \delta R_W$

$$D(\mu) = \frac{\alpha}{4\pi} C_A \left[2\mathrm{L}_M \mathrm{L}_Q - \mathrm{L}_M^2 - 2\mathrm{L}_M + 2 - \frac{5\pi^2}{6} + 2 f_S(1,1) \right] + \delta R_W$$

- The $\frac{1}{\epsilon}$ terms give the anomalous dimension:

$$\gamma_{\mathrm{SCET}} = \frac{\alpha}{4\pi} C_A (4\mathrm{L}_Q - 4) + 2\gamma_{\mathrm{W}}$$

- Regulator Independent (could use analytic regulator)
- The single log term appears

Form Factor

- putting the pieces together:

$$F(Q) = C(Q)\exp\left[\int_Q^M \frac{d\mu}{\mu}\gamma_{\text{SCET}}\right]\left(D_0(M) + D_1(M)\log\frac{Q^2}{M^2}\right)$$

- using EFT technology, we have summed the following logarthims

$$\left(\frac{\alpha}{4\pi}\right)^n\left[\mathrm{L}^{2n},\ \mathrm{L}^{2n-1},\cdots,\ \mathrm{L}^2,\ \not{\mathrm{L}}^1,\ \not{\mathrm{L}}^0\right]$$

Goldstone Boson Equivalence Theorem

- Gives the relationship between W_L^A and φ^A production.

  Cornwall, Levine and Tiktopoulos

  Lee, Quigg, and Thacker

  Chanowitz, Gaillard

- Follows from BRST invariance.

$$\frac{k^\mu}{M_{\text{phys}}}\langle \underline{W}^\mu \cdots \rangle = iA(k^2)\langle \underline{\varphi} \cdots \rangle,$$

- At one loop there is a non-trivial, gauge dependent modification factor.

$$A = \frac{1}{M_{\text{phys}}}\frac{k^2+\xi\Gamma_L^{WW}}{\xi(M-\Gamma^{W\phi})}, \qquad A(k^2) = 1 \text{ at tree level}$$

  Bagger,Schmidt

  Yao, Yuan

  He, Kuang, Li

Goldstone Boson Equivalence Theorem

- At $E \gg M$,

$$\epsilon_L^\mu \sim \frac{k^\mu}{M_{\text{phys}}} + O\left(\frac{M_{\text{phys}}}{E}\right)$$

- LSZ reduction formula gives the matrix elements.

$$\underbrace{\epsilon_L^\mu \langle \underline{W^\mu} \cdots \rangle \sqrt{R_W}}_{\langle W_L \cdots | S | \cdots \rangle} = i \underbrace{\frac{A(M_{\text{phys}}^2)}{M_{\text{phys}}^2} \sqrt{\frac{R_W}{R_\varphi}}}_{E} \underbrace{\langle \underline{\varphi} \cdots \rangle \sqrt{R_\varphi}}_{\langle \varphi \cdots | \text{"}S\text{"} | \cdots \rangle} + \cdots$$

- Treat φ^A as a physical particle–evaluated at M_{phys}^2 not at ξM^2

$$\langle \varphi \cdots | S | \cdots \rangle = \langle T \underline{\varphi} \cdots \rangle \sqrt{R_\varphi}$$

Goldstone Boson Equivalence Theorem

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$$\epsilon_L^\mu \sim \frac{k^\mu}{M_{\text{phys}}} + O\left(\frac{M_{\text{phys}}}{E}\right)$$

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$$\underbrace{\epsilon_L^\mu \langle \underline{W^\mu} \cdots \rangle \sqrt{R_W}}_{\langle W_L \cdots | S | \cdots \rangle} = i \underbrace{\frac{A(M^2_{\text{phys}})}{M^2_{\text{phys}}} \sqrt{\frac{R_W}{R_\varphi}}}_{E} \underbrace{\langle \underline{\varphi} \cdots \rangle \sqrt{R_\varphi}}_{\langle \varphi \cdots | \text{“}S\text{”} | \cdots \rangle} + \cdots$$

- Treat φ^A as a physical particle–evaluated at M^2_{phys} not at ξM^2

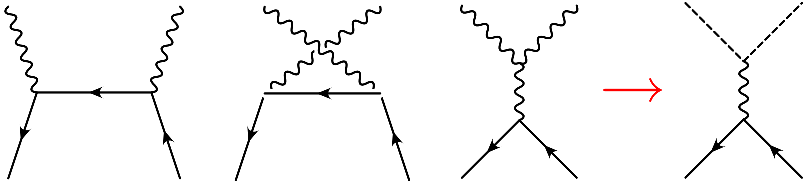
$$\langle \varphi \cdots | S | \cdots \rangle = \langle T \underline{\varphi} \cdots \rangle \sqrt{R_\varphi}$$

The Equivalence Theorem

- Generalization to multiple W_L's is straight forward.

$$\langle \underbrace{W_L^{A_1} \cdots W_L^{A_n}}_{n \text{ copies}} \cdots | \, S \, | \cdots \rangle = i^n E^n \, \langle \underbrace{\varphi^{A_1} \cdots \varphi^{A_n}}_{n \text{ copies}} \cdots | \, S \, | \cdots \rangle + \cdots$$

- The equivalence theorem holds even if the leading terms cancel against each other as in $qq \to WW$:



Even more on the Equivalence Theorem

$$\langle W_L^{A_1} \cdots W_L^{A_n} \cdots | \, S \, | \cdots \rangle = i^n E^n \langle \varphi^{A_1} \cdots \varphi^{A_n} \cdots | \, S \, | \cdots \rangle + \cdots$$

$$E = \frac{1}{M_{\text{phys}}} \frac{k^2 + \xi \Gamma_L^{WW}}{\xi (M - \Gamma^{W\phi})} \sqrt{\frac{R_W}{R_\varphi}}$$

- $\langle W_L \cdots | \, S \, | \cdots \rangle$ is gauge invariant

  $\langle \varphi \cdots | \, S \, | \cdots \rangle$ is not gauge invariant
- E makes the RHS gauge invariant.
- E does not run.

$$\mu \frac{\partial E}{\partial \mu} = 0$$

E in $\overline{MS}$ scheme

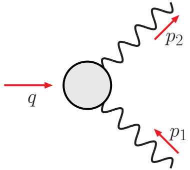
- Feynman Gauge, for $M_h = 2M$, and $\overline{MS}$ scheme:

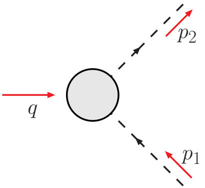
$$E = 1 + \frac{g^2}{16\pi^2}\left(-\frac{179}{24} + \frac{47\pi}{12\sqrt{3}} + \frac{9}{8}\log 2\right)$$

- E provides additional contribution to the low scale matching.
- In the literature, a scheme is always chosen so that $E = 1$.

longitudinal Gauge Boson Production

- $W_L W_L$ production (Breit Frame):



$= i^2 E^2 \times \Bigg\{$  $\Bigg\}$

$$\mathcal{O} = H^\dagger H, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi^2 + i\varphi^1 \\ h - i\varphi^3 \end{pmatrix}$$

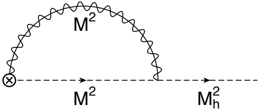
- High scale matching and γ as in unbroken case. (Chiu et al. 07)

$$C(\mu) = \frac{\alpha}{4\pi} C_F \left(-\mathrm{L}_Q^2 + \mathrm{L}_Q - 2 + \frac{\pi^2}{6}\right)$$

$$\gamma_{\mathrm{SCET}} = \frac{\alpha}{4\pi} C_F (4\mathrm{L}_Q - 8)$$

Collinear Diagrams at $\mu \sim M$

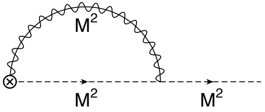
$$W^\dagger H_{\text{SCET}} = H_{\text{SCET}} + \frac{1}{\sqrt{2}} \begin{pmatrix} I_n^{(\varphi)}(\varphi^2 + i\varphi^1) \\ I_n^{(h)} h - i I_n^{(\varphi)} \varphi^3 \end{pmatrix}$$



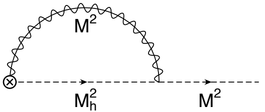
h

$$f_S\left(\frac{m_{\text{ext}}^2}{M^2}, \frac{m_{\text{int}}^2}{M^2}\right)$$

$$I_n^{(h)} = \frac{\alpha}{4\pi} C_F \left[2\mathrm{L}_M \log \frac{p^- \delta}{\mu^2} - \mathrm{L}_M^2 - \mathrm{L}_M + 1 - \frac{\pi^2}{2} + f_S\left(\frac{M_h^2}{M^2}, 1\right) \right]$$



φ^A



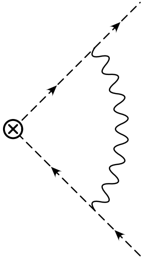
φ^A

$$I_n^{(\varphi)} = \frac{\alpha}{4\pi} C_F \left[2\mathrm{L}_M \log \frac{p^- \delta}{\mu^2} - \mathrm{L}_M^2 - \mathrm{L}_M + 1 - \frac{\pi^2}{2} + \frac{2}{3} f_S(1,1) + \frac{1}{3} f_S\left(1, \frac{M_h^2}{M^2}\right) \right]$$

Low scale matching

- The soft graph is universal:

$$I_s = \frac{\alpha}{4\pi}\left[-2\mathrm{L}_M \log\frac{\delta_1\delta_2}{\mu^2} + \mathrm{L}_M^2 - \frac{\pi^2}{6}\right]$$



- At $\mu \sim M$, the doublet breaks into custodial SU(2) representations.

$$\begin{aligned} D^{(\varphi\varphi)}(\mu) &= 1 + I_n^{(\varphi)} + I_{\bar{n}}^{(\varphi)} + I_s + \delta R_\varphi \\ D^{(hh)}(\mu) &= 1 + I_n^{(h)} + I_{\bar{n}}^{(h)} + I_s + \delta R_h \end{aligned}$$

$$[H^\dagger_{\mathrm{SCET}} W][W^\dagger H_{\mathrm{SCET}}] \quad \rightarrow \quad \frac{1}{2} D^{(\varphi\varphi)}(\mu)\varphi^A\varphi^A + \frac{1}{2} D^{(hh)}(\mu) hh$$

Complete the table

Add to the Table I of Chiu et al (PRD 77, 053004 (2008)).

...	$\gamma_{\rm SCET}(\mu)$	$D(\mu)$
$\bar{\psi}\psi$	$C_F(4\mathsf{L_Q}-8)+\gamma_\psi$	$C_F(2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-4\mathsf{L_M}+4-\frac{5\pi^2}{6})+\delta R_\psi$
$\chi^\dagger\chi$	$C_F(4\mathsf{L_Q}-4)+\gamma_\chi$	$C_F(2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-2\mathsf{L_M}+2-\frac{5\pi^2}{6})+\delta R_\chi$
$B_\perp B_\perp$	$C_A(4\mathsf{L_Q}-4)+\gamma_W$	$C_A(2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-2\mathsf{L_M}+2-\frac{5\pi^2}{6}+2f_s(1,1))+\delta R_W$
$\varphi^A\varphi^A$	$C_F(4\mathsf{L_Q}-4)+\gamma_\varphi$	$C_F(2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-2\mathsf{L_M}+2-\frac{5\pi^2}{6}+\frac{4}{3}f_s(1,1)+\frac{2}{3}f_s(1,\frac{M_h^2}{M^2}))+\delta R_\phi$
hh	$C_F(4\mathsf{L_Q}-4)+\gamma_h$	$C_F(2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-2\mathsf{L_M}+2-\frac{5\pi^2}{6}+2f_s(\frac{M_h^2}{M^2},1))+\delta R_h$

- EFT results are independent of the γ-matrix struture
- Similar result for scalars.
- We can compute any process by considering the Sudakov form factor and weighting with the appropriate group theory. (see Chiu et al. PRD 78)

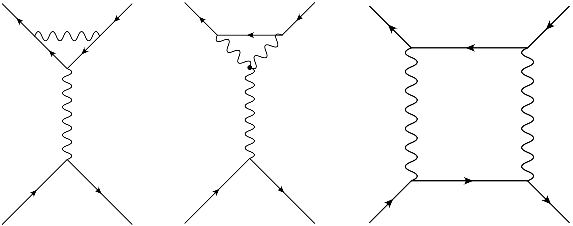
Multiple Particles

high scale matching

- Full theory processes match onto EFT operators:

$$\langle \cdots | \, S \, | \cdots \rangle = \sum_i C_i(Q) \exp\left[\int_Q^M \frac{d\mu}{\mu}\gamma_{\mathrm{SCET}}\right] D_i(M)\langle \mathcal{O}_i \rangle$$

- $C_i(Q)$ is numerically small. 2% QCD and 0.2% for EW



Multiple Particles

running

- Full theory processes match onto EFT operators:

$$\langle \cdots | S | \cdots \rangle = \sum_i C_i(Q) \exp\left[\int_Q^M \frac{d\mu}{\mu}\gamma_{\text{SCET}}\right] D_i(M)\langle \mathcal{O}_i \rangle$$

- The running provides the largest contribution.

  67-97% for QCD corrections at 1-5 TeV

  5% for EW corrections at 1 TeV

  30% for EW corrections at 5 TeV

Multiple Particles

low scale

- Full theory processes match onto EFT operators:

$$\langle \cdots | \, S \, | \cdots \rangle = \sum_i C_i(Q) \exp\left[\int_Q^M \frac{d\mu}{\mu} \gamma_{\text{SCET}} \right] D_i(M) \langle \mathcal{O}_i \rangle$$

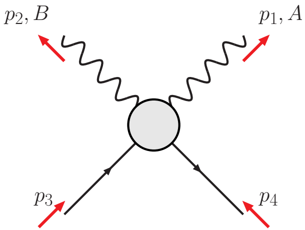
- There is no low scale matching for QCD

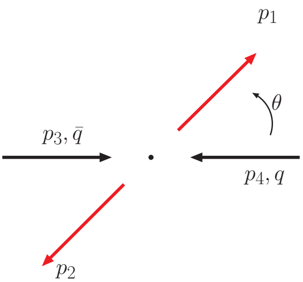
  EW corrections are 2%

  Not very dependent on Q

- γ_{SCET} and $D(\mu)$ are matrix valued and the ordering is crucial.

$q\bar{q} \to W_T W_T$





- SCET operators: Marcantonini, Stewart

$$\mathcal{O} = [\bar{\xi}_4 W_4] \Gamma G^{AB} [W_3^\dagger \xi_3] B_{1\perp}^{A\ \mu} B_{2\perp}^{B\ \nu}$$

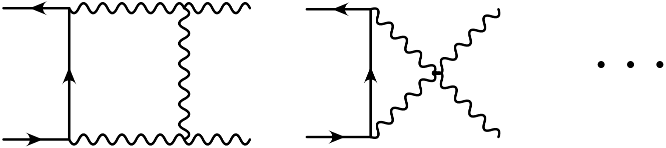
$$\begin{aligned} \mathcal{O}_1 &= \gamma^\lambda \delta^{AB} \mathbf{1} \\ \mathcal{O}_2 &= \gamma^\lambda i\epsilon^{ABC} T^C \\ \mathcal{O}_3 &= \gamma^\lambda \gamma^\sigma \gamma^\rho \delta^{AB} \mathbf{1} \\ \mathcal{O}_4 &= \gamma^\lambda \gamma^\sigma \gamma^\rho i\epsilon^{ABC} T^C \end{aligned}$$

$q\bar{q} \to W_T W_T$: High scale matching

- Tree matching (common factor of ig^2):

$$C_1 = -\frac{1}{2t} p_{3\mu} g_{\lambda\nu} + \frac{1}{2u} p_{3\nu} g_{\lambda\mu}$$
$$C_2 = \frac{1}{s}(p_{2\lambda} - p_{1\lambda}) g_{\mu\nu} - \frac{1}{t} p_{3\mu} g_{\lambda\nu} + \frac{1}{u} p_{3\nu} g_{\lambda\mu}$$
$$C_3 = \frac{1}{t} p_{1\lambda} g_{\sigma\mu} g_{\rho\nu}$$
$$C_4 = \frac{1}{u} p_{2\lambda} g_{\sigma\nu} g_{\rho\mu}$$

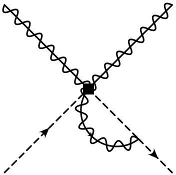
- QCD graphs are most important for one loop matching. (in progress)

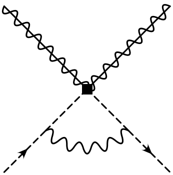


- EW matching can be done, but numerically small.

$q\bar{q} \to W_T W_T$: EFT anomalous dimension

- There are 4 collinear diagrams and 6 soft diagrams.





- The matrix anomalous dimension (overall $\frac{\alpha}{4\pi}$ removed):

$$\gamma_{\text{SCET}} = \gamma_1 \mathbf{1} + \gamma_s$$

$$\gamma_1 = C_A(4\mathrm{L}_Q - 4) + 2\gamma_W + C_F(4\mathrm{L}_Q - 8) + 2\gamma_\psi$$

$$\gamma_s = \begin{pmatrix} 0 & -4\mathrm{L}_{t/u} \\ -8\mathrm{L}_{t/u} & 2C_A\mathrm{L}_{ut/s^2} \end{pmatrix}$$

$q\bar{q} \to W_T W_T$: low scale matching

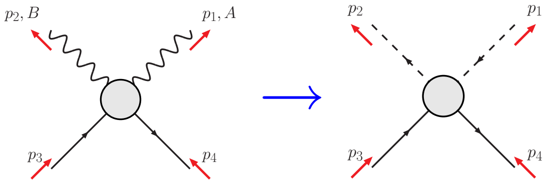
- The matrix low scale matching (overall $\frac{\alpha}{4\pi}$ removed):

$$D(\mu) \quad = \left(D^{(\psi\psi)} + D^{(WW)}\right)\mathbf{1} + \begin{pmatrix} 0 & -2\mathsf{L}_{t/u} \\ -4\mathsf{L}_{t/u} & C_A \mathsf{L}_{ut/s^2} \end{pmatrix} \mathsf{L}_M$$

- Similar result worked out for QCD, Kidonakis.
- Kuhn, Metzler, Penin performed similar calculation for $e^+e^- \to WW$.
  - Method of regions.
  - Same toy theory with $M_h = M$
  - QCD inspired evolution equation

$q\bar{q} \to W_L W_L$

- Use Equivalence Theorem and consider instead $q\bar{q} \to \bar{H}H$



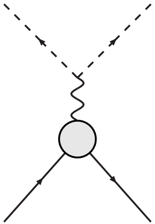
- At $\mu \sim Q$, match onto:

$$\mathcal{O}_1 = [\bar{\xi}_4 W_4]\gamma^\mu T^A [W_3^\dagger \xi_3][H_2^\dagger W_2](i\mathcal{D}^\dagger_{2\mu} - i\mathcal{D}_{1\mu}) T^A [W_1^\dagger H_1] + \text{h.c.}$$

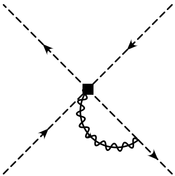
$$i\mathcal{D}_i = \mathcal{P}_i - g(\bar{n}_i \cdot W_{n_i,q})\frac{n}{2}$$

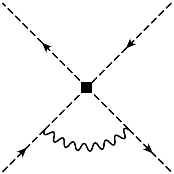
- Only QCD is important at one loop.

$$C(\mu) = \frac{ig^2}{s} F_{\text{QCD}}(q^2)$$



$q\bar{q} \to W_L W_L$: running





- Mixing results in two color structures:

$$\mathcal{O}_1 = T^A \otimes T^A, \qquad \mathcal{O}_2 = \mathbf{1} \otimes \mathbf{1}$$

- The matrix anomalous dimension (overall $\frac{\alpha}{4\pi}$ removed):

$$\gamma_{\text{SCET}} = \gamma_1 \mathbf{1} + \gamma_s$$

$$\gamma_1 = C_F(4\text{L}_Q - 4) + 2\gamma_\chi + C_F(4\text{L}_Q - 8) + 2\gamma_\psi$$

$$\gamma_s = \begin{pmatrix} 2C_A \text{L}_{ut/s^2} & 8\text{L}_{t/u} \\ \frac{3}{2}\text{L}_{t/u} & 0 \end{pmatrix}$$

$q\bar{q} \to W_L W_L$: running

$$W_L W_L \qquad \gamma_1 = C_F(4\mathrm{L}_Q - 4) + 2\gamma_\chi + \cdots$$
$$W_T W_T \qquad \gamma_1 = C_A(4\mathrm{L}_Q - 4) + 2\gamma_W + \cdots$$

Important numerical implications.

$q\bar{q} \to W_L W_L$: low scale matching

- At $\mu \sim M$, the operator breaks into SU(2) custodial invariants.

$$T^A \otimes T^A \to \quad \hat{\mathcal{O}}_1 = T^C \otimes \frac{i}{2}\epsilon^{ABC}\varphi^A\varphi^B \qquad \hat{\mathcal{O}}_2 = T^C \otimes \frac{i}{2}\varphi^C h$$

$$\mathbf{1} \otimes \mathbf{1} \to \quad \hat{\mathcal{O}}_3 = \mathbf{1} \otimes \frac{1}{2}\varphi^A\varphi^A \qquad \hat{\mathcal{O}}_4 = \mathbf{1} \otimes \frac{1}{2}hh$$

- The matching for these operators is:

$$D(\mu) = \quad D^{(\psi\psi)}\mathbf{1}_{4\times 4} + \mathrm{Diag}(D^{(\varphi\varphi)}, D^{(\varphi h)}, D^{(\varphi\varphi)}, D^{(hh)})$$

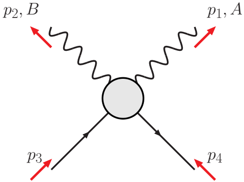
$$+ \begin{pmatrix} C_A \mathsf{L}_{ut/s^2} & 4\mathsf{L}_{t/u} \\ \frac{3}{4}\mathsf{L}_{t/u} & 0 \end{pmatrix} \mathsf{L}_M \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

longitudinal Gauge Boson Production

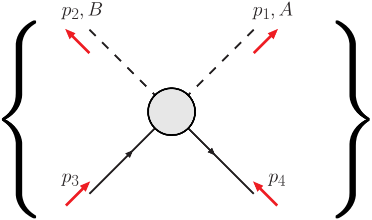
- Take the matrix element of

$$\hat{\mathcal{O}}_1 = T^C \otimes \frac{i}{2}\epsilon^{ABC}\varphi^A\varphi^B \qquad \hat{\mathcal{O}}_3 = \mathbf{1} \otimes \frac{1}{2}\varphi^A\varphi^A$$

- Equivalence theorem

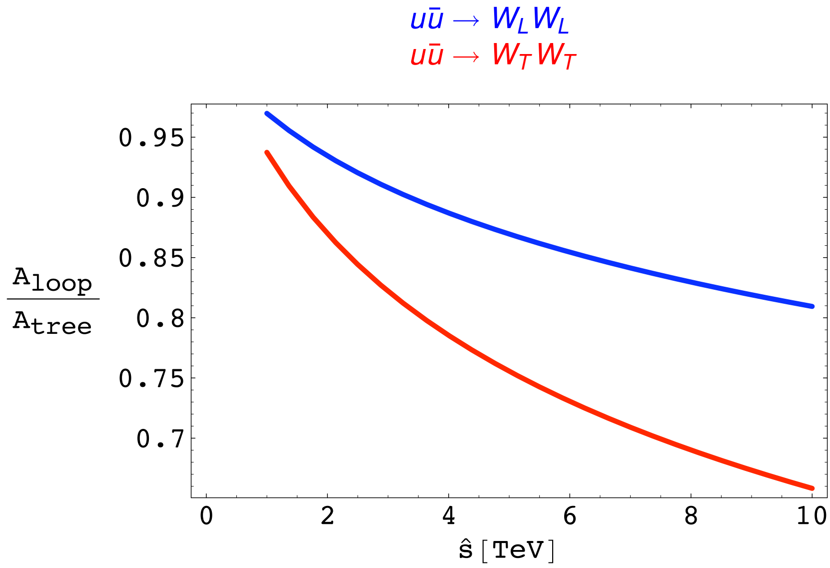


$$= i^2 E^2 \times$$



Numerics

EW only



Summary

- We have demonstrated the use of EFTs to sum the EW logarithms in gauge boson production.
- We can now use EFT's to compute EW radiative corrections for any energetic inital and final state.
- Extension to the standard model is straightforward.