NNLO corrections to $\bar{B} \to X_u l \bar{\nu}$ in the shape-function region

Ben Pecjak

Johannes Gutenberg-Universität Mainz

SCET Workshop 2009

March 24, 2009

Based on work with Christoph Greub and Matthias Neubert


[redacted] MESON
Don't Talk
B

Outline

- motivation
- partial decay rates in $\bar{B} \to X_u l \bar{\nu}$ with SCET
- numerical results at NNLO in α_s
- conclusions

$|V_{ub}|$ and $b \to ul\bar{\nu}$ decays (PDG '08)

Method	Complication	$\lvert V_{ub}\rvert \times 10^3$
Exclusive ($\bar{B} \to \pi l\bar{\nu}$)	form factors	3.5 ± 0.6
Inclusive ($\bar{B} \to X_u l\bar{\nu}$)	exp. cuts $\leftrightarrow$ shape functions	4.1 ± 0.4

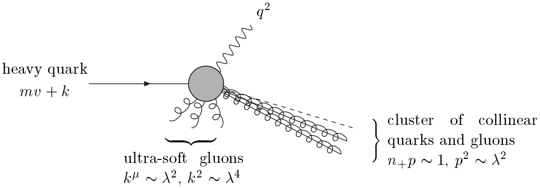
agree within errors, but inclusive tends to be higher

Kinematic cuts and the shape-function region

$$|V_{cb}|^2 \approx 100|V_{ub}|^2$$

- experiments need cuts $M_X < M_D$ to eliminate charm background
- measurements typically in shape-function region

$$\Lambda_{\rm QCD} \ll (M_X \sim \sqrt{m_b \Lambda_{\rm QCD}}) \ll (m_b, 2E_X)$$

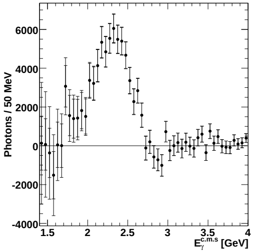


The shape function

Neubert; Bigi, Shifman, Uraltsev, Vainshtein ’93

$$S(\omega,\mu) = \int \frac{dt}{2\pi}\, e^{i\omega t} \langle \bar{B}|(\bar{h}_v Y_s)(tn)(Y_s^\dagger h_v)(0)|\bar{B}\rangle$$

- think of S as a parton distribution function (PDF) for B meson
- experimental information from E_γ spectrum in $\bar{B} \to X_s\gamma$



Factorization in the shape-function region

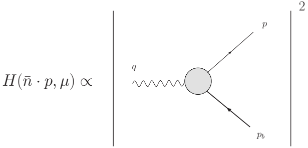
$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum h \cdot j \otimes S^{\Lambda} + \frac{1}{m_b} \sum h \cdot J^{\Lambda} \otimes S + \dots$$

- hard-jet-soft factorization
  Korchemsky, Sterman '94; Akhoury, Rothstein '95; SCET papers
- $(H \cdot J)$ at NLO in α_s (one loop)
  Bauer, Manohar '03; Bosch, Lange, Neubert, Paz '04
- subleading shape-functions (S^{Λ}) at tree level
  Lee, Stewart; Bosch, Neubert, Paz; Beneke, Campanario, Mannel, BP '05
- subleading jet functions (J^{Λ}) at one loop
  Paz '09

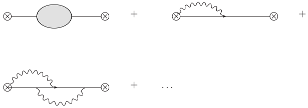
Today: $H \cdot J \otimes S$ at NNLO in α_s

H and J at NNLO

- H from matching $b \to u$ current
  Bonciani, Ferroglia; Asatrian, Greub, BP; Beneke, Huber, Li; Bell ’08



- J from cut quark propagator in light-cone gauge
  Becher, Neubert ’06



Large logs and resummation

$$d\Gamma \sim H(m_b, \mu_f) J(M_X, \mu_f) \otimes \hat{S}(\mu_f)$$

- in limit $m_b \gg M_X$ there are "large" logs $\ln m_b / M_X$
- have model for $\hat{S}$ at low μ_0, but needed at arbitrary μ_f

Standard solution in SCET: derive and solve RG-equations

$$\begin{aligned} H(m_b, \mu_f) &= U_H(\mu_f, \mu_h) H(m_b, \mu_h \sim m_b) \\ J(M_X, \mu_f) &= U_J(\mu_f, \mu_i) \otimes J(M_X, \mu_i \sim M_X) \\ \hat{S}(\mu_f) &= U_S(\mu_f, \mu_0) \otimes \hat{S}(\mu_0) \end{aligned}$$

- large logs are "resummed" into the evolution factors $U_{H,J}$
- solution for J relies on Laplace transform technique
  Becher, Neubert '06

Master formula for partial decay rates

$$\Gamma_u\Big|_{\rm cut} \sim |V_{ub}|^2 \int_{\rm cut} dP_+\, dy\, U(\mu_h, \mu_i, \mu_0) H(y, m_b, \mu_h) y^{-2a_\Gamma(\mu_h,\mu_i)}$$

$$\widetilde{j}\left(\ln \frac{m_b y}{\mu_i} + \partial_\eta, \mu_i\right) \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)} \int_0^{P_+} d\hat{\omega} \left[\frac{1}{P_+ - \hat{\omega}} \left(\frac{P_+ - \hat{\omega}}{\mu_i}\right)^\eta\right]_* \hat{S}(\hat{\omega}, \mu_0)$$

- partial rates formally independent of μ_h, μ_i
- $\mu_h = \mu_i = \mu$ is fixed-order perturbation theory $[H \cdot J](\mu) = C(\mu)$
- need model for $\hat{S}(\hat{\omega}, \mu_0)$ at some low scale μ_0

Numerical evaluation of partial rates

Arbitrary partial rates at NNLO in α_s can be obtained:
Greub, Neubert, BP, in preparation

- cut on maximum $P_+ = E_X - |\vec{P}_X|$
- cut on maximum hadronic invariant mass
- cut on minimum lepton energy
- combinations of these

Will study

- $P_+ < 0.66$ GeV
- $E_l > 2.0$ GeV

Input for partial rates

HQET parameters

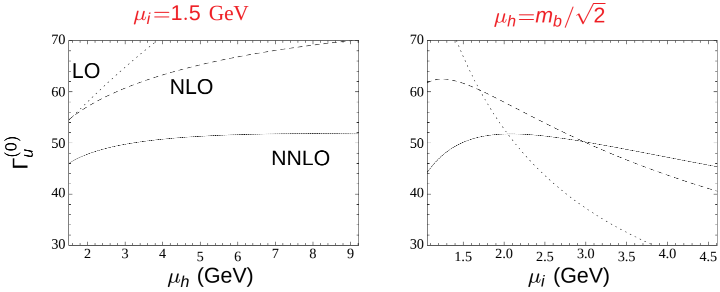
- $m_b \equiv m_b^{\rm SF} = 4.71$ GeV, $\mu_\pi^2 \equiv \mu_\pi^{2,\rm SF} = 0.2\,\mathrm{GeV}^2$

Shape-function model

$$\hat{S}(\hat{\omega}, \mu_0) = \mathcal{N}(b, \Lambda)\hat{\omega}^{b-1}\exp\left(-\frac{b\hat{\omega}}{\Lambda}\right)$$

- $(b, \Lambda)(\mu_0)$ can be tuned to $B \to X_s\gamma$ data
- also constrained by shape function moment relations
  - first moment $\leftrightarrow m_b$
  - second moment $\leftrightarrow \mu_\pi^2$
- moment constraints implemented at NNLO in α_s at $\mu_0 = 1.5$ GeV

Partial rate for $P_+ < 0.66$ GeV

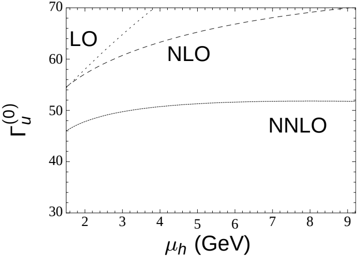


- reduced dependence on μ_h, μ_i at NNLO
- large negative shift between NLO and NNLO
- largest uncertainty associated with μ_i (usually fixed at $\mu_i = 1.5$ GeV)

Comparison with fixed-order perturbation theory

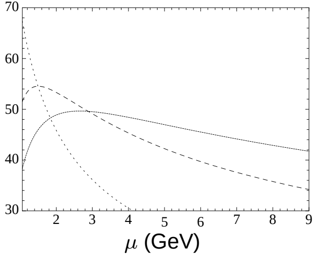
resummed

$\mu_i = 1.5$ GeV



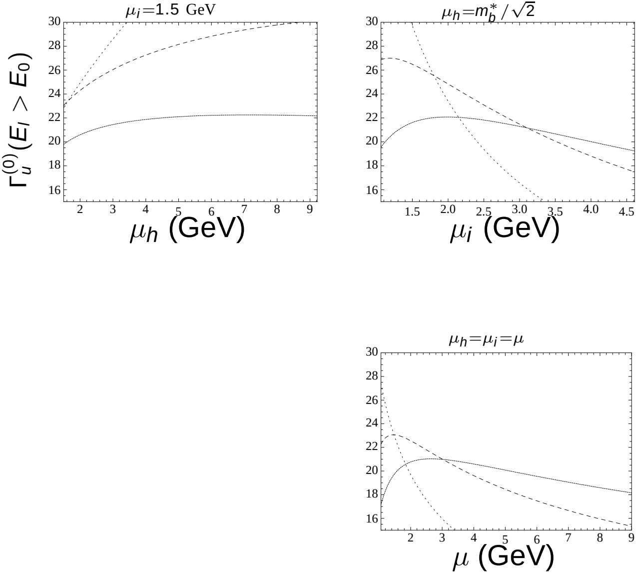
fixed order

$\mu_h = \mu_i = \mu$



- fixed-order well behaved
- not clear that resummation is necessary

Partial rate with cut $E_l > 2.0$ GeV



BLNP and $|V_{ub}|$

Most complete numerical implementation in SCET is “BLNP”

Bosch, Lange, Neubert, Paz ’04; Lange, Neubert, Paz ’05

$$\Gamma_u\Big|_{\rm cut} = |V_{ub}|^2 \left[\Gamma_u^{(0)} + \frac{1}{m_b}\Gamma_u^{(1)} + \frac{1}{m_b^2}\Gamma_u^{(2)}\right]_{\rm BLNP}$$

Have included NNLO corrections to $\Gamma_u^{(0)}$ in BLNP “generator”

Net effect is that $|V_{ub}|$ goes up by $\sim$ 10% compared to NLO

Summary

NNLO calculation for partial rates in $\bar{B} \to X_u l \bar{\nu}$ now complete (to leading order in $1/m_b$ in the shape-function region)

Numerical analysis shows that the NNLO corrections

- reduce perturbative uncertainty compared to NLO
- raise $|V_{ub}|$ by roughly 10% compared to NLO compared to current BLNP results