

18.06 Session 1

Problem 1.1: (1.3 #4. *Introduction to Linear Algebra*: Strang) Find a combination $x_1\mathbf{w}_1 + x_2\mathbf{w}_2 + x_3\mathbf{w}_3$ that gives the zero vector:

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{w}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \mathbf{w}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$

Those vectors are (independent)(dependent).

The three vectors lie in a _____. The matrix W with those columns is *not invertible*.

Solution: We might observe that $\mathbf{w}_1 + \mathbf{w}_3 - 2\mathbf{w}_2 = 0$, or we might simultaneously solve the system of equations:

$$\begin{aligned} 1x_1 + 4x_2 + 7x_3 &= 0 \\ 2x_1 + 5x_2 + 8x_3 &= 0 \\ 3x_1 + 6x_2 + 9x_3 &= 0 \end{aligned}$$

Subtracting twice equation 1 from equation 2 gives us $-3x_2 - 6x_3 = 0$. Subtracting thrice equation 1 from equation 3 gives us $-6x_2 - 12x_3 = 0$, which is equivalent to the previous equation and so leads us to suspect that the vectors are dependent. At this point we might guess $x_2 = -2$ and $x_3 = 1$ which would lead us to the answer we observed above:

$$x_1 = 1, x_2 = -2, x_3 = 1 \text{ and } \mathbf{w}_1 - 2\mathbf{w}_2 + \mathbf{w}_3 = 0.$$

Those vectors are **dependent** because there is a combination of the vectors that gives the zero vector.

The three vectors lie in a **plane**.

Problem 1.2: Multiply: $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}.$

Solution:
$$\begin{bmatrix} 1 \cdot 3 + 2 \cdot (-2) + 0 \cdot 1 \\ 6 + 0 + 3 \\ 12 - 2 + 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 11 \end{bmatrix}.$$

Problem 1.3: True or false: A 3 by 2 matrix A times a 2 by 3 matrix B equals a 3 by 3 matrix AB . If this is false, write a similar sentence which is correct.

Solution: The statement is true. In order to multiply two matrices, the number of columns of A must equal the number of rows of B . The product AB will have the same number of rows as the first matrix and the same number of columns as the second:

$$A(m \text{ by } n) \text{ times } B(n \text{ by } p) \text{ equals } AB(m \text{ by } p).$$