

18.06 Session 7

Problem 7.1:

(a) Find the row reduced form of:

$$A = \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 2 & -2 & 11 & -3 \end{bmatrix}$$

(b) What is the rank of this matrix?

(c) Find any special solutions to the equation $Ax = 0$.

Solution:

(a) To transform A into its reduced row form, we perform a series of row operations. First, we multiply the first row by 2 and subtract it from the third row:

$$\begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 2 & -2 & 11 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & -12 & -3 & -21 \end{bmatrix}$$

We then multiply the second row by $\frac{1}{4}$ to make the second pivot 1:

$$\begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & -12 & -3 & -21 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & -12 & -3 & -21 \end{bmatrix}$$

Multiply the second row by 12 and add it to the third row:

$$\begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & -12 & -3 & -21 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Finally, multiply the second row by 5 and subtract it from the first row:

$$\begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -23/4 & 1/4 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) The matrix is of **rank 2** because it has 2 pivots.

(c) The special solutions to $A\mathbf{x} = \mathbf{0}$ are:

$$\begin{bmatrix} 23/4 \\ -1/4 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -1/4 \\ -7/4 \\ 0 \\ 1 \end{bmatrix}$$

Problem 7.2: (3.3 #17.b *Introduction to Linear Algebra: Strang*) Find A_1 and A_2 so that $\text{rank}(A_1 B) = 1$ and $\text{rank}(A_2 B) = 0$ for $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Solution: Take $A_1 = I_2$ and $A_2 = 0_2$

(or for a less trivial example $A_2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$)