18.701 Lecture Notes

2007-09-14

Defn. A <u>partition</u> of a set S is a decomposition of S into nonempty disjoint subsets

$$S = U_1 \bigcup U_2 \bigcup \dots \bigcup U_n$$

$$U_1 \bigcap U_2 = \emptyset \text{ for } i \neq j$$

$$\bar{S} = \{U_1, U_2, \dots\}$$

So we have a surjective map $\pi : S \to \bar{S}$

Defn. Given $f : S \to T$, the <u>fiber</u> of some $t \in T$ is the inverse image of t: $f^{-1}(t) = \{s | f(s) = t\}$

The nonempty fibers of any function partition S.

Defn. An <u>equivalence relation</u> on S, $a \sim b$, is

- transitive: $a \sim b \wedge b \sim c \Rightarrow a \sim c$
- symmetric: $a \sim b \Rightarrow b \sim a$
- reflexive $a \sim a \forall a \in S$

Prop. The equivalence relationships on S correspond bijectively to the partitions on S

Given a homomorphism $\phi : G \to G'$, a, b in the same fiber if $\phi(a) = \phi(b) \iff \phi(a^{-1}b) = 1 \iff a^{-1}b \in \ker\phi$

Say $\ker\phi = N$. $a^{-1}b \in N \Rightarrow b = an, n \in N$

Defn. A <u>left coset</u> of N in G:

$$aN = \{x \in G | x = an, n \in N\}$$

Let $H \subset G$ Define the left cosets aH. These partition G The corresponding equivalence relationship: $a \cong b \iff a^{-1}b \in H \to b \in aH$

All the cosets have the same order.

Thm. *$|G| = |H|$(# cosets) $= |H|[G : H]$*

Cor. *$|H|$ divides $|G|$*

Cor. *$|[G : H]|$ divides $|G|$*

Cor. *Suppose $|G| = p$ prime. Every subgroup is either $\{1\}$ or $G \Rightarrow$ and G is cyclic*