A Message from our Sponsor, Five Rings Capital
Our Contestants (in alphabetical order)

- Adithya Balachandran
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- Adithya Balachandran
- Daniel Chen

Sponsored by Five Rings Capital
HMMT March 2021 Integration Bee Finals
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- Gopal Goel
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  - If 0 or 4 people get the integral - no points for anyone
  - If 1 person gets the integral, then +3 for them, -1 for others
  - If 2 people get the integral, then +2 for them, and -2 for others
  - If 3 people get the integral, then +1 for them, and -3 for the one person that did not.
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- If there is a tie at the end, we will have a tie-breaking integral estimation question.
Specifications

- We will use the same conventions from the qualifiers:
  - All logarithms are base $e$.
  - The constant of integration may be omitted.
  - You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.
  - If you feel that one of our answers is wrong, please let us know ASAP and we will double check it.
  - For the spectators - feel free to follow along and try these integrals as well!
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Any questions before we start?
Problem 1

Evaluate the following Integral (in terms of $\alpha, \beta \neq 0 \in \mathbb{R}$):

$$\int_{0}^{1} \frac{1}{(\alpha x + \beta (1 - x))^2} \, dx$$
Solution 1

\[ \frac{1}{\alpha \beta} \]
Evaluate the following integral:

\[ \int e^{2x} \tan^{-1}(e^x) \, dx \]
Solution 2

\[ \frac{1}{2} \left( (e^{2x} + 1) \tan^{-1}(e^x) - e^x \right) + C \]
Problem 3

Evaluate the following integral:

$$\int \frac{x^2 + 2020 \cdot 2021}{(x \sin x + 2021 \cos x)^2} \, dx$$
Solution 3

\[
\frac{2021 \sin x - x \cos x}{x \sin x + 2021 \cos x} + C
\]
Evaluate the following Limit:

\[
\lim_{n \to \infty} \frac{\int_0^\infty (x - 1)^n e^{-x} \, dx}{\int_0^\infty x^n e^{-x} \, dx}
\]
Solution 4

\[-\frac{1}{e}\]
Problem 5

Evaluate the following Integral:

\[ \int_{0}^{6} \sqrt[3]{x + \sqrt[3]{x + \sqrt[3]{x + \cdots}} } \, dx \]
Solution 5

\[
\frac{39}{4}
\]
Evaluate the following integral:

\[ \int \frac{1}{x^2 - x\sqrt{x^2 - 1}} \, dx \]
Solution 6

\[ \sqrt{x^2 - 1} - \tan^{-1}(\sqrt{x^2 - 1}) + x + C \]
Problem 7

Evaluate the following Integral:

\[ \int_{0}^{\infty} \frac{\arctan(x) - \arctan(\pi x)}{x} \, dx \]
Solution 7

\[- \frac{1}{2} \pi \log \pi\]
Problem 8

Evaluate the following Integral:

\[ \int \sqrt{x} \log(x + 1) \, dx \]
Solution 8

\[
\frac{2}{3} x^{3/2} \log(x + 1) - \frac{4}{9} x^{3/2} + \frac{4}{3} \sqrt{x} - \frac{4}{3} \arctan \sqrt{x} + C
\]
Evaluate the following integral:

$$\int_{0}^{\pi} \frac{x \sin^{2020}(x) \cos^{2020}(x) + \sin^{2020}(x)}{\cos^{2020}(x) + \sin^{2020}(x)} \, dx$$
Solution 9

\[ \frac{\pi^2}{4} \]
Problem 10

Evaluate the following integral (in terms of $0 < \theta < \frac{\pi}{2}$):

$$\int_{-\infty}^{0} \frac{\cos(\theta x)}{\cos^x(\theta)} \, dx$$
Solution 10

\[
\frac{- \log \cos \theta}{\log^2 \cos \theta + \theta^2}
\]
Problem 11

**Estimate** the following integral:

\[ \int_0^1 e^{-x^3} \, dx \]
Solution 11

\[ \approx 0.807511 \]