

Using Spreadsheets to Parameterize Spur Gear Design for Laser Cutters

by

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SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2002

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Abstract:

The purpose of this Thesis is to design a simple system for cutting rudimentary Spur Gears on a Laser Cutter, using only a few basic parameters of the gears. The Laser Cutter could quickly and accurately cut the outline of a gear from wood or plastic; the low durability of the material would not matter, as long as the gear is being used for a mockup or simple test. But, the gears would still need to mesh correctly, and the Laser Cutter would need to be able to cut many complimentary sets of gears for the prototype of a typical design project.

To this end, we created an Excel spreadsheet which, when given the Number of Teeth, Pressure Angle, and Diametral Pitch of the desired gear, produces a suitable business graphic of that gear. That graphic can then be printed to a plot file which the Laser Cutter can use to cut out the gear.

The process involved several complications, both to model the spur gears parametrically as a spreadsheet graphic and to convert the graphic into laser cutter instructions of the appropriate scale and resolution. Once resolved, however, the process provides a convenient and rapid method for prototyping sets of spur gears.

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Acknowledgements:

I am most grateful to my advisor, Woodie Flowers, who initially suggested the topic of automating spur gear construction on a laser cutter. Since then, he has also been extremely supportive of this Thesis, both with his suggestions, and his encouragement.

I would also like to thank Richard Fenner, for his help accessing the Pappalardo Lab's Laser Cutter throughout the time I was working on this project, and Steve Chomyszak, for his insights into spur gear conventions. Finally, I would like to thank my mother and father for their constant support, and occasional suggestions, that helped me finally finish this thesis.

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List of Abbreviations: [cf. Figure 2.4]

Φ:	Pressure Angle of a gear
P:	Diametral Pitch of a gear
N:	Number of Teeth on a gear
s:	Shaft Diameter of a gear
D:	Pitch Diameter of a gear
p:	Circular Pitch of a gear
t:	Circular Tooth Thickness
τ:	Tooth Angle
a:	Addendum of a gear's tooth
d:	Dedendum of a gear's tooth
D_R:	Root Diameter of a gear
D_B:	Base Diameter of a gear
b:	Backlash of a gear
HPGL/1:	Hewlett Packard Graphics Language (Level 1)
S:	Sector Angle of a gear
R:	Radius to a point on a gear.
α:	String Angle, for a point on an involute
L:	String length, for a point on an involute

Glossary: [cf. Figure 2.4]

Line of Action:	A line that passes through the point where the teeth of two gears mesh. The Line of Action is perpendicular to the surfaces of both teeth.
Pitch Point:	The point at which the Line of Action intercepts the line connecting the centers of two meshing gears.
Pitch Circle:	A circle, centered on the center of the gear, that touches the Pitch Point.
Circular Pitch:	The arc length between a point on one tooth, and the corresponding point on the next tooth, measured along the Pitch Circle.
Pressure Angle:	Angle between the Line of Action, and a line tangent to the line connecting the centers of two meshed gears.
Circular Tooth Thickness:	The arc length between points on either side of a tooth, measured along the Pitch Circle.
Tooth Angle:	The angle spanned by the Circular Tooth Thickness.
Diametral Pitch:	Ratio of the number of teeth on a gear to the diameter of the Pitch Circle..
HPGL/1:	A file format used to drive Plotters.
Addendum:	Radial distance between the Pitch Circle and the top edge of a tooth.
Dedendum:	Radial distance between the Pitch Circle and bottom edge of a valley
Root Diameter:	The diameter of the circle that contains the bottom (or "roots") of the valleys
Base Diameter:	The diameter of the circle from which the involute curve of the teeth is generated
Backlash:	The distance between the trailing edge of a meshing tooth on one gear and the leading edge of the meshing tooth on the other gear. Backlash measures the tolerance of a gear.
Sector Angle:	The angle between a point on one tooth, and the corresponding point on the next tooth. The angle of the Circular Pitch.
String Angle:	The angle at which a string used to create an involute would cross the X-axis.
String Length:	The length of a string, used to create an involute, which has unwrapped itself from the pitch circle.

Chapter 1 -- Introduction

Spur Gears are very useful in numerous applications. Not only can they transfer velocity and torque from one shaft to another, but, by using different size gears, they can alter the ratio between velocity and torque as they transfer them; a gear with many teeth driving a gear with fewer teeth will have less torque, but greater velocity and visa versa. Unfortunately, Spur Gears require a very specific shape for their teeth to work smoothly. Even a simple mockup gear would require a complex surface in order to function properly. Without the calculations required to create these surfaces, two gears would not mesh together smoothly, making it difficult to test the gears. But, since a mockup does not need to work for a long period of time, it can be made of lighter, easily cut materials. These lighter materials could be handled by a laser cutter if suitable instructions were developed so that the proper shape of the gear teeth could be computed.

The tooth of a Spur Gear is based on a mathematical shape known as an involute. Since each tooth can be described by a series of mathematical equations, it is possible to define a gear in terms of a few key parameters, such as the Number of Teeth and the Diametral Pitch. These parameters make it easy to tell if two gears can mesh together. Similarly, by specifying the parameters first, it would be simple to design a gear for any given application from scratch.

Because Spur Gears are essentially two-dimensional shapes, they could be cut out quickly using a Laser Cutter. Laser Cutters use a laser beam to slice two-dimensional shapes out of flat material, so the silhouette of a Spur Gear would be easy to make using a Laser Cutter. Unfortunately, most Laser Cutters do not have built in software to cut gears; they are driven by a series of simple Move-To and Draw-To commands, tracing out straight lines or elliptical arcs across the material. Therefore, an involute must first be converted into an approximation using these simple commands in order for the Cutter to understand it. If a Laser Cutter is to be useful in creating mockups, it must be able to cut these gear designs quickly and easily.

This Thesis explores the use of an Excel spreadsheet to store the algebraic formulae that convert basic spur gear parameters into numerous points on the corresponding curves. These points can then be represented as a "business graphic" of the desired gear outline, essentially plotting each point on a Cartesian axis. The graphic can then be converted to a plot file the Laser Cutter can use to cut the gear out of a flat material.

Chapter 2 discusses the involute curve and the terminology and mathematical equations of spur gears. Chapter 3 explains the techniques used by the spreadsheet to model the gears. Chapter 4 presents the results and conclusions.

Chapter 2 -- Background

The ultimate goal of this work is a system which can quickly design and manufacture a simple Spur Gear using new computer-driven tools like the Laser Cutter. While many programs exist to model non-linear surfaces, they do not work easily on the unusual design of a gear's tooth. Since spur gears are such a common element of mechanical design, the ability to create them quickly and easily (even if only for mockups and prototypes) would be very useful.

Section 2.1: Involute Curves

A spur gear is a device designed to transmit rotary motion from one axis to another; by altering the radii of two gears, the first gear may transmit either greater torque or greater speed to the second. The teeth of a gear transfer the torque from one gear to the other, but the shape of the teeth determines the efficiency of the transfer. If the contacting surfaces of the two gears grow too far apart, the gears will slip. If, on the other hand, the contact surfaces are crushed together, energy will be wasted in deforming the gears and shafts. Thus, the contact point between the two gears must remain at a constant distance from the center of each gear. Figure 2-1 shows two gears meshing correctly.



Figure 2-1 Gear teeth meshing.

In order to maintain that constant distance, gear teeth are shaped using an involute curve. An involute curve can be created by unwrapping a string from around a round shape while pulling the string taut. The path drawn by the loose end of the string is the involute. Figure 2-2 shows an involute drawn by this method.

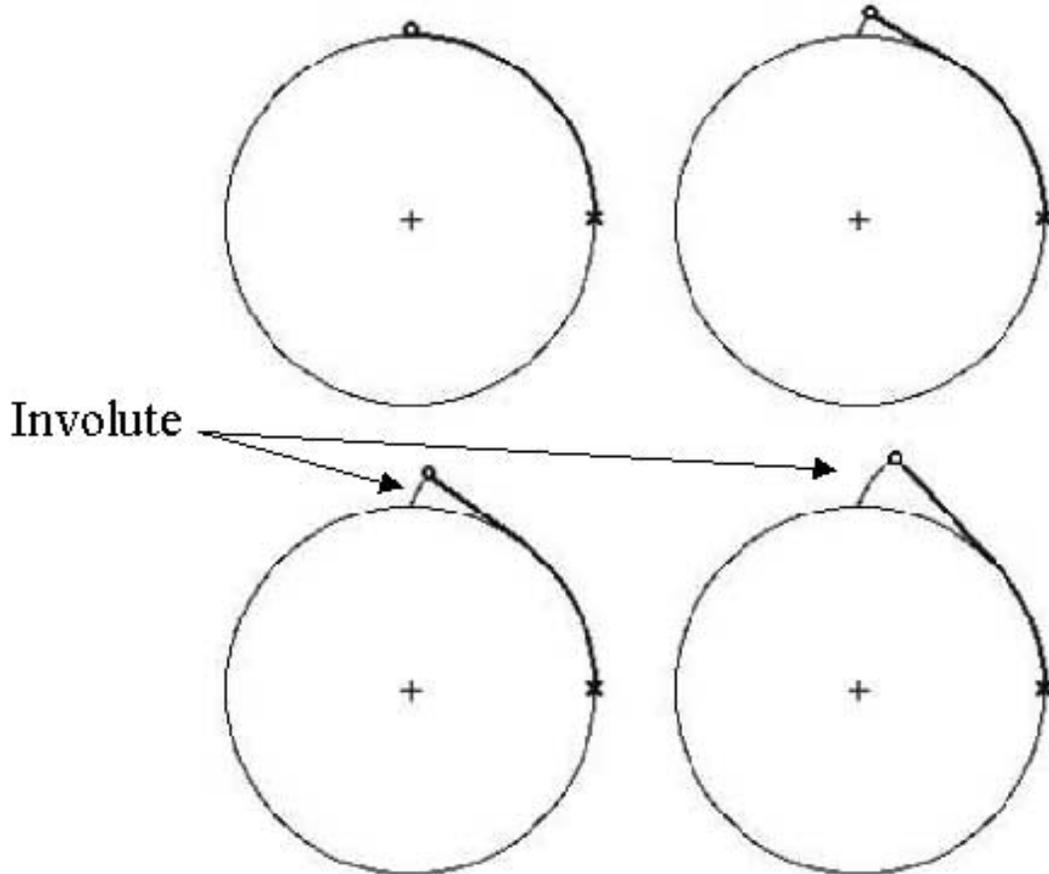


Figure 2-2 Creation of an Involute Curve.

As the string unwinds, the tip moves farther and farther from the contact point on the circle. Thus the radius of the curve is constantly increasing, but, unlike a spiral, the point about which the radius is swung is also moving.

Because the string is always tangent to the curve of the base circle, a line drawn perpendicular to any point on the involute will still be tangent to the base circle. It is this property of involutes that is valuable to Spur Gears; the force transmitted by the tooth of one gear is always perpendicular to the surface of the meshing tooth on the second gear. Additionally, as the involute shape rotates, the contact point will maintain a constant distance from the center of the gear. This radial distance, called the **Pitch Circle**, allows the torque to be transmitted cleanly, without shearing the gears or deflecting the gear shafts. Figure 2-3 shows two meshing gears, and their Pitch Circles.

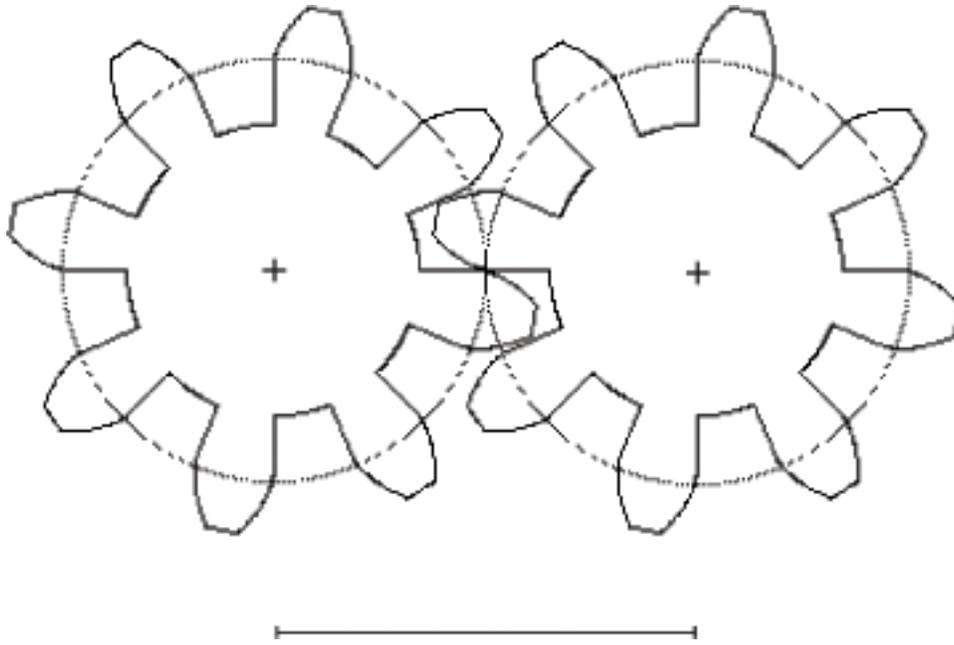
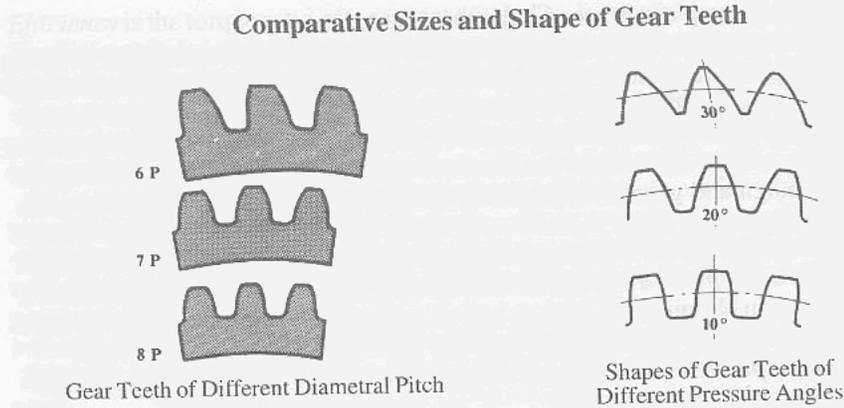


Figure 2-3 The gear teeth meet at their Pitch Circles.

Section 2.2: Spur Gear Terminology

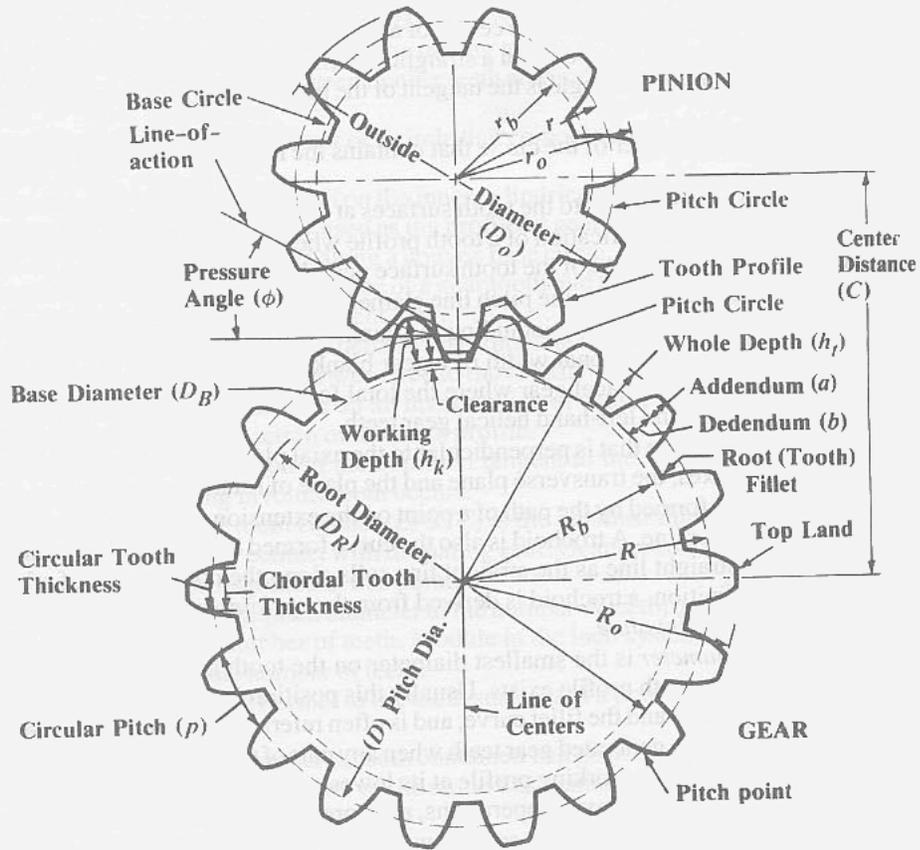
Spur gears have their own set of nomenclature to describe most features of the teeth. A brief overview of the different variables is provided in The Machinery's Handbook, 26th Edition. It has been reproduced here as Figure 2-4, and is followed by more detailed descriptions.



Gear Teeth of Different Diametral Pitch

Shapes of Gear Teeth of Different Pressure Angles

Nomenclature of Gear Teeth



Terms Used in Gear Geometry from Table 1 on page 2004

Figure 2-4 Gear terminology, from The Machinery's Handbook, 26th Edition.

Two gears will contact each other at a single point on their teeth, transmitting the force in a direction perpendicular to the involute at the point of contact. That direction is the **Line of Action**; two correctly meshing gears will maintain a constant Line of Action while they rotate. The Line of Action intersects the line connecting the centers of the two gears at the **Pitch Point**.

Therefore, the **Pitch Circle** of a gear is a circle, centered on the central axis of the gear, which passes through the Pitch Point. While two meshing gears of different sizes will share the same Line of Action and Pitch Point, they would each have a Pitch Circle of differing radii.

The relative size of a gear's teeth can be measured along the Pitch Circle. Thus the **Circular Pitch, p** , represents the circumferential distance between corresponding points on two adjacent sets of teeth, measured on the Pitch Circle. The Circular Pitch measures the arc between each set of tooth and valley. Similarly, the **Circular Tooth Thickness, t** , is the distance between two points, one on each side of the tooth, again measured along the pitch circle circumferentially. It represents the size of a tooth itself.

The **Pressure Angle, Φ** , is defined as the angle between the Line of Action and a line tangent to the Pitch Circle at the Pitch Point. Together, the Pressure Angle and the Circular Pitch can define the key geometry of a gear: a gear will mesh correctly with another gear if they share the same Pressure Angle and Circular Pitch. Once the Pressure Angle and Circular Pitch have been set, the actual diameter of the gear is simply a function of the number of teeth the gear will have [the exact formula may be found in Chapter 3]. The **Diametral Pitch, P** , is the ratio of the number of gear teeth to the diameter of the pitch circle.

Section 2.3: Manufacturing Spur Gears on a Laser Cutter

While a gear of given parameters can mesh with another such gear, the surfaces must be fairly precise to ensure a seamless contact. Moreover, gears usually require careful design (and strong materials) to control the stresses created by the constant contact with another gear. However, if the gear were only to be used for a brief time, or in a low stress situation (a 2.007 prototype, or 2.009 mockups, for example), then they could be made out of weaker materials like wood or plastic. For these applications, the strength of the gear would be less important than the accuracy of its surface, and the speed with which it could be made. Laser Cutters are one of the machines that could create these "test gears" quickly and accurately.

Laser Cutters have a specific set of properties which affect what they can cut. The cutter uses a laser beam to heat materials in a very small area. This effectively burns through materials like wood and plastic, without the need for a cutting tool. Therefore, the Laser Cutter can make very fine edges and corners, or create a cut thinner than any machine tool could. Because shiny surfaces could reflect the beam, and because high thermal conductivity will slow the rate at which the laser burns through the material, metals (especially copper) cannot be cut by most laser cutters. Figure 2–5 shows a photo of the laser cutter used for this project, while Figure 2–6 shows a schematic of how the laser cutter operates.

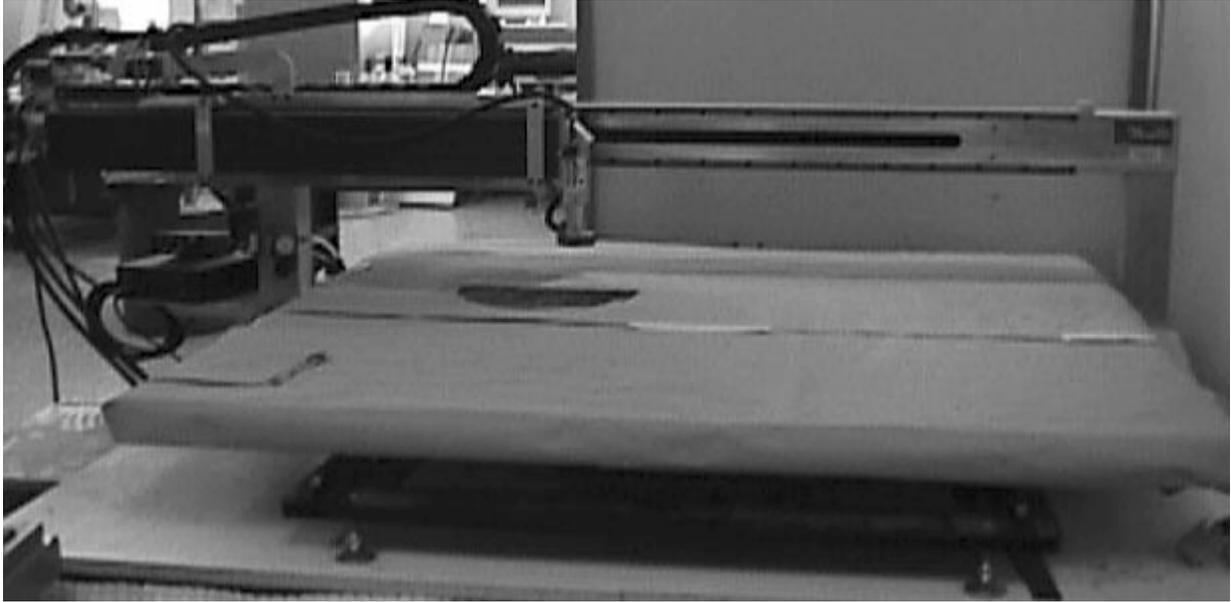


Figure 2-5 The Pappalardo Lab's Laser Cutter

As shown in Figure 2-6, the object to be cut is placed on a flat surface, which can translate in one direction while the laser can translate at a right angle above it. By maneuvering both the laser and the table, the beam can cut out a wide variety of shapes.

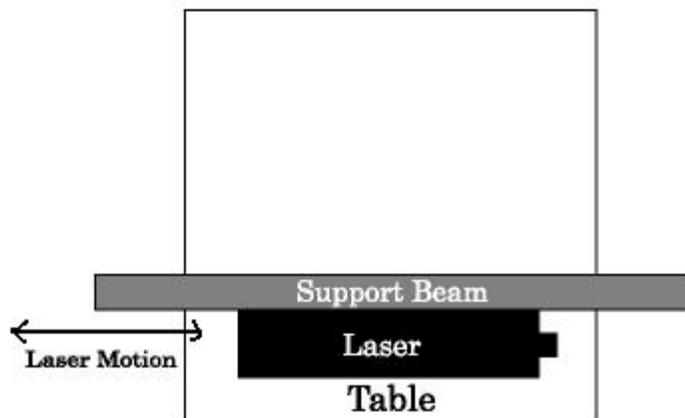
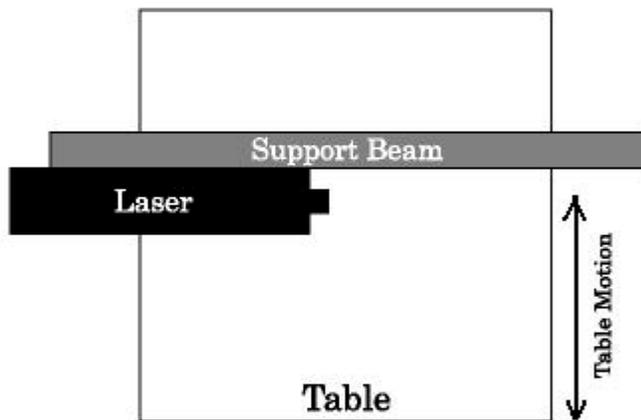
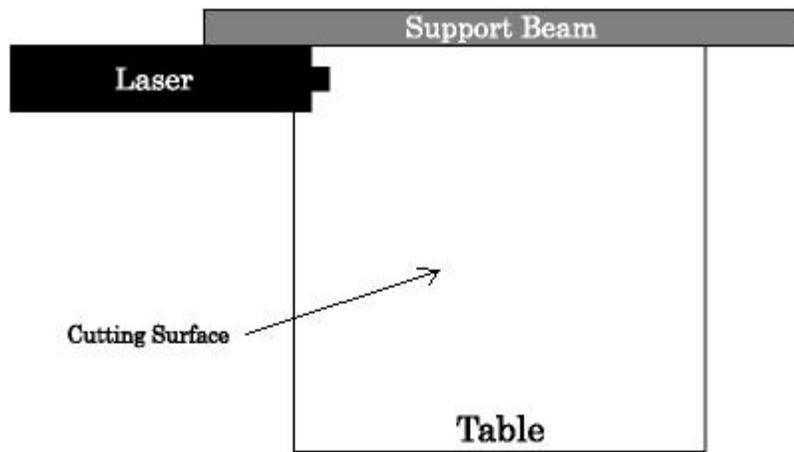


Figure 2-6 The motion of the Laser Cutter

Like many modern systems, the table and laser are designed to be controlled by a computer. The Laser Cutter is driven by a language called Hewlett Packard Graphics Language (Level 1), or HPGL/1. In the simplest sense, HPGL is a standardized way of representing "move to" and "draw to" commands, which produce the overall shape of the object. HPGL is designed to describe lines and simple 2D arcs, not complex surfaces or textures; it was created to control large plotters, which move a pen continuously across large sheets of paper.

The basic outline of a spur gear is a continuous set of curves. If the involute teeth could be modeled in an HPGL/1 file, then the Laser Cutter could produce a gear made of wood or plastic. Since a spur gear can be used for many simple applications, it would be very convenient to quickly cut out a lot of gears, representing many different configurations.

Unfortunately, HPGL was not written with involute curves as a standard shape; the commands in HPGL are designed to approximate complex shapes with a series of lines and ellipses. Therefore, a Laser Cutter could only cut out a spur gear if the shape of the teeth were defined in terms of the more simple HPGL shapes. While the shape of a given gear could be copied onto a computer, then digitally converted into an HPGL/1 file, that shape would work only for the given gear. The shape could not be easily adjusted to model a gear with fewer teeth, or a larger Diametral Pitch; another shape would have to be generated from scratch, and converted into HPGL format. If, however, a set of basic parameters could be converted into the location of numerous points on the contour of a gear, then these points could be connected into line segments and curves. These simple shapes can then be represented in an HPGL format in order to drive the laser cutter.

Because of the mathematical nature of the involutes, a spreadsheet program, like Microsoft's Excel, could be used to compute the points, although that is not quite the purpose for which spreadsheets are designed. Spreadsheets were created to serve as large-scale Algebraic engines, manipulating large sets of numbers based on other input numbers. In that sense, they are very good at constructing a single involute curve from the basic parameters (N , Φ , P). But they were not meant to iterate the process of generating these numbers: once they had created the curves of the first tooth, they cannot simply duplicate that tooth N times by rotating it around the center of the gear until the entire gear modeled. A spreadsheet will store numbers and algebraic formulae, then compute the functions of those numbers in a specified manner. It would be relatively easy, for example, to compute many points along the silhouette of one tooth by creating two columns with formulae for the X and Y components of each point. The difficulty lies in needing an additional pair of columns to store the data for the next tooth; the number of teeth on a gear must be known in advance, in order to create a sufficient number of columns to hold the data.

It is possible to "pretend" to have this knowledge by simply creating columns for a very large number of teeth; the spreadsheet could then create any desired gear with that number of teeth. Should the gear actually require fewer teeth, those extra columns can be filled with 0s (essentially creating several dots in the center of the gear, rather than a tooth). This technique is inefficient, since the spreadsheet will be very large even when the gear has only a few teeth, but it will work.

Because every tooth on a gear is identical in shape, the coordinates which define that shape differ only in the angle which determines where each tooth starts. On a gear with four teeth, for example, the first tooth might begin directly "North" of the origin, while the second would be "East," the third "South," and the fourth to the "West." But, if the gear were rotated 90 degrees clockwise, then the first tooth would be East, the second South and so forth, but the

gear would look exactly the same. Similarly, if a drawing of the first tooth were to be duplicated and then rotated 90 degrees, as shown in Figure 2–7, it would look like the first and second tooth of the gear. By repeating this process, each tooth on the gear could be simply by using the image of the first tooth. Using this process, we can derive the coordinates for each point of each tooth once the spreadsheet has calculated all of the coordinates for one of the teeth.

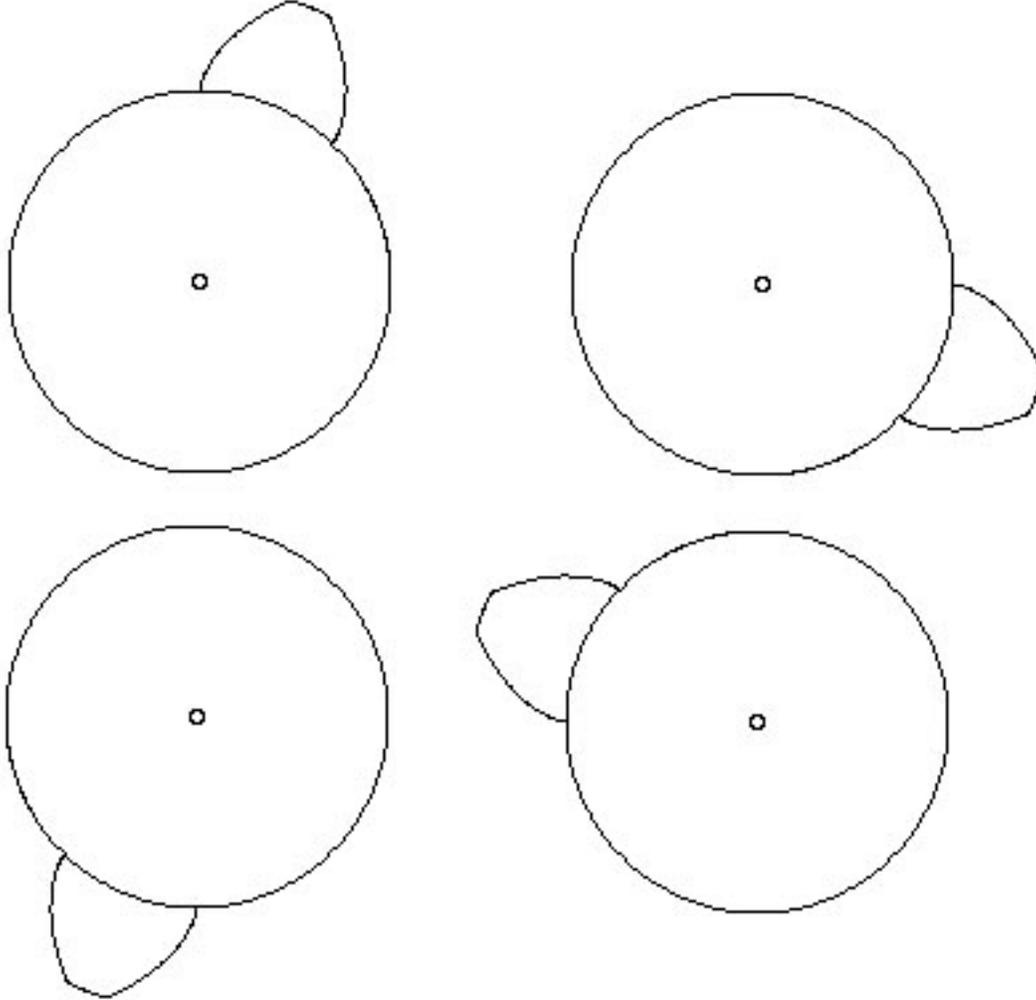


Figure 2–7 Modeling a gear by replicating a single tooth.

To derive the coordinates of the rotated points, we need to know the angle of rotation about a known point. Thus, it is important to know the angle between the beginning of one tooth, and the beginning of the next. This distance is the Circular Pitch, and the angle it spans is the **Sector Angle, S**, which can be derived from the Circular Pitch (in radians):

$$S = \frac{P}{D_R/2}. \quad [\text{Eq. 2–1}]$$

Using the Sector Angle, the coordinates for each tooth on the gear can be generated from the coordinates of the first tooth. The easiest method would be to iterate the process: tooth *m* would be rotated by $(m-1)*S$ radians, with *m* running from 1 to *N* teeth. Since spreadsheets like Excel

cannot iterate in this way, the spreadsheet will instead number each of the column sets from 0 to $N-1$. These columns will contain the results of rotating the original tooth coordinates by a number of degrees equal to the Column Number times the Sector Angle.

As a further simplification, it is possible to model both sides of a tooth on a single set of involute coordinates. Just as any particular tooth can be created by rotating a base tooth by a certain angle, one side of a tooth can be created by “reflecting” the other side across the centerline of that tooth. Using this method, the spreadsheet would create four sets of columns for each tooth; the sections of tooth generated by these columns are illustrated in Figure 3–4. The first pair of columns would contain the coordinates for each point on one side of the tooth. The second pair would contain the coordinates for each point on the top of the tooth. The third would contain the coordinates for the opposite side of the tooth. A fourth column pair would contain the coordinates for the arc of the valley between this tooth and the next tooth. To save space, however, the spreadsheet actually combines the first two sets into a single pair of columns which will map the first side and top of a tooth.

Even though the coordinates necessary to describe the gear have been calculated, spreadsheets cannot convert these data directly into instructions for a laser cutter. They can, however, plot the data as a chart, and then “print” that chart into an HPGL/1 file, and that file could then be used to drive the laser cutter. Since this file will need to be recompiled every time a new gear is designed, the spreadsheet will only be useful if it can accept the new design easily, and in turn create the new HPGL file quickly. As long as the spreadsheet creates its map of the gear from only a few parameters, it will be very easy to adjust those parameters and create a new gear design; by entering only a few numbers on the spreadsheet, the entire gear can be mapped, then printed to a file which may be quickly cut on the laser cutter. The next chapter will develop the mathematical models needed for the spreadsheet to model gears correctly, based on three basic gear parameters which can easily be entered into the cells of a small table in the spreadsheet.

Chapter 3 – Modeling Gears in a Spreadsheet

In order to model the gear, we will need to understand both the mathematics behind a gear itself, as well as the formulae needed to draw a gear in a spreadsheet.

Section 3.1: Mathematical Parameters for Spur Gears

As explained in Chapter 2, the gear design can be modeled via a spreadsheet, based on the Pressure Angle (Φ), Diametral Pitch (P), and the Number of Teeth (N). Based on these three input parameters, the spreadsheet will calculate the eight derived parameters which are needed to compute the outline of the gear. (Refer to Fig. 2–4 for an illustration of each of these parameters).

The **Pitch Diameter (D)** is

$$D = N/P. \quad [\text{Eq. 3-1}]$$

The Pitch Diameter is simply the diameter of the Pitch Circle.

The **Circular Pitch (p)** is

$$p = \pi/P. \quad [\text{Eq. 3-2}]$$

The **Circular Tooth Thickness (t)** is

$$t = \pi/(2.01 * p). \quad [\text{Eq. 3-3}]$$

The angle spanned by **t** is the **Tooth Angle**, τ .

The **Addendum (a)** and **Dedendum (d)** are

$$a = 1/P, \quad [\text{Eq. 3-4}]$$

$$d = 1.25/P. \quad [\text{Eq. 3-5}]$$

The Addendum is the radial distance between the Pitch Circle, and the tip of the tooth. The Dedendum is the radial distance between the Pitch Circle and the bottom the valley. Thus the total "height" of the tooth is the sum of the Addendum and Dedendum.

The **Root Diameter (D_R)** and the **Base Diameter (D_B)** are

$$D_R = \frac{N - 2.5}{P}, \quad [\text{Eq. 3-6}]$$

$$D_B = D * \cos(\Phi). \quad [\text{Eq. 3-7}]$$

The Base Diameter is the diameter of the circle from which the involute curve of the teeth is generated. The Root Diameter is the diameter of the circle that contains the bottom (or "roots") of the valleys.

Finally, the **Backlash (b)** is the distance between the trailing edge of a meshing tooth on one gear and the leading edge of the meshing tooth on the other gear. In a perfectly made system, the Backlash could be reduced to zero; the tooth would then be in contact with the other gear on both sides at once. In reality, a small gap is allowed to separate those surfaces, to

compensate for small imperfections in the size of the gears. Backlash is usually based on a number of variables, based on longstanding convention. For the purpose of this Spreadsheet, we set the Backlash to a fixed portion of the size of the gear's teeth. In the spreadsheet, the Circular Tooth Thickness is always 95% of the "Circular Valley Thickness.". Therefore, the Backlash is

$$b = \frac{.05 * p}{1.95}. \quad [\text{Eq. 3-8}]$$

When the Root Circle is greater than the Base Circle, the involute curve is truncated, and an arc along the Root Circle connects the valley to the next tooth. When the Base Circle is greater than the Root Circle, the end of the involute is connected to the Root Circle by a radial line. Once that line contacts the Root Circle, then an arc connects to the point at which a radial line from the next involute would contact the Root Circle. These lines create an undercut in the gear, allowing the tips of the other gear's teeth to move within the otherwise narrowed valley area. Now that we understand the relations between the basic parameters, we must create a spreadsheet which will use them.

Section 3.2: Creating the Gear Model in the Spreadsheet

The spreadsheet will contain numerous sets of columns; each column will hold the X or Y coordinates of each point along a specific curve in the gear. Once the appropriate portion of each curve is charted, the gear model will be complete. Figures 3-1 and 3-2 show the beginning of the spreadsheet itself; the entire spreadsheet uses more than 35,000 cells. The three basic parameters are input in the top left of the spreadsheet, as shown in Figure 3-1. Using Equations 3-1 to 3-8, the spreadsheet can calculate all the variables needed to model the gear; Figure 3-3 illustrates many of these additional variables. Using the basic parameters and the other derived numbers, the sheet then calculates the gear's surface in the many columns which make up the rest of the spreadsheet. The gear itself is displayed as a graphic on a separate sheet.

In addition to the three basic parameters, the spreadsheet also requests a fourth parameter: the diameter of the shaft (s) about which the gear will sit. Since the shaft will be the axis about which the gear will rotate, it is important that the shaft be correctly centered on the gear. Therefore, the spreadsheet will automatically draw a circular hole of the appropriate size for the laser cutter to cut out. The [Shaft Angle], [Shaft Radian], [HX] and [HY] columns shown in Figure 3-2 create the curve for this hole. The Shaft Diameter parameter, however, will not actually affect the teeth of the gear, and is therefore not considered one of the basic parameters which will influence the rest of the spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Inputs															
2	Pressure Angle	(H)	14.5													
3	Diagonal Pitch	(P)	2													
4	Number of Teeth	(N)	12													
5	Shoof Diameter	(H)	1													
6																
7	Primary Derived Values															
8	Pitch Diameter	(D)														
9	Outside Pitch	(M)	1.57379033													
10	Circular Tooth Thickness	(S)	8.75149075													
11	Addendum	(A)	0.5													
12	Dedendum	(B)	0.25													
13	Root Tooth Thickness	(T)	8.75149075													
14	Root Diameter	(D4)	4.75													
15	Boor Diameter	(D4)	8.09869284													
16	Backlash	(B)	8.04827683													
17																
18																
19																
20	Other Useful Derived Numbers															
21	Teeth/Inch Angle		8.26170038													
22	Root TV Ratio		8.25109558													
23	Tooth Angle		8.26851218													
24	Valley Angle		4.98220937													
25	Total Addendum		3.5													
26	Roots Limit	(B4)	8.07760993													
27	Derived Angle Limit	(B4)														
28																
29																
30																
31	Huge Reams of Calculated Numbers															
32	Angle (Deg):	Angle CX:	Angle CY:	Used to Find Root/Lat/abs:												
33	90	1.57	1.7732E-18	2.90444292												
34	98	1.57	1.7732E-18	2.90444292												
35	88	1.55	0	0.856093109	0.01745328	0.05080953	2.80444292	1.7732E-18	2.90444292	1.7732E-18	2.90444292	0.0000	0.0000	0.0000	2.2750	0.0000
36	89	1.54	0	0.11079418	0.03480959	0.1012638	2.80267261	4.1173E-85	3.90821187	2.8062	0.0000	0.0000	0.0000	0.0000	2.8062	0.0000
37	87	1.52	0	0.15207428	0.05235988	0.15300688	2.80048248	0.00161384	2.80042154	2.8084	0.0000	0.0000	0.0000	0.0000	2.8084	0.0000
38	86	1.5	0	0.20279837	0.08880317	0.2026037	2.80738764	0.00332926	2.81151225	2.8115	0.0000	0.0000	0.0000	0.0000	2.8115	0.0000
39	85	1.46	0	0.25348846	0.09726646	0.25118888	2.80529804	0.00664291	2.81348117	2.8155	0.0000	0.0000	0.0000	0.0000	2.8155	0.0000
40	84	1.47	0	0.30415255	0.10471976	0.30358698	2.80512088	0.01110156	2.82032468	2.8203	0.0000	0.0000	0.0000	0.0000	2.8203	0.0000
41	83	1.45	0	0.35484464	0.12217395	0.35386255	2.80279264	0.02176287	2.82822032	2.8280	0.0000	0.0000	0.0000	0.0000	2.8280	0.0000
42	82	1.43	0	0.40553874	0.13962634	0.40427073	2.80177109	0.03263025	2.83281689	2.8326	0.0000	0.0000	0.0000	0.0000	2.8326	0.0000
43	81	1.41	0	0.45622863	0.15707983	0.45435407	2.80088441	0.04374386	2.84015432	2.8401	0.0000	0.0000	0.0000	0.0000	2.8401	0.0000
44	80	1.4	0	0.50692892	0.17453283	0.50435122	2.80031791	0.05513157	2.84824328	2.8483	0.0000	0.0000	0.0000	0.0000	2.8483	0.0000
45	79	1.38	0	0.55761301	0.19198932	0.55419394	2.80008872	0.06692576	2.85747777	2.8575	0.0000	0.0000	0.0000	0.0000	2.8575	0.0000
46	78	1.36	0	0.6083051	0.20943951	0.60386164	2.80007387	0.07988548	2.86744762	2.8675	0.0000	0.0000	0.0000	0.0000	2.8675	0.0000
47	77	1.34	0	0.65899719	0.22689328	0.65326975	2.80003224	0.01126036	2.87824435	2.8783	0.0000	0.0000	0.0000	0.0000	2.8783	0.0000
48	76	1.33	0	0.70968929	0.2443481	0.70384834	2.80001866	0.01403986	2.88986793	2.8899	0.0000	0.0000	0.0000	0.0000	2.8899	0.0000
49	75	1.31	8.75172514	2.80547643	0.76038138	0.75172514	2.80547643	0.01725313	3.00227781	3.0023	0.0000	0.0000	0.0000	0.0000	3.0023	0.0000
50	74	1.29	0.8857287	2.79182873	0.81107347	0.7925288	2.79182873	0.02180181	3.01491807	3.0156	0.0000	0.0000	0.0000	0.0000	3.0156	0.0000
51	73	1.27	0.84817693	2.77753299	0.86179596	0.84817693	2.77753299	0.02660642	3.02848848	3.0296	0.0000	0.0000	0.0000	0.0000	3.0296	0.0000
52	72	1.26	0.89752222	2.76218607	0.91246766	0.91246766	2.76218607	0.03167342	3.04245428	3.0444	0.0000	0.0000	0.0000	0.0000	3.0444	0.0000
53	71	1.24	0.94599817	2.74620473	0.96314875	0.94599817	2.74620473	0.03691815	3.05697902	3.0600	0.0000	0.0000	0.0000	0.0000	3.0600	0.0000
54	70	1.22	0.99327798	2.72972899	1.01384164	0.99327798	2.72972899	0.04267826	3.07263791	3.0763	0.0000	0.0000	0.0000	0.0000	3.0763	0.0000
55	69	1.2	1.04086925	2.71351068	1.04086925	2.71351068	2.71351068	0.04850121	3.08930258	3.0934	0.0000	0.0000	0.0000	0.0000	3.0934	0.0000
56	68	1.19	1.08802347	2.69829529	1.11522802	1.08802347	2.69829529	0.05430081	3.11127208	3.1132	0.0000	0.0000	0.0000	0.0000	3.1132	0.0000
57	67	1.17	1.13466028	2.67395389	1.16591811	1.13466028	2.67395389	0.06012246	3.12911431	3.1299	0.0000	0.0000	0.0000	0.0000	3.1299	0.0000
58	66	1.15	1.18134338	2.65334064	1.21661521	1.18134338	2.65334064	0.065901464	3.14818059	3.1490	0.0000	0.0000	0.0000	0.0000	3.1490	0.0000
59	65	1.13	1.22747092	2.632231924	1.26738223	1.22747092	2.632231924	0.071660488	3.16780432	3.1689	0.0000	0.0000	0.0000	0.0000	3.1689	0.0000

Figure 3-1 Beginning of the Excel Spreadsheet Part 1

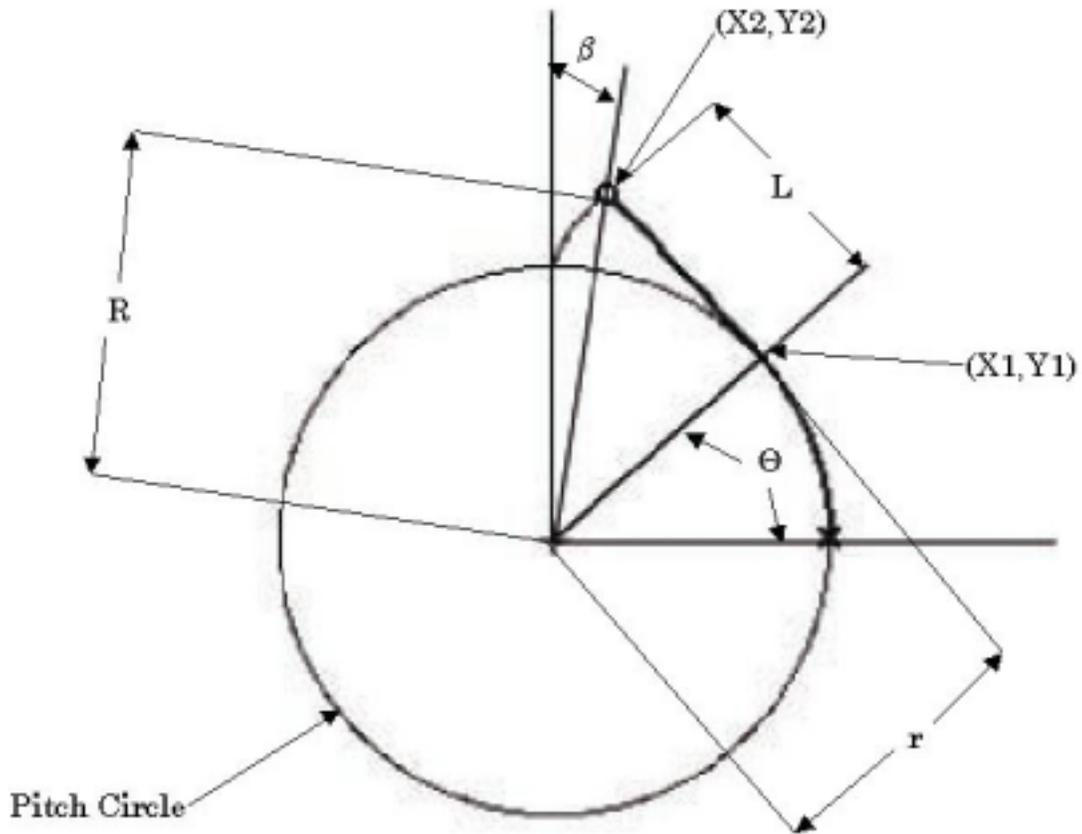


Figure 3-3 Variables used by the Excel Spreadsheet.

Section 3.2.1: Defining the Sections of the Gear Tooth

Figure 3-4 shows the six distinct curves into which a gear tooth is broken in order for the spreadsheet to compute its profile. Each of these curves is computed by a different set of equations in the final spreadsheet.

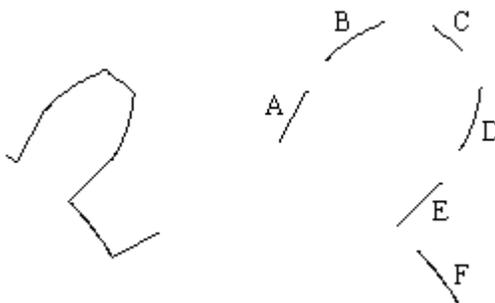


Figure 3-4 Different Curves Used to Compute the Gear Tooth

Curve A is the surface between the involute on one side of the tooth and the valley between teeth. Because it does not mesh with other teeth surfaces directly, its exact shape is not important. Therefore, it is represented by the spreadsheet as a radial line from the end of the

valley to the end of the involute curve. The equations used to compute Curve A are explained in Section 3.2.5.

Curve B is the surface of one side of the tooth which will mesh with another tooth. This curve is the involute drawn about the Base Circle. The equations used to compute Curve B are explained in Section 3.2.2.

Curve C forms the top of the tooth, an arc connecting the end of one involute with the end of the involute on the opposite side of the tooth. Again, its exact shape is less important since it will not be in direct contact with another surface. The equations used to compute Curve C are explained in Section 3.2.3.

Curve D is the meshing surface on the opposite side of the same tooth. It is also an involute, and is the mirror image of Curve B. The equations used to compute Curve D are explained in Section 3.2.4.

Curve E is separates the opposite side involute (Curve D) from the valley between teeth. Like Curve A, it is a radial line from the end of the involute to the Root Diameter. The equations used to compute Curve E are explained in section 3.2.5.

Curve F is the valley that separates the current tooth from the next tooth one. Since it will not mesh directly with the surface of another gear, it is simply an arc on the root diameter. The equations used to compute Curve F are explained in Section 3.2.5.

Section 3.2.2: Modeling the Basic Involute Curve

The first few columns of the spreadsheet (shown in Figure 3–1) are used to create the pitch circle. The first column, [Angle (Deg)], iterates an angle, ϕ , through 360 degrees; each row in the column represents a different value of the angle. ϕ is iterated in one degree steps (as seen in Figure 3–1) for most of the circle; from 30° until 0°, however, ϕ is iterated in half degree steps in order to give the curves which will be derived from it more points. Without these extra data points, the graphic used to drive the laser cutter would not have a smooth, well defined curve. The [Angle (Rad)] column simply converts the angle into radians, Θ . Using Θ , the [X1] and [Y1] columns generate the X and Y coordinates for numerous points in the Pitch Circle.

Therefore

$$[X1]_i = D/2 * \cos(\Theta_i), \quad [\text{Eq. 3-9}]$$

$$[Y1]_i = D/2 * \sin(\Theta_i). \quad [\text{Eq. 3-10}]$$

i represents the rows in each column; i is effectively iterated through each entry in a column, starting at the top, and working down.

Once we have the coordinates of the Pitch Circle, we can use them to generate an involute. The [String Lng] column contains the length, L , of the imaginary string creating the involute. Thus

$$L_i = (\pi/2 * r) - (\Theta_i - \pi). \quad [\text{Eq. 3-11}]$$

Next, the [String Angle] column computes the String Angle, α . The String Angle is the compliment of Θ , so

$$\alpha_i = \pi/2 - \Theta_i. \quad [\text{Eq. 3-12}]$$

Finally, the [X2] and [Y2] columns calculate the coordinates for the points on the involute curve itself. The equation for the involute is

$$[X2]_i = [X1]_i - L_i * \cos(\alpha_i), \quad [\text{Eq. 3-13}]$$

$$[Y2]_i = [Y1]_i + L_i * \sin(\alpha_i). \quad [\text{Eq. 3-14}]$$

Unfortunately, these coordinates represent a complete involute, which will extend much further than the actual gear tooth will require.

In order to trim the involute to the proper length, we must know where it should end. The tip of the gear is at a radius determined by the Addendum plus the Pitch Circle radius. Thus, the [Xlim] and [Ylim] columns hold the coordinates for the gear's involute, but once the last point is reached (the point at the radius of the tip of the tooth), they repeat that last point. It is necessary to truncate the involute by repeating the last point, rather than by using fewer points, because the size of the gear itself is variable. A larger tooth will need more points to define it smoothly, and the spreadsheet cannot know in advance how many points will be needed. Therefore, it must use many more points than are ever likely to be needed, and ignore the excess by repeating the last needed point. The formulae used for this are

$$\begin{aligned} & \text{if } ([X2]_i^2 + [Y2]_i^2) > (r+a)^2, \text{ then} \\ & [X\text{lim}]_i = [X\text{lim}]_{i-1}, \text{ else} \quad , \quad [\text{Eq. 3-15}] \\ & [X\text{lim}]_i = [X2]_i \end{aligned}$$

$$\begin{aligned} & \text{if } ([X2]_i^2 + [Y2]_i^2) > (r+a)^2, \text{ then} \\ & [Y\text{lim}]_i = [Y\text{lim}]_{i-1}, \text{ else} \quad . \quad [\text{Eq. 3-16}] \\ & [Y\text{lim}]_i = [Y2]_i \end{aligned}$$

The final point on this involute, ($\mathbf{X2_f}$, $\mathbf{Y2_f}$), can be used to calculate the angle, β_f .

$$\beta_f = \arctan\left(\frac{X2_f}{Y2_f}\right). \quad [\text{Eq. 3-17}]$$

Because the involute starts at (0,0) and ends at ($\mathbf{X2_f}$, $\mathbf{Y2_f}$), β_f represents the arc distance that the side of a gear tooth will span. This will be very useful when calculating the coordinates for the other side of the tooth.

Section 3.2.3: Modeling the Top Segment of the Tooth

The next task is to create the top of the tooth (Curve C in Figure 3-4). The top is simply a circular arc between the ends of the two involutes, at radius \mathbf{R} from the origin.

$$R = \frac{D}{2} + a \quad [\text{Eq. 3-18}]$$

Because the [Xlim] and [Ylim] columns contain more points than are needed to define the involute, the last several points are all the same. To save space, those entries could actually be replaced with a new set of points which will define the top surface of the tooth. Therefore, the [X2+] and [Y2+] columns, rather than repeating the last point on the involute, form the top arc:

$$\begin{aligned}
& \text{if } ([X \text{ lim}]_i^2 + [Y \text{ lim}]_i^2) > (r + a)^2, \text{ then} \\
& [X 2+]_i = R * \sin(\beta_i), \text{ else} \quad , \quad \text{[Eq. 3-19]} \\
& [X 2+]_i = [X \text{ lim}]_i
\end{aligned}$$

$$\begin{aligned}
& \text{if } ([X \text{ lim}]_i^2 + [Y \text{ lim}]_i^2) > (r + a)^2, \text{ then} \\
& [Y 2+]_i = R * \cos(\beta_i), \text{ else} \quad . \quad \text{[Eq. 3-20]} \\
& [Y 2+]_i = [Y \text{ lim}]_i
\end{aligned}$$

This arc, however, must also be long enough to create variable sized teeth. So, just like the involute, it will also continue past the end of the gear's top, and must be truncated.

The [X2+L] and [Y2+L] columns truncate the arc by repeating the final point over again. We can calculate the angle of the final point, β_{fa} , by

$$\beta_{fa} = \tau - \beta_f . \quad \text{[Eq. 3-21]}$$

This formula works because the tooth is symmetric, so the arc of one side of the tooth will be the same length as the arc spanned by the other side. Thus, the equations for the first side and top of the tooth are

$$\begin{aligned}
& \text{if } (\beta_i < \beta_{fa}), \text{ then} \\
& [X 2 + L]_i = [X 2+]_i, \text{ else} , \quad \text{[Eq. 3-22]} \\
& [X 2 + L]_i = [X 2 + L]_{i-1}
\end{aligned}$$

$$\begin{aligned}
& \text{if } (\beta_i < \beta_{fa}), \text{ then} \\
& [Y 2 + L]_i = [Y 2+]_i, \text{ else} . \quad \text{[Eq. 3-23]} \\
& [Y 2 + L]_i = [Y 2 + L]_{i-1}
\end{aligned}$$

Section 3.2.4: Modeling the Mirrored Involute

Next, the spreadsheet calculates the points for the far side of the tooth (Curve D in Figure 3-4).

The [X3] and [Y3] columns generate the base involute for the opposite side of the tooth. The formula is identical to the one used to create the first involute, but it has been reflected about the Y-axis. Thus

$$[X 3]_i = (-1) * ([X 1]_i - L_i * \cos(\alpha_i)) , \quad \text{[Eq. 3-24]}$$

$$[Y 3]_i = (-1) * ([Y 1]_i + L_i * \sin(\alpha_i)) . \quad \text{[Eq. 3-25]}$$

Again, the involute must be truncated, so the [X3+] and [Y3+] columns are

$$\begin{aligned}
& \text{if } ([X 3]_i^2 + [Y 3]_i^2) > R^2, \text{ then} \\
& [X 3+]_i = [X 3+]_{i-1}, \text{ else} \quad , \quad \text{[Eq. 3-26]} \\
& [X 3+]_i = [X 3]_i
\end{aligned}$$

$$\begin{aligned}
& \text{if } \left([X3]_i^2 + [Y3]_i^2 \right) > R^2, \text{ then} \\
& [Y3+]_i = [Y3+]_{i-1}, \text{ else} \\
& [Y3+]_i = [Y3]_i
\end{aligned} \tag{Eq. 3-27}$$

Finally, the involutes must be rotated such that they start at the end of the tooth, rather than the beginning. Thus the [X4] and [Y4] columns are

$$[X4]_i = [X3+]_i * \cos(\tau) + [Y3+]_i * \sin(\tau), \tag{Eq. 3-28}$$

$$[Y4]_i = [Y3+]_i * \cos(\tau) - [X3+]_i * \sin(\tau). \tag{Eq. 3-29}$$

This completes the model of the tooth, since it now has both sides and its top.

Section 3.2.5: Modeling the Valley Between Teeth

Once the spreadsheet has completely modeled the tooth itself, it must model the valley between that tooth and the next one (Curve F in Figure 3-4). The floor of the valley is simply a circular segment, along the Root Diameter, that starts when one tooth ends, and ends when the next tooth begins. Therefore it starts at τ , and ends at **S**, the **Sector Angle**:

$$S = \frac{P}{R}. \tag{Eq. 3-30}$$

S represents the total angle spanned by both a tooth and a valley. The valley floor is modeled in the [PCX] and [PCY] columns,

$$\begin{aligned}
& \text{if } (\beta_i < S), \text{ then} \\
& \left[\begin{array}{l} \text{if } (\beta_i > \tau), \text{ then} \\ [PCX]_i = D_R/2 * \sin(\beta_i), \text{ else} \\ [PCX]_i = D_R/2 * \sin(\tau) \end{array} \right], \text{ else ,} \\
& [PCX]_i = [PCX]_{i-1}
\end{aligned} \tag{Eq. 3-31}$$

$$\begin{aligned}
& \text{if } (\beta_i < S), \text{ then} \\
& \left[\begin{array}{l} \text{if } (\beta_i > \tau), \text{ then} \\ [PCY]_i = D_R/2 * \cos(\beta_i), \text{ else} \\ [PCY]_i = D_R/2 * \cos(\tau) \end{array} \right], \text{ else .} \\
& [PCY]_i = [PCY]_{i-1}
\end{aligned} \tag{Eq. 3-32}$$

The ‘if’ statements are designed to truncate the Valley curve. If β is less than the Tooth Angle (τ), then the Valley has not started yet, and it will map a point to the start of the Valley. If β is greater than the Sector Angle (**S**), then the next tooth has begun, so it will map a point to the same coordinates as the preceding point. If β is between those two values, it will map a point to the valley curve.

This arc must still be connected to the involutes, if the Root Diameter is less than the Base Diameter. In that case, we will use a radial line to connect the involute to the arc (Curves A and E in Figure 3–4), by changing the first entry in the involute columns to lie on the Root Circle rather than the Pitch Circle. Therefore, the first rows of the [X2+], [Y2+], [X3] and [Y3] columns will be changed:

$$[X2+]_1 = \frac{D_R}{2} * \sin(\beta_1), \quad [\text{Eq. 3-33}]$$

$$[Y2+]_1 = \frac{D_R}{2} * \cos(\beta_1), \quad [\text{Eq. 3-34}]$$

$$[X3]_1 = \frac{D_R}{2} * \sin(\beta_1), \quad [\text{Eq. 3-35}]$$

$$[Y3]_1 = \frac{D_R}{2} * \cos(\beta_1). \quad [\text{Eq. 3-36}]$$

With the entire sector now modeled, the spreadsheet must replicate and rotate that sector to completely model the gear.

Section 3.2.6: Rotating the First Tooth to Model the Entire Gear

To completely model the gear, the spread sheet has 30 sets of six columns. The [T1Xn] and [T1Yn] columns hold the coordinates for the first involute side and top of the **n**th tooth. The [T2Xn] and [T2Yn] columns model the opposite side involute for the **n**th tooth. Lastly, the [CX] and [CY] columns model the valley between the **n**th tooth and the (**n**+1)th tooth. Each of these columns simply rotates the points calculated previously about the origin by a certain angle; the angle is determined by the tooth number, **n**. Each set of columns represents a given value of **n** as it ranges from 0 to **N**-1, using the formulae

$$[T1Xn]_i = [X2+L]_i * \cos(S * n) + [Y2+L]_i * \sin(S * n), \quad [\text{Eq. 3-37}]$$

$$[T1Yn]_i = [Y2+L]_i * \cos(S * n) - [X2+L]_i * \sin(S * n), \quad [\text{Eq. 3-38}]$$

$$[T2Xn]_i = [X4]_i * \cos(S * n) + [Y4]_i * \sin(S * n), \quad [\text{Eq. 3-39}]$$

$$[T2Yn]_i = [Y4]_i * \cos(S * n) - [X4]_i * \sin(S * n), \quad [\text{Eq. 3-40}]$$

$$[CXn]_i = [PCX]_i * \cos(S * n) + [PCY]_i * \sin(S * n), \quad [\text{Eq. 3-41}]$$

$$[CYn]_i = [PCY]_i * \cos(S * n) - [PCX]_i * \sin(S * n). \quad [\text{Eq. 3-42}]$$

Any set of columns for **n** larger than **N**-1, i.e. columns representing more teeth than a given gear actually has, will be filled with 0 instead. These columns will map a point at the origin, rather than a tooth.

Section 3.2.7: Plotting the Gear from the Business Graphic

The graphic for any given gear is created by plotting Curves A, B, C, D, E, and F for each of the N teeth in the spur gear. A sample graphic is shown in Figure 3–5 for a 30 tooth gear (the maximum number allowed by the spreadsheet), with a Diametral Pitch of 5.

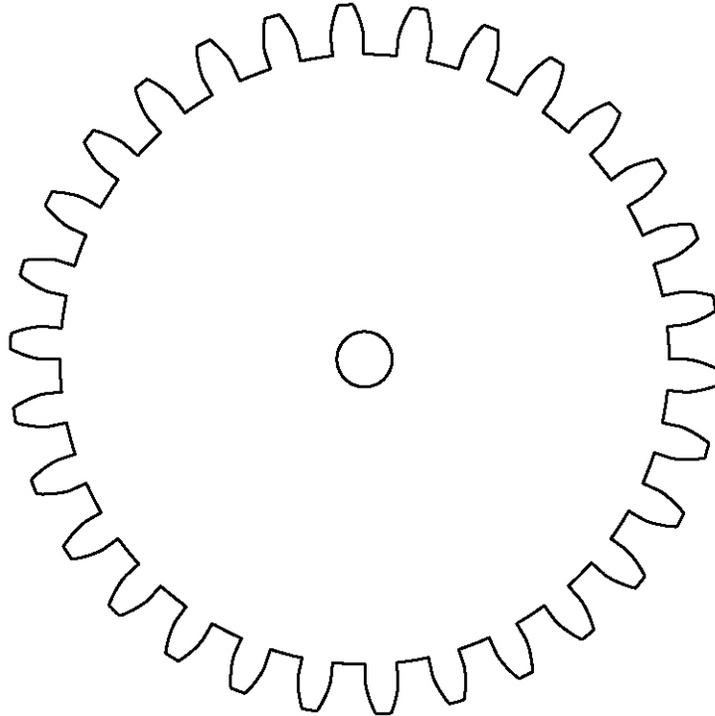


Figure 3–5 Business Graphic of a 30 Tooth Gear with a Diametral Pitch of 5, and a Pressure Angle of 14.5°.

Within the spreadsheet, the gear can be changed simply by altering the three input parameters; different gears created by changing these parameters are shown in Chapter 4. Because, however, so many cells are dependant on those parameters, the computer will need time to recalculate all of the values in the spreadsheet. The amount of time needed depends on the computer being used. On an older 300 MHz Pentium II system (with around 64 megabytes of RAM), the spreadsheet needed several seconds to recalculate changes. Using a fairly modern 1 GHz Pentium III machine (with 256 megabytes of RAM), however, changes take less than a second to recalculate. We therefore believe that even the baseline computers available now should be able to recalculate the entire spreadsheet in a relatively short amount of time.

Once the graphic is finalized, it must still be converted into a set of HPGL/1 commands which can be used to drive the Laser Cutter. Because the Laser Cutter cannot be controlled directly by Excel, we cannot simply ‘print’ the graphic to the Laser Cutter. The next best choice is to print the graphic to another device, such as a plotter, which can understand HPGL/1 commands; therefore, the computer running the spreadsheet also requires a driver for an HPGL/1 plotter. Modern plotters use the newer HPGL/2 drivers, which the Laser Cutter cannot understand. Therefore, we will need the drivers for an older generation of plotters, which will need to be installed on modern Windows computers. These drivers can be found on older Windows disks, or ordered from Hewlett Packard; fortunately, the installation only needs to be done once. Using these older drivers, the graphic can be printed to a file on the hard disk. Since that file will contain an HPGL/1 description of the gear, it can now be used to drive the laser

cutter. Although any future changes to the gear in the spreadsheet would require the creation of a new plot file, the process of converting it into an HPGL/1 file is now fairly straightforward.

Chapter 4 – Results and Conclusions

The spreadsheet is capable of creating gears with up to 30 teeth. Figure 4–1 shows the outline of a gear with 12 teeth, a Diametral Pitch of 4, and a Pressure angle of 14.5°.

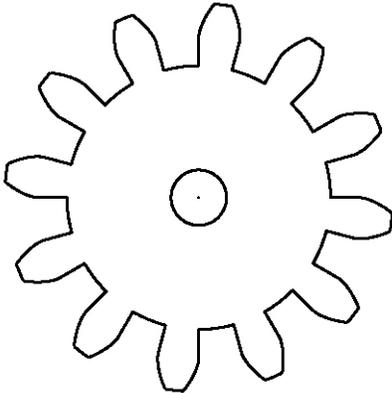


Figure 4–1 Outline created by Excel for a 12 Tooth Gear with a Diametral Pitch of 4, and a Pressure Angle of 14.5°.

Just by changing the number of teeth in the spreadsheet, we can generate a new gear, shown in Figure 4–2.

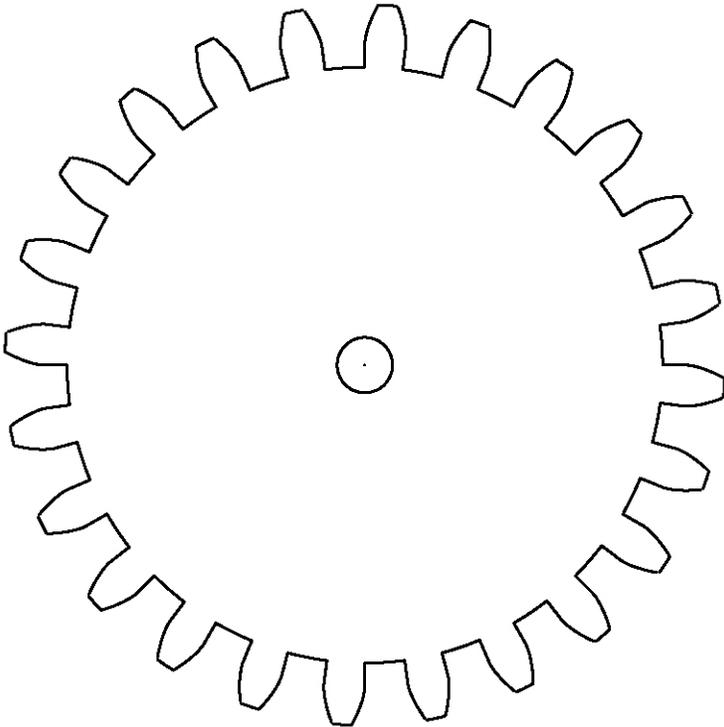


Figure 4–2 Outline created by Excel for an 24 Tooth Gear with a Diametral Pitch of 4, and a Pressure Angle of 14.5°.

Because they share the same Diametral Pitch and Pitch Angle, these two gears would mesh together. It is also possible to create totally different sets of gears by adjusting all three of the parameters, as seen in Figure 4–3. This is a 15 tooth gear with a Diametral Pitch of 2.5, and a Pressure Angle of 20°.

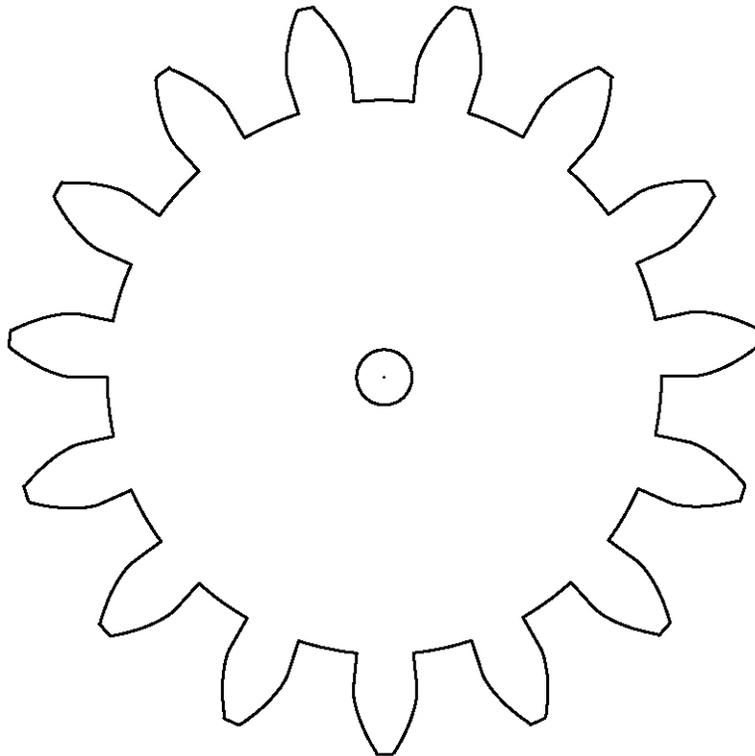


Figure 4–3 Outline created by Excel for a 15 Tooth Gear with a Diametral Pitch of 2.5, and a Pressure Angle of 20°.

Discussion

Since the spreadsheet can draw gears quickly and easily, it will be very useful in creating sets of gears for any projects that require accurate shapes and rapid prototyping of interlocking gears. Another good point is that Excel is a very common program. It is bundled with most new Windows machines, and remains very backwards compatible. So the spreadsheet will likely remain useable for a long time.

There are, unfortunately, several shortcomings to the spreadsheet setup. The most basic flaw is that any adjustments to the design cannot be immediately reflected in the Laser Cutter program. Instead, the new parameters must be entered into the Excel sheet, then transferred to a plot file. While the Excel spreadsheet automatically graphs the gear (providing a visualization of what the new gear will look like), there is an extra step in the HPGL/1 file transfer before the design becomes an actual model.

A second problem arises from the need to scale the gears correctly. The Diametral Pitch is the ratio of the number of teeth to the diameter of the Pitch Circle. In the real world, that diameter is measured in inches (or meters), giving the Diametral Pitch units of reciprocal inches

(or reciprocal meters). In Excel, however, the diameter of the Pitch Circle is simply measured in “units,” which represent the undefined distance between 0 and 1 on a graph. Because Excel was not created to draw scale models, it is difficult to have one unit in the graphic correspond exactly with one inch on the Laser Cutter. This means that while the plot file will create a properly proportioned gear, the gear may not be the right size. Fortunately, the Laser Cutter controller allows the user to set a scaling factor when cutting. By measuring a gear that was cut and comparing it to its desired dimension, the scaling factor can be determined. Once this scaling factor has been set, any further gears created by the spreadsheet can readily be cut to proper scale.

It is important that this scale be adjusted in the laser cutter controls rather than in the Excel spreadsheet, to help control the quality of the graphic. If the Excel graphic is too small (10 inches scaling to only one “unit,” for example), the gear plot will not have enough detail, and the final gear will have a very bumpy surfaces. On the other hand, if the scale is too large (such as 10 units representing one inch), the graphic will have so much detail that the plot file will take a prohibitively long time to be cut on the laser cutter. The spreadsheet used in this project settled on a 1:1 ratio (one inch represents one unit); this provided enough detail to cut a smooth gear, in a fairly short amount of time.

The final problem comes from the age of the user interface for the Laser Cutter; at some point, future computers will not be able to run or interact with the program. HPGL/1 is already obsolete, and the user interface of the laser cutter cannot understand HPGL/2. Because newer versions of the Windows operating system do not seem to include drivers for old HPGL/1 plotters, eventually, new methods of controlling the Laser, or creating the gears will be needed. If the laser cutter can learn to accept HPGL/2 commands, however, it will be possible to simply save the graphic of the gear to a file which can drive the laser cutter. It may even be possible to control the laser cutter as a printer, allowing the user to “print” the gear directly from Excel.

Chapter 5 -- Future Research

The spreadsheet program does achieve the primary goal of enabling the rapid prototyping of sets of interlocking spur gears. There are, however, many options left open to future research. The easiest improvement would be to expand the spreadsheet to allow gears with more than 30 teeth. Using the same techniques developed in this project, the gear could have an arbitrarily large number of teeth. While creating new columns for the extra teeth is very easy, this process is tedious because each new set of columns must be “plotted” on the graphic. This involves editing each separate curve segment in each new tooth to the appropriate format (a thin black line graph, with no markers at the data points), rather than default Excel setting. If someone were to write a macro program to automate this task, the spreadsheet could easily handle much larger numbers of teeth, limited mostly by the capabilities of the computer. As computers become faster and more powerful, larger spreadsheets become more practical.

The next option would be a system to design pairs of gears at once. No gear operates alone; they are always designed to mesh with another gear. Although the Circular Pitch and Number of Teeth allow gears created separately to be used together, a graphical interface showing the gears interacting could help the user visualize and tune the gears before cutting them out. This would require a more flexible graphical display which could show multiple gears meshing together, rather than a single, unmoving, gear.

Another possible topic would be to compensate for the stress generated within the gears. The current spreadsheet simply cuts gears as ordered, without giving any consideration to the integrity the finished gear will have. For example, gears intended for rigorous use have small fillets on the base of the teeth, to help disperse the stress generated by the force on each tooth. A new method of designing gears could add similar features to help alleviate the stress generated by its intended workload. That method could also predict when the forces would be too great for the gear design specified, and warn the user in advance.

Unfortunately, HPGL/1 drivers have been phased out in favor of the new HPGL/2 standard, and have become more difficult to locate. Therefore, it is necessary to retain the HPGL/1 plotter drivers we used for this project. If, however, those drivers become incompatible with future versions of Windows, we will need to find another way to transfer the gear graphic to the Laser Cutter. The best alternatives would be to somehow convert an HPGL/2 file into an HPGL/1 compatible format, or to save the Excel graph in another format which can easily be converted into HPGL/1. Since there is no currently available software for downgrading an HPGL/2 file, the next best option seems to be saving the graphic as a PostScript file. This file can be read by several packages such as Corel Draw, which are able to export drawings into HPGL/1 compatible plot files. The process is more tedious, and it can reduce the quality of the curves, but it is possible.

A further improvement would be to design a new interface. Although the Excel spreadsheet is very useful for calculating numbers, it is still not the ideal means of designing gears. A graphical interface, which could show what each variable represented, would be better. Such a system would be especially helpful if it could store the most common parameters for standard gears, and allow the user to select one of them rather than having to know them on his own. The Excel spreadsheet is also not designed with drawing scale figures in mind, so the user may have to cut a series of test gears in order to make gears of a precise size. Therefore, a new program, written from scratch to design these gears, could make use of the equations and ideas of the spreadsheets, while vastly improving the user interface of the procedure. As an added

feature, the program could output directly to an HPGL file (eliminating the need to convert formats), or to any number of other useful formats.

Chapter 6 -- Bibliography:

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