Allaz and Vila for asymmetric firms

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Abstract

1 Notation and results

Consider two firms \((i, j)\) producing a reproducible commodity with marginal costs, respectively, \(c_i\) and \(c_j\) with \(c_i < c_j\). Hence firm \(i\) can be considered the "large firm" in the sense that produce more and obtain higher profits in the pure-spot (Cournot) game. Firms attend the spot market by simultaneously choosing quantities \(q^i\) and \(q^j\). As always, we assume that the spot price is given by the linear inverse demand function

\[ p^s = p(q^i + q^j) = a - (q^i + q^j)\]

Before the opening of the spot market firms are also free to simultaneously buy or sell forward contracts that call for delivery of the good at the spot market that follows. Forward transactions are denoted by \(f^i\) and \(f^j\) and the forward price by \(p^f\).

At the opening of the spot market, and given \(f^i\) and \(f^j\), firm \(i\) solves

\[
\max_{q^i} \pi^{s,i} = p^s \cdot (q^i - f^i) - c_i q^i = (a - q^i + q^j)(q^i - f^i) - c_i q^i
\]

Using both FOC's, we find that the subgame perfect spot quantities are given by

\[
q^i(f^i, f^j) = \frac{a - 2c_i + c_j + 2f^i - f^j}{3} \tag{1}
\]

\[
q^j(f^i, f^j) = \frac{a - 2c_j + c_i + 2f^j - f^i}{3} \tag{2}
\]

Note that we are assuming that cost differences are not too large so both firms produce under pure-spot trading, that is,

\[a - 2c_j + c_i > 0\]
or
\[ c_j - c_i \equiv \Delta < a - c_j \]
Note that this restriction is equivalent as to say that \( c_j \) is below the monopoly price that \( i \) would charge if it were the only firm in the market, i.e., \( c_j < (a + c_i)/2 \).

Anticipating the equilibrium outcome (1)–(2), at the forward stage firm \( i \) solves (arbitrage profits vanish, i.e., \( p^f = p^s \))
\[
\max_{f^i} p^f f^i + \pi^{i,f}(q^i(f^i, f^j), q^j(f^i, f^j)) = p^s q^i(f^i, f^j) - c_i q^i(f^i, f^j)
\]
with \( p^f = p^s = p(q^i(f^i, f^j) + q^j(f^i, f^j)) \). Applying the envelop theorem
\[
\partial p^f \partial q^i f^i + \partial p^f \partial q^j f^i + p^f + \partial p^s \partial q^j (q^i - f^i) - p^s = 0
\]
which reduces to
\[
-\frac{2}{3} f^i + \frac{1}{3} q^j(f^i, f^j) = 0
\]
Replacing \( q^j(f^i, f^j) \) we finally obtain the (equilibrium) forward quantities
\[
f^{i*} = \frac{a - 3c_i + 2c_j}{5}
\]
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\]
But for \( f^{i*} \) and \( f^{j*} \) be equilibrium quantities we require (note that \( f^{j*} \) could eventually be negative)
\[
q^j(f^{i*}, f^{j*}) > 0 \quad \text{and} \quad q^j(f^{i*}, f^{j*}) > 0
\]
Since the restriction is more demanding for \( j \), we basically require
\[
q^j(f^{i*}, f^{j*}) = \frac{2}{5} [a - 3c_j + 2c_i] > 0
\]
or
\[
\Delta < \frac{a - c_j}{2} \quad \text{(3)}
\]
Note that \( f^{j*} < 0 \) is the same as \( q^j(f^{i*}, f^{j*}) < 0 \), so we cannot have an equilibrium outcome where \( j \) goes long in an effort to increase prices.

## 2 Discussion

Let us start by looking at firm \( i \)'s Stackelberg outcome in the absence of forward trading. Firm \( i \) solves
\[
\max_{q^i} (a - q^i - q^j(q^i)) q^i - c_i q^i
\]
where \( q^j(q^i) \) is firm \( j \)'s reaction to \( i \), which is given by

\[
q^j(q^i) = \frac{a - q^i - c_j}{2}
\]

Solving we obtain (superscript \( s \) stands for Stackelberg leader and \( f \) for follower)

\[
q^{is} = \frac{a + c_j - 2c_i}{2}
\]

\[
q^{if} = \frac{a - 3c_j + 2c_i}{4}
\]

This solution will be valid as long as \( q^{if} > 0 \), or

\[
\Delta < \frac{a - c_j}{2}
\]

which is condition (3). So, if \( \Delta > (a - c_j)/2 \), it is Stackelberg optimal for \( i \) to leave \( j \) outside the market. In such a case, \( i \) will produce \( q^i = a - c_j \) leading to an equilibrium price equal to \( c_j \). Therefore, when firms are sufficiently different, i.e., condition (3) does not hold, the opening of the forward market allows the large firm to implement its Stackelberg solution with a price a bit below \( c_j \) and leaving firm \( j \) outside the market.

Unlike the exhaustible-resource case, here it is the large firm the one that strictly benefits from the opening of the forward market. In that sense, there is no prisoners’ dilemma for the large firm.

Couple of points. Note that if \( \Delta > (a - c_j)/2 \), firm \( i \) implements its Stackelberg outcome by selling \( f^i > f_{\min} \) to keep away \( j \) from the spot market (if \( \Delta = (a - c_j)/2 \) then \( f_{\min} = f^{is} \)). The final quantities will be

\[
q^i(f^i = f_{\min}, f^j = 0) = a - c_j
\]

\[
q^i(f^i = f_{\min}, f^j = 0) = 0
\]

Finally, what if \( \Delta < (a - c_j)/2 \)? Since the cost difference is not that large, we know that in equilibrium both firms will operate if there is only one forward opening preceding the spot market. But what if there are two or more forward openings preceding the spot market? Before going into the analysis, note that in this case it is not Stackelberg optimal for \( i \) to be the only one serving the market.

This is my take on this. Consider two forward openings 1 and 2. In addition, suppose that if we look for an interior solution we would obtain that

\[
q^i(F^i, F^j) > 0
\]
\[ q^j(F^i, F^j) < 0 \]

where \( F^i = f^i_1 + f^i_2(f^i_1, f^j_1) \) is the aggregate equilibrium contracting and \( f^j_2(f^i_1, f^j_1) \) is the subgame quantity given \( f^i_1 \) and \( f^j_1 \). This cannot be an equilibrium since \( q^j < 0 \), so we cannot have both firms selling forwards at \( t = 1 \). Neither can we have no firm selling forward at \( t = 1 \). Why? Because firm \( i \) has incentives to sell forwards in an effort to appropriate a larger share of the oligopoly rents that result from the subgame that starts with the next forward opening. So in equilibrium it must happen, I think, that \( f^j_1 = 0 \) and \( f^i_1 > 0 \) such that \( q^j(F^i, F^j) > 0 \), and the equilibrium price would be strictly above \( c_j \).

What if we add a third forward period to the example above, say \( t = 0 \)? Nothing should happen since we conjecture that \( j \) signs nothing at the subgame that starts at \( t = 1 \). As \( c_j \) gets closer to \( c_i \), there will be a larger number of forward openings in which both firms sign forwards in equilibrium but \( i \) will always signs in one extra forward. In addition, in equilibrium \( j \) will always be active as long as \( \Delta < (a - c_j)/2 \).