Energy Sharing and Frequency Regulation in Energy Internet via Mixed $H_2/H_\infty$ Control with Markovian Jump

Haochen Hua, Yuchao Qin, Zicheng He, Liuying Li and Junwei Cao

Abstract—In this paper, the problem of mixed optimization for energy sharing and frequency regulation in a typical energy Internet (EI) scenario where energy routers (ERs) interconnected AC microgrids (MGs) is investigated. Continuous-time Markov chains are introduced to describe the switching paths in the power dynamics of MGs. Such that the modelling of considered EI system could be closer to the real-world engineering practice. Advanced parameter estimation techniques are integrated into the proposed method to achieve better modelling accuracy and controlling performance. Based on the parameters of MG power dynamics, the mixed $H_2/H_\infty$ controllers are obtained via stochastic control theory. The feasibility and efficacy of the proposed approach are evaluated in numerical examples.

Index Terms—Energy Internet, Microgrids, $H_2/H_\infty$ control, Markov jump.

I. INTRODUCTION

OVER the past decades, human beings have faced great challenges such as environmental pollution, global warming, especially the energy crisis. Consequently, much attention has been paid on the RESs such as wind power, solar power and hydropower [1]. In order to integrate DERs into the utility grids, the operational architecture called MG is utilized. Typically, a MG consists of not only the DERs mentioned above, but also conventional power generation devices, energy storage devices and local loads. A variety of RESs in the MG system may cause intermittence, nonlinearity and uncertainty in power deviations. These features could make the energy management of MG very challenging [2], [3].

With the development of information and communication technologies, EI is proposed to deal with such issues [4]. Within the demonstrating projects of EI, multiple MGs are interconnected through ERs to share information and energy cooperatively [5]. These MGs usually work in the grid-connected mode. In contrast, they should also function well in the islanded mode (also known as the off-grid mode) considering the expensive cost of energy delivery and potential outage of the main grids [6]. Aiming at realizing a reliable and efficient operation of the EI, a new class of energy control scheme is desired. The stabilization and optimization problems in smart power systems have already been extensively studied in the past.

Firstly, regarding the stability of power systems in the EI scenario, there have been tremendous amount of research outputs investigating the stabilization of MG systems from different perspectives. For distributed DC MGs, a system-level stability analysis method is proposed in [7]. Taking into account the uncertainty existing in the power deviations of RESs and loads, a class of robust energy scheduling approach is introduced for MG systems [8]. Similarly, in [9], with the application of advanced robust control techniques, a novel robust voltage stabilization strategy is proposed for DC MG such that the time delay and modelling errors can be properly addressed. To regulate the frequency deviation induced by DERs, the $H_\infty$ and $\mu$-synthesis control methods have been applied to ensure the robustness of an islanded MG [10]. Taking modelling uncertainties into account, the system stabilizing issue of MG incorporating WTs has been studied in [11]. In [12], the two-level control strategy involving centralized controllers and multiple droop controllers enables MGs to function in both grid-connected and islanded modes. Updating the stability criterion of EI, authors in [13] design an impulsive feedback control method for consuming the fault
energy, thus stabilizing the EI system. To maintain the stability of the energy sharing functionality in the EI system, in [14], the robust $H_{\infty}$ control method is proposed for ERs such that the short-term energy storage utilization can be appropriately achieved. For further results regarding robust control in the field of MGs, readers can consult [15]–[18], and the references therein.

Besides, research on the optimal energy control and management in the field of EI has been popular in recent years. In [19], a criterion is formulated to assess the rationality of utilizing the connected DERs. The desired controller is obtained by solving the coupled differential Riccati equations. Notably that multiple-layer optimization can be applied as an effective tool to solve the optimal control problems of power systems with various RESs [20]. By installing controllers in MTs and ERs, the bottom-up energy management principle for EI is achieved accompanied by the lifetime extension of BESs [21]. Recently, the significant growth of demand-side resources in EI has motivated the research of optimal energy flow control in the case of the high operating expense [22]–[24].

There is also a great amount of research on optimization problems in the multi-microgrid setting. Take into account the time-of-use electricity price mechanism, a particle swarm optimization based optimal scheduling method is proposed in [25] for the coordinated power dispatching in multi-microgrid systems. For the energy management problem in multi-microgrid, a sequential operation based optimal control method is utilized in [26] to improve the system efficiency. In [27], [28], the model predictive control techniques are adopted for coordinated management tasks in multi-microgrid scenarios. To achieve the utilization of RESs effectively, the coordinated power dispatching and energy sharing problem in networked MG systems are discussed in [25], [29], [30]. Also, with the advances in deep learning, the application of reinforcement learning methods in the optimization problems in power systems has attracted much attention [31]–[34].

The mixed $H_2/H_{\infty}$ control problems considering both criteria of optimization and robustness are raised naturally by deeply exploring the robust and optimal control issues in EI. Such mixed $H_2/H_{\infty}$ control problems have been well investigated in both frequency domain and time domain [35]–[41]. Nevertheless, these aforementioned works still have deficiencies. The control approaches proposed in [36] and [37] lack of consideration for the nonlinearity and stochasticity of MG system. DERs such as PVs and WTs are not explicitly considered in [39]. It may lead to results inaccuracy and could be less applicable. In [40], linear feedback controllers are obtained without considering system constraints. To facilitate the maturity and application of EI, solutions to the mixed $H_2/H_{\infty}$ control problems considering system complexity should be the foreground.

In this paper, we propose a class of mixed $H_2/H_{\infty}$ controller for short-term operation cost management and frequency regulation of AC MGs in EI. The considered application scenario of EI is assumed to function without access to the main power grid. First, the dynamical EI system is formulated as SDEs with Markovian switching (also known as Markov jump) in system parameters. Then, the problem of short-term operation cost optimization and system stabilization is formulated as a mixed $H_2/H_{\infty}$ control problem mathematically. Eventually, the control issue is solved by stochastic optimization methods.

The importance and main technical contributions made in this work can be summarized as below:

- This work is investigated theoretically under the scope of a generalized off-grid EI topology in which each AC MG is allowed to be composed of PVs, WTs, FCs, MTs, BESs and loads. In particular, the power dynamics of all these components are considered from the control perspective. Markov jump SDEs and system disturbance inputs are adopted in the power modelling of renewable power generation devices (WTs and PVs) and loads. It is highlighted that with such a new model, the stochasticity and uncertainty of WTs, PVs and loads can be better represented.

- A class of mixed $H_2/H_{\infty}$ controller is designed for the considered EI. The $H_2$ performance refers to the optimal short-term operation cost management, including three aspects: the cost of utilizing BESs, the extra cost involved by controllers, and short-term operation cost of ERs for the adjustment of power transmission among MGs. The $H_{\infty}$ performance refers to each MG’s AC bus frequency stabilization against external disturbance inputs. It is notable that there has been few work taking all of these criteria into consideration simultaneously.

- Based on typical system parameters, numerical simulations for four interconnected MGs demonstrate the feasibility of our proposed method. The performances of the proposed mixed $H_2/H_{\infty}$ control method are compared with the results when there is no controller employed. The comparison shows that the controller proposed in this paper is effective.

The rest of the paper is organized as follows. Section II describes the modelling for system dynamics of the considered EI. Section III formulates the mixed $H_2/H_{\infty}$ control problem and introduces the approach to solving it. Section IV provides some simulations. Finally, we conclude our paper in Section V.

II. ENERGY INTERNET DYNAMICAL SYSTEM MODELLING

In this section, the short-term dynamical system of EI is formulated as continuous SDEs with Markovian switching in system parameters.

A. The EI Topology and MG Components

In this work, in order to show the effectiveness of the designed controller for common engineering scenarios, a generalized version of off-grid EI including $m$ interconnected AC MGs is considered. Such EI topology is illustrated in Fig. 1.

In Fig. 1, $m$ AC MGs are interconnected via multiple ERs. For illustrative purpose, each individual MG is assumed to be composed of WTs, PVs, FCs, MTs, BESs and loads. We focus on power dynamics of these devices.
where \( T_{ PV}^k(r_k^t) \), \( T_{ WT}^k(r_k^t) \), \( T_{ L}^k(r_k^t) \), \( \sigma_{ PV}^k(r_k^t) \), \( \sigma_{ WT}^k(r_k^t) \) and \( \sigma_{ L}^k(r_k^t) \) are system time-invariant parameters following Markov jumps. For notation simplicity, time \( t \) for all the equations throughout this paper is omitted.

Based on real power data and corresponding climate condition; see, e.g., [44], the paths of Markov jumps can be obtained via parameter estimation approaches with the technique proposed in [45]. In the similar way, system parameters \( T_{ PV}^k(r_k^t) \), \( T_{ WT}^k(r_k^t) \), \( T_{ L}^k(r_k^t) \) and \( \sigma_{ PV}^k(r_k^t) \), \( \sigma_{ WT}^k(r_k^t) \), \( \sigma_{ L}^k(r_k^t) \) could be obtained.

In the considered EI system, controllers are set in MTs, FCs and ERs only. ODEs have been used to model power dynamics of MTs, FCs, BESs, ERs and oscillations of AC bus frequencies in many works; see, e.g., [10], [46]. In this paper, the ODE-based modelling approach is also adopted.

Let us denote \( u_{MT}, u_{FC}, u_{ER} \) as control inputs for MTs, FCs, ERs, respectively. For the \( k \)-th MG, the power dynamics of MTs, FCs, BESs, ERs are presented in (4), (5), (6) and (7), respectively, and the frequency deviation is expressed in (8).

\[
p_{MT}^k = \frac{1}{T_{MT}^k(r_k^t)}[-p_{MT}^k + b_{MT}^k(r_k^t)u_{MT}^k],
\]
\[
b_{FC}^k = \frac{1}{T_{FC}^k(r_k^t)}[-p_{FC}^k + b_{FC}^k(r_k^t)u_{FC}^k],
\]
\[
p_{BES}^k = \frac{1}{T_{BES}^k(r_k^t)}[-p_{BES}^k + b_{BES}^k(r_k^t)u_{BES}],
\]
\[
p_{ER}^k = \frac{1}{T_{ER}^k(r_k^t)}[-p_{ER}^k + b_{ER}^k(r_k^t)u_{ER}^k] + v_{ER}^k,
\]
\[
j^k = -2\Delta f(r_k^t) + \frac{2}{M^k(r_k^t)}\Delta P^k,
\]

where \( b_{MT}^k(r_k^t), b_{FC}^k(r_k^t), b_{BES}^k(r_k^t) \) are time-variant coefficients for the control inputs, which are determined by the mechanical characteristics of these devices. Due to the communication delay and the limited energy capability of ERs, the power adjustment of ERs might be disturbed against the control input. Thus, in (7), the term \( v_{ER}^k \) is used to represent the disturbances existing in the power transmission of ERs.

We denote \( \Delta P^k \) in (8) as the total power deviation within the AC bus of the \( k \)-th MG. Considering the power balance in each MG, we have

\[
\Delta P^k = P_{PV}^k + P_{WT}^k + P_{MT}^k + P_{FC}^k - P_{L}^k \pm P_{BES}^k + P_{ER}^k.
\]

where \( P_{ex}^k \) is the total energy transmitted from other MGs to the \( k \)-th MG. Based on the topology of ER network in the considered EI system, we are able to assign different numbers as labels for the transmission lines in the ER network. By denoting \( P_{ER}^k \) as the power transmitted via the \( p \)-th transmission line, we are able to calculate \( P_{ex}^k \), based on the topology of the considered EI system. The dynamic model for \( P_{ER}^k \) is presented in (7).

Since power outputs by PVs, WTs and loads vary stochastically according to various factors, e.g., the change of weather in different time of a day, (1) – (3) are only valid for short-term power dynamics of the considered devices, e.g., 5 minutes. In this paper, it is assumed that the short-term dynamics of PVs, WTs and loads can be approximated by linear SDEs.

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Fig. 1. A general EI topology.
with jumping parameters and external disturbance inputs. We assume that there exist several typical parameter sets which could be estimated along with the Markov chains simultaneously. By utilizing the power forecast results obtained with advanced modelling methodologies for DERs and loads; see, e.g., [47], [48], we are able to establish our model for the EI system for a longer period.

C. Dynamical Power Modelling for EI

As long as the power dynamics of each component in EI are formulated in (1) – (8), let us rewrite the dynamical equation of the entire EI system in an explicit formula.

In this paper, it is assumed that, for any MG in the considered EI, there exist switching modes in its power dynamics. Thus, the parameters in (1) – (3) would change when mode alteration occurs. Based on the observation of real-world power data in [44] and the nature of continuous time Markov chain, in most cases, there exist few drastic parameter change in the considered MGs. In this sense, during the time when system parameters for the considered EI stay unchanged, we are able to apply control approaches for stochastic systems with constant parameters.

In this paper, a new control method for the considered EI system is proposed. Firstly, advanced parameter estimation and identification techniques, see, e.g., [42], [43], [45], could be employed to identify the system modes for MGs. Assuming that the identification results are already obtained, based on the results, the entire EI system can be described with a linear SDE with time-invariant parameters within a short period.

Suppose that during \( t \in [0,T] \), no mode change occurs in the EI system, and all the parameters can be regarded as constants. Since the dynamics for MGs and ER network are modelled with linear differential equations shown in (1) – (8), they can be rewritten into an explicit form as follows,

\[
dx = [A(r_t)x + B(r_t)u + C_t]dt + D(r_t)dW,
\]

in which,

\[
x = [P_{PV}^1, P_{WT}^1, P_L^1, P_{MT}^1, P_{FC}^1, P_{BES}^1, f^1, \ldots, P_{PV}^n, P_{WT}^n, P_L^n, P_{MT}^n, P_{FC}^n, P_{BES}^n, f^n, P_{ER}^1, \ldots, P_{ER}^n]^T
\]

is system state,

\[
u = [u_{MT}^1, u_{FC}^1, \ldots, u_{MT}^n, u_{FC}^n, u_{ER}^1, \ldots, u_{ER}^n]^T
\]

is system control input,

\[
v = [v_{ER}^1, \ldots, v_{ER}^n]^T
\]

is system disturbance input,

\[
W = W_{PV}^1 = W_{WT}^1 = W_L^1 = \ldots = W_{PV}^n = W_{WT}^n = W_L^n
\]

is Weiner process. In (10), \( A, B, C \) and \( D \) are system parameters obtained from individual dynamic models of the EI system.

III. THE MIXED \( H_2 / H_\infty \) CONTROL APPROACH

In this section, the problem of short-term operation cost optimization and system stabilization in EI is formulated as the mixed \( H_2 / H_\infty \) control problem.

First, we formulate the problem of short-term operation cost optimization as a \( H_2 \) control problem. We define the main short-term operation cost of EI as the summation of the following three aspects: the cost of utilizing BESs, extra cost involved by controllers and power transmission cost via any pair of interconnected MGs. The cost function of \( H_2 \) performance is defined as follows,

\[
J_1 = \mathbb{E} \left[ \int_0^T [\varepsilon_1 \sum_k (P_{BES}^k)^2 + \varepsilon_2 \sum_p (P_{ER}^p)^2 + \varepsilon_3 \sum_k [(u_{MT}^k)^2 + (u_{FC}^k)^2] dt] \right]
\]

where constants \( \varepsilon_1, \varepsilon_2 \) and \( \varepsilon_3 \) are weighting coefficients, \( \mathbb{E} \) stands for mathematical expectation. The detailed explanation for each explicit term in (11) is as follows.

Since long-term charging or discharging of BESs would lead to losses of battery’s service life [49], a rational utilization of BESs is urged, in the sense that BESs shall be used only when necessary. One typical example of irrationally utilizing BESs is given as follows for illustrative purpose. For any MG in EI, if the amount of power generated by PVs and WTs is constantly large, and its interconnected MGs are not lack of power, the BESs in the considered MG are still discharging unnecessarily. We claim such energy management strategy to be irrational. In order that RESs in MGs can be utilized with priority, BESs shall be regarded as the supplementary power supplier. Meanwhile, any unnecessary large-scale power input/output via BESs shall be avoided. In (11), the term \( \mathbb{E} \left[ \int_0^T \varepsilon_1 \sum_k (P_{BES}^k)^2 dt \right] \) stands for the cost of utilizing BESs. It is notable that such formulation has been used in many works; see e.g., [49], [50].

On the other hand, irrational utilization of ERs would also lead to additional costs [21]. Besides, according to the bottom-up principle in EI, power supply-demand balance should be achieved within local MGs with priority, and only if the local power balance cannot be maintained, energy routing within wide area network shall be implemented. For detailed explanation and discussion on the bottom-up principle in EI, readers can refer to [21], and the references therein. In (11), the term \( \mathbb{E} \left[ \int_0^T \varepsilon_2 \sum_p (P_{ER}^p)^2 dt \right] \) stands for the cost of utilizing ERs. By minimizing the value of such term, the adjustment for energy exchange via ERs within the whole considered EI scenario is minimized, which is beneficial for the achievement of bottom-up energy management principle.

In real engineering scenarios, the additional costs introduced by the controllers themselves are inevitable. Generally, strong controllers set in MTs and FCs can achieve satisfactory control effects. But the possible situation of over-control might bring damage to these devices, which may result in high costs for equipment maintenance. Thus, the cost brought by controllers shall be restricted properly, which is reflected in setting the term \( \mathbb{E} \left[ \int_0^T \varepsilon_3 \sum_k [(u_{MT}^k)^2 + (u_{FC}^k)^2] dt \right] \) in (11).
As long as the value of $J_1$ is minimized, the optimal energy management strategy for EI is achieved, in the sense that the considered short-term operation cost is controlled to a minimum amount. In addition to the $H_2$ performance, the $H_{\infty}$ performance of EI system is considered.

For the considered system in Fig. 1 electric power is assumed to be transmitted between MGs via DC transmission technology. Hence, the frequency deviation in the AC bus of each individual MG is independent [51]. It is notable that load fluctuation, wind power deviation and solar irradiation disturbance, damping coefficient and inertia constants can significantly influence the stability of frequencies in MGs. To alleviate such frequency fluctuations, the frequency regulation significantly influence the stability of frequencies in MGs. To

We are able to rewrite the entire system with the form shown in [19], [52], the $H_{\infty}$ performance of EI is defined as follows,

$$J_2 = E \left[ \int_0^T \left[ - \sum_p (v_{p,ER}^p)^2 + \gamma^2 \sum_k (\Delta f_k^p)^2 \right] dt \right].$$  

(12)

Next, by considering both $H_2$ and $H_{\infty}$ criteria simultaneously, we formulate the mixed $H_2/H_{\infty}$ control problem which is defined as follows.

We denote $\mathcal{U}$ as the set for all feasible controllers for system (10). Similarly, $\mathcal{V}$ is denoted as the set for all possible disturbance inputs of system (10). If there exist a pair of controller $u^*(x,t)$ and disturbance $v^*(x,t)$ for system (10), such that for any $u \in \mathcal{U}, v \in \mathcal{V}$,

$$J_1(u,v^*) \geq J_1(u^*,v^*),$$  

(13)

$$J_2(u^*,v^*) \geq J_2(u^*,v),$$  

(14)

holds, then $(u^*,v^*)$ is called a $H_2/H_{\infty}$ solution to the mixed $H_2/H_{\infty}$ problem. The inequalities in (13) and (14) indicate that, when $v^*$ is used as the disturbance input for (10), $u^*$ is the best possible controller that shall be able to minimize the $H_2$ performance. On the other hand, when $u^*$ is applied to (10), $v^*$ is the worst case disturbance which will result in the maximum value of the $H_{\infty}$ performance index $J_2$. In this sense, $u^*$ would be the desired mixed $H_2/H_{\infty}$ controller if there exists only one pair of such $H_2/H_{\infty}$ solution.

Once parameters of MGs in the EI system are determined, we are able to rewrite the entire system with the form shown in (10). The $H_2$ and $H_{\infty}$ performance $J_1$ and $J_2$ in (11) and (12) can be rewritten as the following forms,

$$J_1 = E \left[ \int_0^T \left[ x'Mx + \varepsilon uu' \right] dt \right],$$

$$J_2 = E \left[ \int_0^T \left[ \gamma^2 x'Fx - v'v \right] dt \right],$$

where $x,u$ and $v$ are of the same definitions as the ones introduced in (10), and $M, F, \varepsilon$ can be obtained via matrix transforming techniques. The dynamical system (10) of the proposed mixed $H_2/H_{\infty}$ control problem are inconsistent with the required form in classic mixed $H_2/H_{\infty}$ control problem. However, it could be transformed to the compatible one without essential difficulty. We denote $I$ as a vector with proper dimension, and all of its elements are assigned to be 1. By simply expanding the state variable $x$ as $X = [x', I]'$ and expanding the corresponding coefficient matrices with zero matrix $0$ (with proper dimension),

$$\hat{A}(r_t) = \begin{pmatrix} A(r_t) & 0 \\ 0 & 0 \end{pmatrix}, \hat{B}(r_t) = \begin{pmatrix} B(r_t) \\ 0 \end{pmatrix}, \hat{C} = \begin{pmatrix} C \\ 0 \end{pmatrix},$$

$$\hat{D}(r_t) = \begin{pmatrix} 0 & \hat{D}(r_t) \\ 0 & 0 \end{pmatrix}, \hat{M} = \begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}, \hat{F} = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix},$$

we have

$$dX = [\hat{A}(r_t)X + \hat{B}(r_t)u + \hat{C}v]dt + \hat{D}(r_t)XdW.$$  

(15)

As mentioned above, it is assumed that we have already obtained the parameter mode identification results via observation of the system. In this sense, at each time $t$, the state $r_t$ of the Markov chain could be estimated. During the period that the system stays at one certain state $r_t$, the system parameters, i.e., $\hat{A}(r_t), \hat{B}(r_t), \hat{C}$ and $\hat{D}(r_t)$, are actually fixed. So, the mixed $H_2/H_{\infty}$ controller $u(x,r_t)$ could be calculated with Theorem 1 provided in Appendix A.

Such procedure is depicted in Fig. 2. The identification results $r_t$ of the system parameters are obtained based on the measurements from smart meters deployed in the EI system, and many different techniques could be applied in this task. Then, regarding the obtained system parameters, the mixed $H_2/H_{\infty}$ controller $u(x,r_t)$ are calculated according to the mixed robust and optimal control scheme in Theorem 1. Finally, the controller is applied in the EI system. In this manner, the mixed $H_2/H_{\infty}$ control for the Markovian jumping stochastic EI system is achieved.

Notice that dynamic system (10) would only be valid for a limited time period, for each short time segment, the corresponding desired controller can be obtained with the proposed method. By continuously performing the above calculations for all the short time segments, we are able to achieve a long-term optimal and robust performance for the entire EI system.

IV. NUMERICAL EXAMPLES

In this section, we solve the mixed $H_2/H_{\infty}$ control problem based on typical system parameters in real-world engineering scenarios. Based on the modelling for MG dynamics in Section II, it is clear that, for each MG, there exist a negative feedback law in BES power dynamics. Thus, the frequencies of the considered AC MGs would fluctuate within small ranges. Intuitively, without violent disturbance inputs or strong stochastic
deviations in power dynamics of PVs, WTs and loads, the EI system would maintain stable even with an $H_2$ controller for MTs, FCs and ERs. In order to show the effectiveness of the proposed method, the performances under the proposed mixed $H_2/H_{\infty}$ controller are compared with the results when a classic $H_2$ controller is employed. The simulations are implemented based on Python.

For illustrative purpose, we consider an EI composed of four MGs interconnected via ERs, whose specific connection topology is shown in Fig. 1. Each MG consists of PVs, WTs, MTs, FCs, BESs and loads. We presume that the EI works at the balanced state, meaning that the power balance in the EI system is achieved, and the frequency oscillations are mainly related to the stochastic power fluctuation of RESs and loads.

TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s)</th>
<th>Parameter</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_L$</td>
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<td>$\sigma_L$</td>
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</tr>
<tr>
<td>$T_{PV}$</td>
<td>1.3</td>
<td>$\sigma_{PV}$</td>
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<tr>
<td>$T_{WT}$</td>
<td>1.2</td>
<td>$\sigma_{WT}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$T_{MT}$</td>
<td>0.2</td>
<td>$b_{MT}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_{FC}$</td>
<td>0.3</td>
<td>$b_{FC}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_{ER}$</td>
<td>0.2</td>
<td>$b_{ER}$</td>
<td>1.0</td>
</tr>
<tr>
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</tr>
<tr>
<td>$M$</td>
<td>1.8</td>
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</tr>
</tbody>
</table>

The trajectory of system mode transition corresponding to the typical parameters of the considered EI system are shown in Table I. Without loss of generality, it is assumed that the state of the Markov chain for system parameters could be obtained with certain parameter identification approaches at high precision. Thus, we are able to apply appropriate controller in the EI system at different periods. The trajectory of system mode transition corresponding to the numerical simulation setting is illustrated in Fig. 3.

The frequency deviation curves in different MGs with the proposed $H_2/H_{\infty}$ controller are shown in Fig. 4. It is clear that the AC bus frequency deviations in each MG are effectively alleviated. The power dynamics of BESs, MTs and FCs under the proposed $H_2/H_{\infty}$ control scheme are illustrated in Fig. 5 and Fig. 6 respectively. It can be found that, under the proposed control scheme, parts of the drastic power deviations on the AC bus can be properly absorbed by MTs and FCs. Thus, fluctuations in the charge/discharge power of the BESs can be limited, which suggests that the BESs can be protected by adjusting the power outputs of MTs and FCs.

![Fig. 3. State transition of system parameters.](image)

![Fig. 4. Frequency deviations in EI system under $H_2/H_{\infty}$ control.](image)

![Fig. 5. Power of BESs under $H_2/H_{\infty}$ control.](image)

![Fig. 6. Power of MTs under $H_2/H_{\infty}$ control.](image)
Similarly, the power deviations of ERs with disturbance inputs are depicted in Fig. 8, where $P_{ER_{i,j}}$ denotes the power transmitted from $MG_i$ to $MG_j$. According to the curves in Fig. 4 and Fig. 8, one can infer that, the impacts from disturbances in ERs on power bus frequencies in MGs are successfully restricted. At the same time, the power exchange via ERs could help the rational utilization of the power generation devices and BESs. With the proposed $H_2/H_{\infty}$ control scheme applied, the MGs could better utilize the advantages from the ER networks without significant detraction of frequency stability.

To show the advantage of the proposed $H_2/H_{\infty}$ control method over the conventional $H_2$ control method in the frequency regulation problem, in Fig. 9, the frequency fluctuations in $MG_1$ under these two different control strategies are plotted. The notation $\Delta f_i^* \star$ refers to the frequency deviation in $MG_1$ under the $H_2/H_{\infty}$ control scheme $u^*$ proposed in this paper. In the meantime, the frequency deviation in $MG_1$ under a classic $H_2$ controller $u^*$ is illustrated as $\Delta f_i^*$ in Fig. 9. Specifically, with the disturbance input $v$ in (10) omitted, the corresponding $H_2$ controller is obtained via optimizing the weighted sum of objectives $J_1$ and $J_2$, i.e., $J_1 + \gamma^2 J_2$. It is obvious that the proposed $H_2/H_{\infty}$ control method has better frequency regulation performance. As delineated in Fig. 9, the corresponding frequency deviations of $\Delta f_i^*$ have been limited within a relatively smaller range compared with $\Delta f_i^\star$.

In the meantime, the adjustments to the MT power outputs in $MG_1$ when the aforementioned two control methods are applied are illustrated in Fig 10. Clearly, though the proposed $H_2/H_{\infty}$ controller $u^*$ can achieve higher frequency stability for multi-microgrid systems, it would require more drastic and frequent adjustments in controllable generators like MTs, which will thus lead to higher operation costs to the considered energy internet system. In contrast, only moderate level of power adjustments for MTs are conducted by the classic $H_2$ controller $u^*$ in Fig 10. This is related to the property of the $H_2/H_{\infty}$ control scheme $u^*$. By the definition in (13) and (14), $u^*$ only ensures its optimality when the worst disturbance $v^*$ is imposed on the system (10). In this sense, $u^*$ may not guarantee its corresponding operation costs measurement $J_1$ to be the minimum in other cases.

In summary, by evaluating the controllers obtained from Theorem 1 with numerical simulations, the advantages and validity of our proposed method is demonstrated.

V. CONCLUSIONS

In this paper, the frequency regulation problem for a typical EI system is investigated. The dynamics of the considered multi-microgrid system are modeled with SDEs driven by
Brownian motions, and the complex patterns exist in power deviations are modelled as Markovian jump noises. In order to achieve the rational utilization of controllable devices like MTs and ERs as well as stabilizing the frequency fluctuations on AC buses, a novel $H_2/H_{\infty}$ control scheme with Markovian jump is proposed. With the numerical example provided in this paper, the feasibility and efficacy of the proposed control scheme is evaluated. Based on the simulation results presented in this paper, both of the frequency regulation target and the short-term costs minimization target can be properly achieved, which demonstrates the effectiveness of the proposed method.

In this paper, for the Internet layer, we have developed a centralized control method. The proposed strategy should rely on a central controller, and once the control center is under cyber-attack, the security of the whole EI system is risky. Compared with the distributed control method, under which case each interconnected microgrid does not need to disclose full private information with others, more attention should be paid on cyber security when the centralized method is implemented in real engineering scenario. In addition, in the Internet layer, the computation time and cost is also worth consideration, especially when the scale of the control problem is relatively large. This is also a limitation or restriction of the proposed centralized control method. Nevertheless, the performance of decentralized control approaches suffers from problems like low precisions and slow convergence speed as well. Thereby, we should consider both centralized and distributed control methods simultaneously in our future research.

APPENDIX

A. The Mixed $H_2/H_{\infty}$ Control Theorem

Theorem 1 ([15]): For the EI system ([15]), if the coupled differential Riccati equations in ([16]) has one solution $(P_1, P_2, K_1, K_2)$ such that $P_1(T) = 0$, $P_2(T) = 0$, $P_1(0) \geq 0$, $P_2(0) \geq 0$, then the solution to the mixed $H_2/H_{\infty}$ control problem is $u^* = K_2 x$ and $v^* = K_1 x$.

\[
\begin{align*}
\dot{P}_1 &= -\tilde{F} + \tilde{D} P_1 \tilde{D} + \gamma^2 K_1^T K_1 + 2 P_1 \hat{A} + 2P_1 \hat{B} K_2 + 2P_1 \hat{C} K_1, \\
\dot{P}_2 &= \hat{M} + \hat{D} P_2 \hat{D} + \varepsilon K_2^T K_2 + 2 P_2 \hat{A} + 2P_2 \hat{B} K_2 + 2P_2 \hat{C} K_1, \\
K_1 &= -\gamma^{-2} \hat{C}^T P_1', \\
K_2 &= -\varepsilon^{-1} \hat{B}^T P_2'.
\end{align*}
\]

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