

APPLICATION, PREDICTIVE TEST, AND STRATEGY IMPLICATIONS FOR A DYNAMIC MODEL OF CONSUMER RESPONSE*

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This paper describes and evaluates the application of a dynamic stochastic model of consumer response. The model describes, then forecasts, how consumers respond to a new transportation service and to the marketing strategies used during its introduction. The model is estimated on survey data during the first 11 weeks of service. Forecasts over the next 19 weeks are then compared to actual ridership as measured by dispatch records.

The model is simple. At any point in time, consumers are described by a set of 'behavioral states', indicating (1) whether they are aware of the new service (DART) and (2) what mode of transportation was used for their last trip. Behavior is described by movement among behavioral states. E.G., If a car user tries DART, he makes a transition from 'car used for last trip' to 'DART used for last trip'. The transition probabilities and the rate of transition are dependent on marketing strategies (direct mail, publicity), word of mouth, consumer perceptions, availability of a mode, and budget allocation to transportation.

The advantages and disadvantages of the model and the measurements are discussed with respect to predictive ability and managerial utility.

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1. Research Goals

Our ability to model consumer response advances through the interaction of theory, methodology, and practice. In a recent article, Hauser and Wisniewski (1982), we developed a methodology to integrate diverse mathematical models in stochastic brand choice, diffusion of innovations, test market analysis, and some aspects of information flow. Theoretically, the methodology shows promise as a means to build and test practical models of consumer behavior, but the ability of such a model to explain and predict actual consumer behavior in a managerially relevant environment remains an empirical question.

The purpose of this paper is to address the empirical question by applying and evaluating a model based on the methodology. Because this paper is empirical, a number of tradeoffs (sample size, questionnaire design, data collection strategy, behavioral states, explanatory variables, bias corrections, etc.) were necessary to construct the empirical realization of the theoretical model. We made one set of judgements. Other researchers with different goals and philosophies might make different empirical judgments. Thus, we estimate the model based on survey data and compare predictions to actual sales obtained from unobtrusive observation of consumer behavior. In this way, survey errors and specification errors work against good predictive ability. Finally, so that other researchers can test alternative models the raw data are available at cost upon request from the U.S. Department of Transportation as indicated in the acknowledgements.

Since a review of the literature and a detailed technical derivation of the theoretical methodology are published in Hauser and Wisniewski (1982), we do not repeat them here. Instead Appendix 1 briefly summarizes the key results. For a review of transportation demand models see Ben-Akiva and Lerman (1982).

We begin with the research context.

2. Research Context

The model was developed to evaluate the impact of a marketing campaign used to support the introduction of an innovative transportation service in Schaumburg, IL. The Village of Schaumburg is a northwest suburb of Chicago with a population of approximately 51,000 people (16,000 households). Schaumburg covers a 6 mile $\times$ 7 mile area consisting primarily of single family homes but with some newer apartment and condominium buildings. There is no large central business district, but Schaumburg does contain one of the largest shopping malls in the midwest. The existing transportation system consists of commuter rail lines to downtown Chicago and limited conventional bus service (6 vehicles over 6 routes in peak hours, 1 vehicle over 1 route in the off-peak hours) serving approximately 200 roundtrips per day. There are an average of 1.8 automobiles per household in Schaumburg.

The transportation innovation is a demand responsive dial-a-ride service called DART which was funded by the Urban Mass Transportation Administration (U.S. Department of Transportation) in cooperation with the Chicago

area Regional Transportation Authority (RTA) and the Village of Schaumburg. The primary mode of operation is for passengers to call a dispatcher who arranges for them to be picked up and brought to their destination with stops along the way to serve other passengers. The service began officially¹ on October 15, 1979, with four 22-passenger white school buses operating a total of 28 vehicle hours per day. Service was available from 9:00 A.M.–5:30 P.M. Monday through Friday. The fare was 80¢ with half-fare available to the elderly, the handicapped, and students on their way to or from school or school-related activities.

The marketing consisted primarily of newspaper publicity and information brochures (see Appendix 2) distributed at banks, schools, and other locations. On December 1 and March 3, the Village used direct mail promotions based on the information brochures.

There were two levels of managerial goals. On the local level, the transit manager was interested in evaluating and improving his marketing strategy and in determining whether service improvements (more vehicles, longer hours, etc.) were necessary to achieve the ridership goals of the Village. On the national policy level, the DART service was part of a federal program of service and method demonstrations (SMD's). The U.S. Department of Transportation was interested in evaluating the impact of marketing for use in future SMD's. In addition, they shared our goal in testing the predictive accuracy of a simple dynamic model of consumer response. To the extent that the simple model predicts well, an evolution of that model with improved, more efficient data collection could be used for evaluation of potential dial-a-ride sites.

The operating plan (vehicle hours, dispatching strategy) was constant² throughout the model evaluation although system performance varied. Operating changes to improve service were made after the model evaluation period, partially as the result of the model's implications. Thus our explanatory variables include marketing strategy (direct mail, publicity), diffusion phenomena (word of mouth) and measures of the impact of the variation in system performance.

3. Consumer Model

Loyal transit usage does not develop overnight. Consumers must first become aware of the new service. Even if they become aware of the service, they will not all try it immediately. Instead ridership (at least trial) is likely to grow over time as those who are aware of the service try it. Marketing theory suggests these awareness and trial processes are not automatic but influenced by marketing strategies and system performance.

¹There was a two-week, no-fare, no-promotion service from October 1 to October 12 to familiarize the dispatchers and drivers with the mode of operation and with the Schaumburg street system.

²New vehicles replaced the school buses during the predictive period but not the estimation period. Such an unmodeled change in operating strategy represents noise in the predictive period and biases the model against good prediction. Thus if we predict well despite not modeling vehicle change, we could expect a model which includes change to predict as least as well.

Consider a marketing strategy of mailing information brochures to all households in the target market. The impact of this strategy can occur in many ways. First, those who read the brochure will become aware of DART, its hours of operation, and its fare. In addition to communicating the characteristics of the system, the brochure may also contain persuasive messages to influence a consumer's preference for DART. But no one would expect the marketing strategy to cause 100% awareness, encourage 100% trial, or assure 100% repeat. Some consumers may not receive the brochure, some may not read it, some may read it but not seriously consider the information, and some may seriously consider DART and reject it as not filling their needs. Furthermore, the impact will be greatest in the week of the mailing and decay as time passes.

It is possible to describe this process by a complex model of information processing including "behavioral states" for awareness, full information, intent, trial and repeat. (See Bettman [1979].) Instead, we choose a simpler beginning model describing consumers as either unaware of DART or aware of DART. In addition to their level of awareness, at any point in time consumers can also be classified by how they made their last trip. For example, for those aware of DART the last trip was either by DART, by the existing bus system (BUS), or by car. (Models for other communities might also include taxi, walk or bicycle but an earlier survey indicated that the use of these transportation modes in Schaumburg was negligible compared to car and transit.) Since the Village believed that those who were passengers in cars were more likely to switch to transit than those who drove cars, we classify car users as either car drivers or car passengers. Thus we can describe each consumer at each point in time as being in one of the seven "behavioral states" as indicated in Figure 1.

Consumer behavior is movement from one behavioral state to another including self-flows. For example, before a consumer becomes aware of DART he "flows" among the three modes of transportation in the box in the left of Figure 1. Each "flow" is an act of taking a trip by one mode. Over time as the result of advertising, word of mouth, and other variables consumers

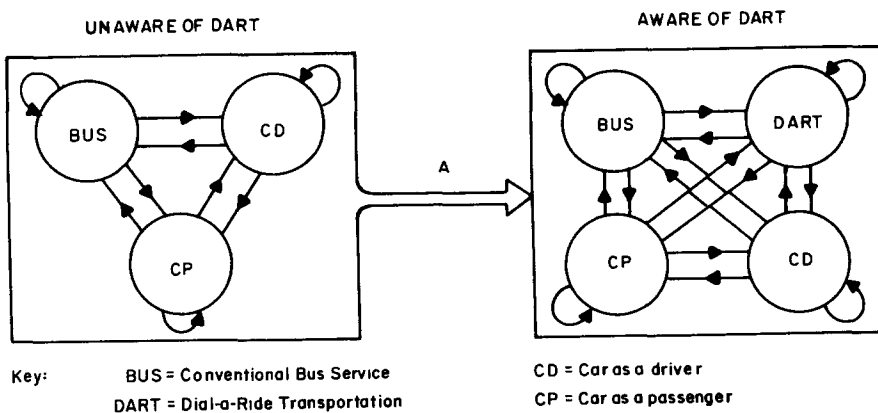


FIGURE 1. Conceptual Representation of Consumer Model.

become aware of DART as presented by the arrow marked 'A' in Figure 1. Once a consumer becomes aware of DART, his behavior is modeled by the four behavioral states in the box in the right of Figure 1. The whole process is continuous in time with flows probabilistically dependent on the explanatory variables.

Analytically we model Figure 1 as a semi-Markov process defined by (1) $f_i(t)$, the probability distribution of times, t , until a consumer who just entered state S_i will leave it, and (2) q_{ij} , the probability that the next state he flows to (from S_i) is state S_j . In marketing terms, $f_i(t)$ is the distribution of inter-purchase times and q_{ij} are the switching probabilities. The methodology can handle either Erlang or negative exponential distributions for $f_i(t)$, but previous work by Lerman (1979) suggests that transportation mode choice, with the possible exception of home-based trips, is best modeled by a negative exponential distribution, $f_i(t) = \mu_i \exp(-\mu_i t)$, where μ_i is the "flow rate" from state S_i .

When $f_i(t)$ is negative exponential,³ the semi-Markov process becomes a continuous time Markov process and can be summarized by a set of flow rates, $a_{ij} = \mu_i q_{ij}$, where $a_{ij} \Delta t$ is the probability that a consumer flows from S_i to S_j in time Δt . Since μ_i and q_{ij} can be recovered from the a_{ij} , i.e., $q_{ij} = a_{ij} / \sum_k a_{ik}$ and $\mu_i = \sum_k a_{ik}$, we deal directly with the flow rates, a_{ij} . Derivations are given in Hauser and Wisniewski (1982). (The notation $\sum^0$ implies special consideration for self-flows and is described in the appendix.)

At this point it is worth digressing to acknowledge the growing debate in marketing science as to whether or not state dependent switching probabilities are necessary to describe consumer behavior. Many researchers believe that behavior is not dependent upon the last product purchased, and hence, zero-order probability models will suffice. In our case, we are dealing with a new product and thus, as least, the awareness and trial-repeat flows are first-order Markov, even if the equilibrium process is zero-order.

We model the flow rate from state to state, a_{ij} , as a linear function of the explanatory variables. For example, the flow from "BUS/Unaware" to "DART/Aware" might depend on the number of direct mail pieces sent out, the amount of newspaper coverage, the availability of DART, and the probability that a consumer prefers DART to BUS. This preference probability is in turn dependent upon consumers' relative perceptions of DART and BUS. Note that in any time period, a consumer can make a number of transitions. E.G., on a given day he may ride the BUS to work, get a ride home, and in the evening read his mail, become aware of DART, and use it for a trip to the shopping mall. Thus during that day he would (1) flow into "BUS/Unaware", (2) flow from "BUS/Unaware" to "Car Passenger/Unaware", (3) flow from "Car Passenger/Unaware" to "Car Passenger/Aware" and (4) flow from "Car Passenger/Aware" to "DART/Aware."

³ Application of the model to frequently purchased consumer products might require Erlang rather than negative exponential interpurchase times. See Jeuland, Bass and Wright (1980), Massy, Montgomery, and Morrison (1970), and Zufryden (1980). The methodology can handle Erlang distributions with a modification in state definitions. See Hauser and Wisniewski (1982) for details.

Analytically, let x_{ijn} be the value of the l th explanatory variable, say the percentage of people receiving a brochure, affecting the i to j flow in period n , and let w_l be the "importance" of the l th variable. We model the impact of the variables as

$$a_{ijn} = \sum_l w_l x_{ijn} . \quad (1)$$

For example, w_1 might be the "importance" of direct mail. Once we know the w_l parameters we can describe the system.

We postulate that flows from "unaware" to "aware" of DART are a function of direct mail, publicity and word of mouth. Flows among usage states (e.g., BUS/Aware to DART/Aware) depend on the relative availability of the modes, on the consumers' budget for transportation, and on consumers' perceptions of the various characteristics of the modes. Perceptions are in turn dependent on marketing and system performance. As described in §5, these hypotheses are based on Brunswik's model (1952) of consumer information processing but will be tested empirically. Mixed flows (e.g., BUS/Unaware to DART/Aware) are dependent upon all of the above variables. (As indicated in Figure 1, the consumer's behavioral state describes (a) his last mode used and (b) whether or not he is aware of DART.)

Estimation of Model Parameters

So far we have described a system that is continuous in time, but it is not feasible empirically to observe each and every transition, i.e., every flow. We can observe a series of discrete snapshots at $t = T_0, T_1, T_2, \dots, T_n$. (These do not need to be equal time intervals.) In each time period, T_{n-1} to T_n , we observe the number of consumers, C_{in} , in each state, S_i , at the start of the period, T_{n-1} , and the number of these, C_{ijn} , who end up in each state, S_j , at the end of the time period. C_{in} and C_{ijn} are readily obtained from panel data or, with some recall bias, from periodic surveys. For example, C_{45n} might be the number of consumers who were BUS users on November 1, and who are DART users on November 15.

Let $\tilde{p}_{ijn} = C_{ijn}/C_{in}$, i.e., $\tilde{p}_{ijn}$ is an estimate of the probability that a consumer is in S_i at T_{n-1} and in S_j at T_n . Let $\tilde{P}_n = \{\tilde{p}_{ijn}\}$ be the matrix of the $\tilde{p}_{ijn}$. Let $\tilde{a}_{ijn}$ be flow rates and let $\tilde{A}_n = \{\tilde{a}_{ijn}\}$ be the matrix of the $\tilde{a}_{ijn}$. Then Hauser and Wisniewski (1982) show that the maximum likelihood estimators, $\hat{w}_l$, of the w_l are approximated by the following regression equation:

$$\tilde{a}_{ijn} = \sum_l \hat{w}_l x_{ijn} + \text{error} \quad (2)$$

where $\tilde{A}_n t_n = \tilde{E}_n [\log \tilde{\Lambda}_n] \tilde{E}_n$, $\tilde{E}_n$ is the matrix of eigenvectors of $\tilde{P}_n$, and $[\log \tilde{\Lambda}_n]$ is a matrix with the logarithms of the eigenvalues of $\tilde{P}_n$ on the diagonal and zeros elsewhere.

Equation 2 allows us to estimate the “importance” weights, $\hat{w}_l$, by first using an eigenstructure computer package to transform the frequency data, C_m and C_{yn} , and then using ordinary regression to obtain the estimates, $\hat{w}_l$. Simulation suggests that good estimates can be obtained if (1) the average C_{yn} is greater than 20 and (2) $t_n = T_n - T_{n-1}$ is short relative to the time it takes the process to reach equilibrium.

When the data is available, equation (2) can also be applied to segments of consumers. For example, transportation behavior is often analyzed with different data collection and models for different trip purposes, i.e., work oriented travel, central business district (CBD) oriented travel, and general purpose travel. We used such a segmentation in Tybout and Hauser (1981). For the Schaumburg application, DART served primarily general purpose travel (There is no CBD in Schaumburg. DART did not serve work oriented trips.) and we felt trip purpose segmentation would not be necessary.

Statistics of Managerial Interest

To forecast we use the estimated importance weights and the observed explanatory variables to estimate the flow rates, i.e., $\hat{a}_{yn} = \sum_l \hat{w}_l x_{lyn}$ where $(\hat{\cdot})$ indicates estimate. The forecast flow probabilities, $\hat{p}_{yn}(t_n)$, are given by an eigenstructure formula given in the appendix.

Although the forecast flow probabilities can describe the system, a manager wants descriptions of consumer response that are more similar to the types of statistics with which he normally deals. Two important summary statistics that the model can provide are (1) cumulative awareness and (2) rides per period. Cumulative awareness is the total number of consumers aware of DART by period n . We compute cumulative awareness by calculating the number of consumers who flow out of the “Unaware” states. We compute rides per period by calculating the number of times any consumer flows into “DART/Aware”, including flows from “DART/Aware” back into “DART/Aware.” While the mathematics of computing these statistics is complex, algebraic formulae have been developed which are readily adaptable to computer analysis. See Appendix 1.

A final managerial question that is addressed by the model is what will happen in the long run if a strategy is continued. We call the long run ridership per period resulting from a stable strategy the “equilibrium” ridership. To compute equilibrium ridership we assume that the same managerial strategy is applied in every period and use the sales formulae to compute what happens as the number of periods becomes very large. The equilibrium ridership also can be calculated with a simple formula based on the estimated flow rates, $\hat{a}_{yn}$.

This completes our brief description of the model. Details will become more clear as we proceed to the empirical model development.

4. Data Collection

The primary data on which our analyses are based are a series of sixteen identical twelve-page mail surveys sent periodically and randomly to

Schaumburg residents and to residents of a neighboring community (Hoffman Estates) who are in the DART service area. The early surveys were mailed out at relatively short intervals (1 week) to be sensitive to rapid behavior changes likely to occur once service began. Later surveys were mailed at longer intervals (2 weeks, then 4 weeks) to enable us to track the behavior changes over the length of the demonstration project and to do so at a reasonable cost of data collection. Archival records of ridership, publicity and mailings were collected to establish some of the explanatory variables and to provide a non-survey test of our model's predictions of behavior. We describe each in turn.

Pre-Analysis

Prior to implementation of service, we performed a pre-analysis to help the Village establish the image they wished to portray for DART. The name of the service⁴ and its core benefit proposition were developed with this analysis. The pre-analysis was based on focus groups, a telephone survey to establish ridership patterns and a mail survey sent to 1500 residents. See Wisniewski (1981) for details.

For our purposes, this pre-analysis, combined with an earlier analysis in Evanston, Il. (Hauser, Tybout, and Koppelman (1981)) provided a measurement instrument with validated scales to measure usage and perceptions of transportation modes. Tybout and Hauser (1981) use archival ridership data to evaluate the Evanston forecasts which were made with a static version of the model in the left box of Figure 1. Their analyses provided further input to the development of the measurement instrument.

Periodic Surveys

In each observation period between September 20, 1979 and April 17, 1980 periodic surveys provided measures of perception and preference, provided the data necessary for the dependent measures used to estimate the dynamic model, and provided self-reported descriptions of media, mail promotion and word of mouth. The surveys were developed based on the pre-analysis, focus groups and extensive pretests. In fact, an abridged version of the survey was used to monitor a June 1979 strategy modification in the conventional bus system. Although that sample was small, the results suggest that the survey questions were sufficiently sensitive to identify changes in behavioral states. We describe the specific survey measures in §5.

Table 1 indicates the mailing dates, the sample sizes, and the response rates of the periodic surveys. The overall response rate of 30.4 percent (of which 91.2 percent were complete and usable) is a moderate response rate for mail

⁴DART (Dial-A-Ride Transportation) was chosen to connote a service that was everywhere (convenient) and provided speedy service. Prior to the analysis, the Village was considering STEP (Schaumburg Transportation Energy Conservation Program). The logo was to be a drawing of people riding in a shoe. Tests indicated that DART communicated the core benefit proposition better than STEP and other potential names.

TABLE 1
Mailing Dates and Response Rates of Periodic Surveys Used to Estimate and Test the Dynamic Consumer Model

Wave Number	Date Mailed	Sample Size	Percent Return	Estimation	Prediction
1	Sept. 20	320	26.7		
2	Oct. 4	400	29.8	✓	
3	Oct. 11	725	27.2	✓	
4	Oct. 18	700	30.0	✓	
5	Oct. 25	600	33.8	✓	
6	Nov. 1	750	36.8	✓	
7	Nov. 15	750	17.1	✓	
8	Nov. 29	750	31.9	✓	
9	Dec. 13	750	22.7	✓	
10	Dec. 27	750	30.9		✓
11	Jan. 10	600	33.7		✓
12	Jan. 24	600	33.7		✓
13	Feb. 7	600	34.7		✓
14	Feb. 21	600	35.3		✓
15	Mar. 21	600	34.3		✓
16	Apr. 17	500	30.2		✓

surveys (Hauser, Tybout, and Koppelman [1981] achieved a 41.2 percent return). Comparisons with a 1978 census in Schaumburg indicate a slight bias toward males (55%) and a slight undersampling of the elderly and students but no other significant differences at the .01 level. A further (but untestable) hypothesis is that there might be a slight bias in return toward those more interested in public transportation. In either case, such biases work against successful prediction of archival ridership and thus make the predictive test a more stringent test of the model.

Table 1 also indicates which surveys provided data for the estimation of the model and which surveys provided data for the predictive tests of the model. Period 1 was prior to implementation. Periods 2 through 9 provided the data to estimate the parameters of the model, the $\hat{w}_i$'s. Periods 10 through 16 provided explanatory variables only. Note that the model is developed based on the first 11 weeks of service and is used to predict ridership for the next 19 weeks.

Archival Data

The explanatory variables for media and promotion and the ridership counts to test the model were obtained by unobtrusive observation. We kept records on when articles on DART appeared in each newspaper, how long they were, and what percentage of Schaumburg residents subscribed to each newspaper. We also noted the dates and the coverage of both direct mail campaigns. Ridership was obtained from dispatch records. For each service call, the dispatcher recorded who rode the system, when they rode it, where they were picked up, and where they were dropped off. Finally, it is important to note that there was no service on holidays (Nov. 22, Dec. 25, Jan. 1).

5. Operationalization of the Consumer Model

To apply the consumer model, we must select measures to operationalize the dependent and the explanatory measures. In the Schaumburg analysis, we make a number of tradeoffs to develop a feasible model.

Behavioral States

We prefer the model in Figure 1, but our data does not contain consumer perceptions of BUS, car driver and car passenger for those consumers who are unaware of DART. Marketing strategies cause awareness and influence consumers to modify their ridership patterns. System performance has its greatest impact on consumers' ridership patterns. Based on discussions with representatives of the Village of Schaumburg and the RTA we selected the five-state model shown in Figure 2 as sufficient for the identification of managerial strategies affecting DART ridership. This model requires 16 independent nonzero flows (excluding self-flows). Since our data averages 200 consumers each making five trips per week, this is well within the simulation limits suggested by prior analysis (Hauser and Wisniewski, 1982).

Dependent Variables for Estimation Purposes

To describe a consumer, we must identify whether a consumer is aware of DART and what mode of transportation he has used last. To identify the percent of consumers, $\tilde{p}_{ijn}(t_n)$, that flow from state to state in period n we must identify which behavioral state describes a consumer at the beginning of an observation period, time T_{n-1} , and at the end of an observation period, time T_n . ($t_n = T_n - T_{n-1}$) Consumers who do not report awareness of DART at given observation time are assigned to the 'unaware of DART' behavioral state. Consumers who report awareness of DART are assigned to one of the other states according to the modes of transportation that they used.

Awareness. Awareness of DART at the end of the period (time T_n) was measured in survey n by a direct awareness question placed among awareness and familiarity questions about four existing services. Awareness of DART at the start of the period (time T_{n-1}) was measured in survey n by a recall question asking consumers when they first learned of DART. See Table 2. Each question is potentially biased. However, we are only measuring changes

	UD	CD/A	CP/A	B/A	D/A
Unaware of DART	✓	✓	✓	✓	✓
Car Driver /Aware		✓	✓	✓	✓
Car Passenger /Aware		✓	✓	✓	✓
BUS/Aware		✓	✓	✓	✓
DART/Aware		✓	✓	✓	✓

FIGURE 2. Behavioral States Used in the Model of Consumer Response to DART (✓ Indicates Potential Flows).

TABLE 2
Survey Measures of Awareness

(A) Direct Measure (time T_n , survey n)				
3. Are you aware of the following bus services in Schaumburg? (please check one box for each service)				
No, I am not aware of this service	Yes, but I am not sure of the details	Yes, I am thoroughly familiar with the details	Yes, and I have used this service	
Senior Citizen Bus	☐	☐	☐	☐
Rush-hour commuter bus to Roselle train station	☐	☐	☐	☐
Dial-A-Ride bus service (DART)	☐	☐	☐	☐
Regular RTA bus service	☐	☐	☐	☐
(B) Recall Measure (time T_{n-1} , survey n)				
6. When did you first learn of Dial-A-Ride bus service?				
☐ this is the first I've heard of the service	☐ within the last 2 weeks			
☐ within the last week	☐ over 2 weeks ago			

TABLE 3
Regression Equation Used to Adjust Recall Measure of Awareness to Direct Measure of Awareness (Dependent Measure is directly measured awareness, time T_{n-1} , survey $n-1$)

	Coefficient	Significance
Recall Awareness	.74	.00
Period Length—1 week	—	—
Period Length—2 weeks	.12	.03
Period Length—4 weeks	.17	.04
Target Sample	.11	.11
Daily Herald	-.08	.04
Voice of Schaumburg	.09	.13
Constant	.15	.15
R^2 (adjusted)	.92	.01

in awareness and are thus mainly concerned with the relative bias, i.e., the difference in bias between the two measures.

If there were no relative bias, the overall percent of consumers aware at the end of the $(n-1)$ th period, time T_{n-1} , should equal the overall percent aware at the beginning of the n th period, also time T_{n-1} . Examination of overall percentages indicates that relative to direct measurement the recall question underestimates awareness at time T_{n-1} .

In an attempt to adjust for the underestimated recall, we developed a regression model (periods 2 through 16) to modify the recall measure to match the direct measurement.⁵ See Table 3. Basically, the model in Table 3 estimates directly measured awareness (time T_{n-1} , survey $n-1$) as 74% of recall awareness (time T_{n-1} , survey n) modified by a series of (0,1) dummy variables that correct for the environment of measurement. "Period length" accounts for the fact that t_n varies from 1 week to 2 weeks to 4 weeks. "Target Sample" accounts for postal error resulting in partial non-delivery of surveys to a neighboring community in the first five periods. "Daily Herald" and "Voice of Schaumburg" account for media publicity which seem to affect the measurement *bias* as well as awareness. Because of the small sample size, we retained all variables even though the significance goes as high as the .15 level. This adjustment is necessary due to a relative bias in recall measurement of membership in a behavioral state. Note that while the coefficient of recall awareness alone is only 74%, the total effect of the regression in Table 3 is that adjusted recall awareness is a number greater than measured recall awareness.

Behavior. Simulation and analytic arguments suggest that the time period of observation, t_n , be short compared to the dynamics of interest. This is no problem for the typical consumer product with interpurchase times on the order of weeks. For transportation service interpurchase times are fractions of

⁵Ideally we would estimate the regression in table 3 on data from periods 2 through 9. Because we have but one observation per period we were forced to estimate the model on periods 2 through 16 for sufficient degrees of freedom. Since this is a measurement adjustment not a model estimation we felt our decision was justified.

days, short compared to the weekly observation periods. We overcome this potential problem by recognizing that the *pattern* of behavior (choice of mode portfolio) is likely to change at a much slower rate. This is particularly true with respect to DART which, for most consumers, is a mode that is used for occasional trips.

For those consumers who were aware of DART, the portfolio of usage of transportation modes was measured by self-reported frequency of use at T_n and at T_{n-1} . Unlike awareness, there was no overall relative bias identified for self-reported frequency of use. See Table 4.

At this point we must make an empirical approximation to the theoretical model. The statistics of interest are C_{ijn} and C_{in} where C_{ijn} equals the number of consumers who were observed in state S_i at time T_{n-1} and in state S_j at time T_n . But the DART manager is more interested in the total number of trips by DART than in the number of trips they make per week. (Note that C_{ijn} is not the number of S_i to S_j transitions nor a state-to-state transition probability, but a statistic resulting from two snapshots of a continuous time probabilistic process. See detailed discussion in Hauser and Wisniewski [1982].)

To implicitly weight consumers by the number of trips they make, the flow percentages, $\tilde{p}_{ijn}(t_n) = C_{ijn}/C_{in}$, were estimated as a weighted average of individual flow percentages. For example, suppose a consumer made 6 BUS trips during the week of November 29 and he made 4 BUS trips and 2 DART trips during the week of December 13. Let $S_4 = \text{BUS/Aware}$ and let $S_5 = \text{DART/Aware}$. See Table 2. Then, we count his contribution to C_{44n} as 4 and his contribution to C_{45n} as 2 where n corresponds to the data period from November 29 to December 13. (In Table 1, $n = 9$.) We count his contribution to C_{4n} as 6. If the consumer made 3 BUS trips and 3 DART trips during the week of December 27, then we count his contribution to $C_{44, n+1}$ as 2, $C_{45, n+1}$ as 2, $C_{54, n+1}$ as 1, and $C_{55, n+1}$ as 1. His contribution to $C_{4, n+1}$ is 4 and his contribution to $C_{5, n+1}$ is 2.

Whether or not this realization of the dependent variable, $\tilde{p}_{ijn}(t_n)$, adequately corresponds to the dynamic process in Figure 2 remains an empirical question. However, at the suggestion of the reviewers, we tested an alternative realization of $\tilde{p}_{ijn}(t_n)$ where each consumer was weighted equally. The comparisons are reported in §6.

Explanatory Variables—Marketing Strategy

The primary marketing strategies used by Schaumburg were direct mail and media publicity.

Archival Measures. We recorded when publicity appeared in each of the five Schaumburg newspapers (Schaumburg Daily Herald, Voice of Schaumburg, Record Newspaper, Chicago Tribune, and Chicago Sun Times) and we know the circulation of each newspaper within Schaumburg. Thus one archival measure is a series of dummy variables which take on (0,1) values depending upon whether or not publicity appeared in that newspaper during the observation period. A more accurate measure would be to weight the

TABLE 4
Survey Measures of Behavior

(A) Direct Measure (time T_n , survey n)

12. Within the last 7 days, how many round trips did you make within Schaumburg Township by each of the following means of transportation for a purpose other than going to a full-time place of employment.
(A round trip starts and ends in the same place; for example, home $\rightarrow$ destination 1 $\rightarrow$ destination 2 $\rightarrow$ home would be 1 round trip.)

	None	1	2	3	4	5-6	7 or more
Car as a driver	[]	[]	[]	[]	[]	[]	[]
Car as a passenger	[]	[]	[]	[]	[]	[]	[]
Dial-A-Ride	[]	[]	[]	[]	[]	[]	[]
Regular RTA buses	[]	[]	[]	[]	[]	[]	[]
Other (please specify)	[]	[]	[]	[]	[]	[]	[]

(B) Recall Measure (time T_{n-1} , survey n)

13. In the week just before this last week (i.e. 8-14 days ago), please estimate how many round trips you made within the Township for a purpose other than going to a full-time job by:

Used, but
don't recall

	None	how often	1	2	3-4	5-6	7 or more
Car as a driver	[]	[]	[]	[]	[]	[]	[]
Car as a passenger	[]	[]	[]	[]	[]	[]	[]
Dial-A-Ride	[]	[]	[]	[]	[]	[]	[]
Regular RTA buses	[]	[]	[]	[]	[]	[]	[]
Other (please specify)	[]	[]	[]	[]	[]	[]	[]

variables by their percent circulation. However, such a series of variables does not take into consideration the overlap in readership among the five newspapers.

To account for overlap in readership we created a composite variable to estimate the “reach” of the publicity, i.e., the percent of consumers who read at least one newspaper containing an article on DART. This variable was computed by standard rules of probability (assuming independent events). For example, if two articles appeared between time T_{n-1} and T_n , one in a newspaper with 60% circulation and one in a newspaper with 20% circulation, then the reach variable takes on a value of .68 representing .60 plus .20 minus the overlap $(.60)(.20)$.

Direct mail was measured as the percent of consumers to whom the brochure was mailed. Finally, we created an alternative “marketing” variable which included the direct mail variable in the calculation of reach.

Survey measures. The archival variables are actionable but may be less accurate than survey measures which can measure whether the consumer actually received and noted the direct mail brochure or the newspaper article. To test this hypothesis we used the alternative operationalizations of the marketing variables appearing in lines 1, 3, 5, and 8 of Table 5.

Explanatory Variables—Word of Mouth

The literature on the diffusion of innovations (Rogers and Shoemaker 1971) suggests that word of mouth can have a major influence on awareness and on the adoption of innovations.

Archival Measures. Bass (1969) has developed a parsimonious model to account for diffusion phenomena in consumer durable goods. This model has been applied in a variety of product categories (Dodds 1973, Nevers 1972) and has predicted well in many of those categories. In that model, Bass operationalizes word of mouth as the number of consumers who have already adopted the innovation. For DART, this operationalization corresponds to the total number of consumers (as measured from the archives) who have tried the system by time T_{n-1} . For forecasting purposes, this variable is endogenous to the model.

Survey Measures. We also tested an alternative operationalization which asked consumers to self-report whether they had received information about DART by “word of mouth from friends or associates.” See Table 5, line 7.

Explanatory Variables—Imbedded ‘Lens’ Model

Marketing strategies and word of mouth should impact awareness flows directly. Changes in usage patterns are more complex. Operating strategies and marketing strategies affect behavior but psychological theory (e.g., Brunswik [1952]) suggests that their impact on behavior is moderated by a series of intervening variables. This conceptual representation of the impact of marketing strategy is called the ‘Lens’ model. It is similar to models developed by Shocker and Srinivasan (1979), Hauser and Urban (1977), and Sternthal and Craig (1982, Chapter 4). See Figure 3. In the ‘Lens’ model, operating

TABLE 5
Alternative (Survey) Operationalizations of Marketing and Word of Mouth Variables

1. Consider <u>only</u> information and events of the <u>last 7 days</u> . How did the following effect your decision to try or use Dial-A-Ride in the last 7 days?	This occurred in the last 7 days, and had:			
	This did not happen in the last 7 days	A Negative Effect on my decision to try/use	No Effect on my decision to try/use	A Positive Effect on my decision to try/use
Mailings to my residence on Dial-A-Ride	☐	☐	☐	☐
Receipt of free-ride or multi-ride coupon(s)	☐	☐	☐	☐
Receipt of brochure/fact sheet	☐	☐	☐	☐
Calls to the Village Transit Manager	☐	☐	☐	☐
Reading newspaper articles on Dial-A-Ride	☐	☐	☐	☐
Saw the Dial-A-Ride buses in operation	☐	☐	☐	☐
Word of mouth from friends/associates	☐	☐	☐	☐
RTA advertising	☐	☐	☐	☐
Gas price increases/shortages	☐	☐	☐	☐
No other means of transportation available	☐	☐	☐	☐
Other _____	☐	☐	☐	☐

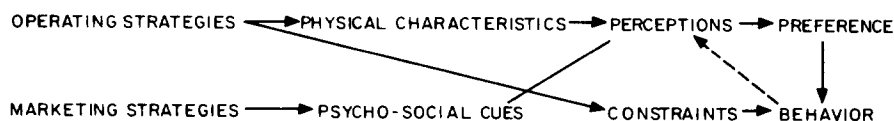


FIGURE 3. "Lens" Model Relating Operating and Marketing Strategies to Behavior Through Preference and Availability Constraints.

strategies and marketing strategies affect *objective* reality (e.g., physical characteristics of the transportation system such as travel time) and psycho-social cues (e.g., advertising). These influence *subjective* reality as represented by consumers' perceptions of the various means of transportation. Based on these perceptions, consumers form their preferences. Behavior is then based on preference but moderated by situational constraints such as availability or budgets. Feedback loops such as the dotted line in Figure 3 are also possible. For an application and evaluation of the conceptual model for transportation services and musical programs see Tybout and Hauser (1981) and Holbrook (1981) respectively.

In situations where operating strategies vary, we could measure and estimate the full 'Lens' model for diagnostic purposes or the reduced form model (strategies → behavior) for predictive purposes. In our situation operating strategies are constant throughout the estimation period. Physical characteristics such as travel time vary but were not directly measured due to cost considerations.⁶ Instead we used direct survey measures of perceptions and constraints to reflect the variation in system performance. In this way, we develop a model which can evolve through the addition of an external model linking operating and marketing strategies to perceptions. Such models are feasible and have been developed in other contexts. See Green, et al. (1981), Green and DeSarbo (1978), Hauser and Simmie (1981), Holbrook (1981), Neslin (1978), and Urban and Hauser (1980, pp. 248–255).

Perceptions. Perceptions are measured by having consumers evaluate car as a driver, car as a passenger, RTA bus, and DART on the series of eighteen agree/disagree scales developed to measure the constructs identified in the preanalysis. Table 6 shows the first of these eighteen scales as it appeared in the periodic surveys.

Since the eighteen scales are redundant measures of the perceptual constructs we use factor analysis⁷ to reduce the eighteen scales to four perceptual constructs: 'convenience', 'ease-of-use', 'safety', and 'general opinion'. 'Convenience' reflects the ability to "come and go as I wish", on time performance, the hassle in arranging for use, and the fit to the consumer's schedule. 'Ease-of-use' reflects whether the mode is not tiring, enjoyable, and easy to use in bad weather. 'Safety' includes the fear of crime and accidents. 'General

⁶The direct measurement of physical characteristics for all four modes of transportation is a non-trivial, labor intensive, expensive data collection process.

⁷The factor analysis is run across stimuli and subjects. The four factor solution was selected on eigenvalue and scree rules (Rummel 1970, pp. 361–365) and ease of interpretation. It explains 63.3% of the total variance.

TABLE 6
Example Scale to Measure Perceptions

		Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
It would require a lot of effort to travel around Schaumburg by:	Car as a driver	[]	[]	[]	[]	[]
	Car as a passenger	[]	[]	[]	[]	[]
	Dial-A-Ride	[]	[]	[]	[]	[]
	Regular RTA bus	[]	[]	[]	[]	[]

opinion' is a series of scales to measure beliefs about personal and social norms. Factor scores based on the eighteen scales are the explanatory variables used to represent the four perceptual constructs.

Preference. Preference is measured in the surveys by having consumers rank order the four modes of transportation in terms of preference. To use this construct in the model we want to be able to predict preference as a function of the measured perceptions. To accomplish this we use the multinomial logit model (McFadden 1980) which predicts the probability that a consumer with a given set of perceptions prefers a given mode. The model is estimated with maximum likelihood techniques where the dependent variable is a (0, 1) variable that is equal to the 1.0 if and only if consumer c prefers mode j to all other modes. See McFadden (1980). The explanatory variables are the four perception measures and a series of alternative specific constants that are included to insure consistent estimates. (An alternative preference model would be monotonic regression with rank order preference as the dependent variable. Our experience, e.g. Tybout and Hauser (1981), is that preference logit predicts better than preference regression in a transportation context).

As shown in Table 7, the estimated model does quite well in predicting preference based on the measured perceptions (82% of the consumers correctly predicted, 63% of the uncertainty explained).⁸ This model is estimated based on survey periods 2 through 9.

To create the explanatory variable that is used in the dynamic consumer model we use the logit equation to compute the estimated probability, L_{cj} , that consumer c prefers mode j . That is,

$$L_{cj} = \exp\left(\sum_k \beta_k y_{cjk} + \delta_j\right) / \sum_l \exp\left(\sum_k \beta_k y_{clk} + \delta_l\right) \quad (3)$$

where β_k are the estimated logit coefficients (Table 7) and y_{cjk} is the factor score estimating consumer c 's perceptions of mode j for perceptual dimension

⁸Percent correctly predicted is the percent of consumers who prefer the mode that the model predicts has the highest probability of being preferred. Percent uncertainty explained is an information-theoretic measure equal to the "information" provided by the model divided by the total information explainable. The latter is equal to the entropy which is the total uncertainty of the probabilistic system. See Hauser (1978).

TABLE 7
Preference Estimation Via Logit Analysis (Dependent measure is first preference)

	Coefficient	t-statistic
Convenience	.86	5.7
Ease of use	.64	5.8
Safety	.55	3.5
General Opinion	1.28	10.1
Constants		
Car Driver	2.07	10.2
Car Passenger	—	—
BUS	.68	2.4
DART	+	+
Percent Correctly Predicted	82.2	—
Percent Uncertainty Explained	62.5	—

+ Insignificant at the .05 level and dropped from the final model. The car passenger constant is arbitrarily set to zero since the constants measure relative effects.

k_i is the alternative specific constant for mode i . The coefficients are estimated based on data in periods 2-9. The same coefficients are used to forecast for periods 10-16.

Constraints. The final variable we need based on the 'Lens' model is a measure of the constraints faced by the consumer. Previous transportation research has identified availability as a major constraint on choice of mode. We operationalized availability with the scale in Table 8a. Economic theory suggests that a consumer's budget allocation to a product category is a major

TABLE 8
Constraints on Choice

(A) Availability Constraints				
12. In general, for your trips for a purpose other than going to a place of full-time employment, how readily available is: (please give your opinion for each means of travel)				
	Usually readily available	Usually available with some difficulty	Usually available, but with great difficulty	Usually not available
Car as a driver	[]	[]	[]	[]
Car as a passenger	[]	[]	[]	[]
Dial-A-Ride	[]	[]	[]	[]
Regular RTA buses	[]	[]	[]	[]
Other	[]	[]	[]	[]
(please specify)				
(B) Budget Allocation to Transportation				
10. Do you consider transportation costs to be:				
[] a large part of your yearly household expenses	[] a moderate part of your yearly household expenses	[] a small part of your yearly household expenses		

constraint on choice (Blackorby, Primont and Russell 1975). We operationalize budget allocation with the categorical scale in Table 8b.

Preference Inertia. Neslin (1976) showed empirically that consumers exhibit an inertia factor when considering innovative services, i.e., their delay in trying a new service is more than could be explained by their relative preferences among the new and existing services. We operationalize inertia by a (0, 1) variable which is equal to 1.0 for all flows into DART/Aware and to 0.0 otherwise. If this variable is significant and negative then it could represent an otherwise not modeled hesitancy on the part of consumers to try the innovation. If this variable is significant and positive it could represent an otherwise not modeled bias to try the innovation.

6. Model Estimation

§5 provides measures of the dependent variables, $\tilde{p}_{ijn}(t_n)$, and the explanatory variables. (To review, $\tilde{p}_{ijn}(t_n)$ is the percentage of consumers who are in S_i at time T_{n-1} and in S_j at time T_n where $t_n = T_n - T_{n-1}$; x_{ijlnc} is the value of the l th explanatory variable takes on for the i to j flow in period n for consumer c . We choose the macro-flow model described in Hauser and Wisniewski (1982) and hence use representative variables, $r_{ijn} = (1/C_{in}) \sum_c x_{ijlnc}$ where C_{in} is the number of consumers in state S_i at the beginning of period n .) We use the regression approximation to obtain the estimates, $\hat{w}_l$, of the parameters of the system. (Review equation (2).) The dependent variables in the regression are the $\tilde{a}_{ijn}$'s obtained from the C_{ijn} 's and C_{in} 's. The independent variables are the representative variables, r_{ijn} , which measure the average effect of marketing strategy, word of mouth, preference, constraints, and preference inertia. These variables take on different values for different i - j combinations. E.g., for preference,

$$r_{ijn} = \bar{L}_{jn} / \bar{L}_{in}, \quad \text{where} \quad \bar{L}_{jn} = (1/C_{in}) \sum_c L_{cjn}.$$

Figure 4 illustrates the model graphically. The regression model is indicated by the dotted-line box. The dependent variables in the regression are the $\tilde{a}_{ijn}$ which are mathematical transformations of the $\tilde{p}_{ijn}(t_n)$. For example, Table 9 illustrates $\tilde{P}_n$ and $\tilde{A}_n$ for period 8. The explanatory variables are publicity reach, direct mail, word of mouth, preference probability, availability, budget allocation, a constant affecting flows from unaware, and a constant affecting all other factors. In turn, preference probability is an output of a logit model that is based on 'convenience', 'ease of use', 'safety', and 'opinions'. The implicit assumption is that the measured constructs are the result of marketing and operating strategies. Figure 4 is a conceptual representation of the regression. As discussed earlier, some variables affect only specific flows and are thus zero for all other flows. (E.g., the marketing variables have a direct affect on only flows out of awareness, while the "Lens" variables affect all flows.) Finally, forecasts of cumulative awareness, ridership, and equilibrium ridership are mathematically derived from forecasts of the flow rates. Our model is a "share" model. We assume that the total number of trips (counting

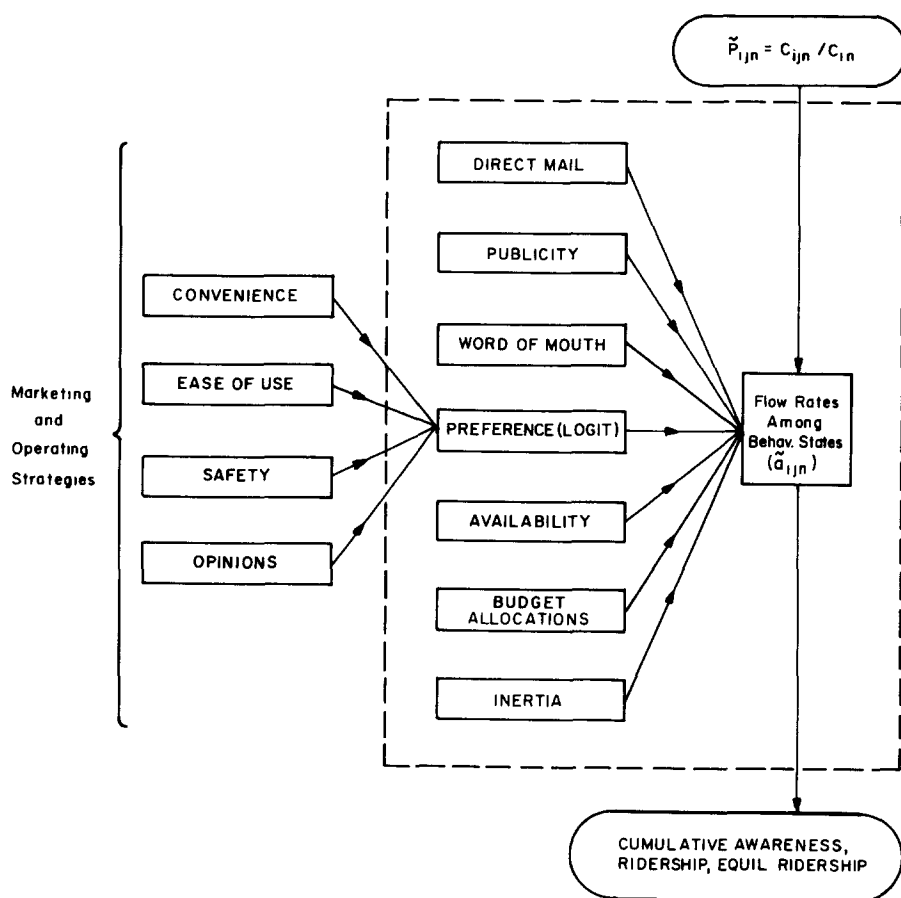


FIGURE 4. Graphical Representation of the Constructs in the Combined Models (The Regression Equation is Indicated by the Dotted Lines).

TABLE 9
Example P_n and A_n Matrices (Actual data for period 8).

	UD	CD/A	CP/A	B/A	D/A
$\tilde{P}_n$					
UD	.983	.013	.004	.000	.000
CD/A	—	.862	.130	.007	.002
CP/A	—	.580	.407	.008	.005
B/A	—	.698	.093	.209	.000
D/A	—	.732	.220	.000	.049
$\tilde{A}_n$					
UD	— .035	.026	.009	.000	.000
CD/A	—	— .493	.461	.025	.007
CP/A	—	—	2.02	— 2.11	.040
B/A	—	—	3.03	.159	— 3.17
D/A	—	—	4.41	1.84	— 6.14

all modes of transportation) is constant throughout the estimation period. (Any misspecification resulting from this last assumption biases our model against good fit and good prediction.)

The regression is run across periods 2 through 9 and across ten of the sixteen independent,⁹ nonzero flows in Figure 2. In other words, there are eighty, $80 = (8 \text{ time periods}) \times (10 \text{ independent, nonzero flows})$, draws of the dependent variable. For a more graphical illustration of the general regression see Table 3 in Hauser and Wisniewski (1982). As defined earlier, some of the explanatory variables are equal to zero for some of the flows. Based on the interpretations of the consumer model given in §5 and the simulation results described in the appendix, the sample size and observation period length should be sufficient to obtain reasonable estimates, $\hat{w}_i$, of the w_i .

Model Development

According to the theory of §5, the marketing variables, publicity and direct mail, and word of mouth should impact flows *out of* awareness, i.e., all flows in the first row of Figure 2. The "Lens" model variables, preference, availability, budget allocation, and inertia should impact all flows. (The self-flows on the principal diagonal of Figure 2 are not independent of the other flows and are not included in the regression.) Examination of the correlation matrix indicated that publicity and direct mail were collinear, but all other variables were within the simulation limits. Since the collinearity in the marketing variables was structural—whenever the Village mailed out brochures, the newspaper ran a news item—we use marketing reach to replace publicity and direct mail. The complete model is shown in Table 10 as model 1 where we have chosen archival variables for marketing reach and word of mouth.

While Model 1 fits the data well (adjusted $R^2 = .82$), many of the variables are not significant at the .10 level. We delete these variables and reestimate the model. See Model 2 in Table 10. All nonconstant variables are now significant at the .10 level and a comparable fit is obtained (adjusted $R^2 = .82$). Comparison of the two models indicates that Model 1 is not significantly better than Model 2 at the .10 level of statistical significance. [$F(4, 71) = 1.36$].

As a further test of the descriptive fit, we compared ridership predicted by the model with the ridership observed in the archival data. This is by no means a guaranteed fit. First, the dependent measure in the regression is flows among behavioral states. Ridership is a structural output of the model dependent upon the adequacy of the probabilistic model in representing behavior. Second, the model estimation is based on survey data while the archives are based on observed behavior.

As shown in Figure 5, the model fits the archival data reasonably well, successfully predicting the increase during November 1–14 (due to publicity),

⁹In the estimation periods the sample size, C_m , for flows *out of* the low share modes, BUS and DART, were below the simulation limits. This can cause problems due to insignificant random fluctuations. For example, notice the negative, but insignificant, off-diagonal elements for flows out of BUS and DART in table 9. Flow *into* these modes were based on sufficient sample sizes. This step reduces the number of observations but should not bias the estimates.

TABLE 10
Model Development

	(1) Full Model	(2) Selected Model
Impacts on Awareness Flows		
Marketing Reach	.99*	.62*
Word of Mouth	-.00	
Constant	.04	.03
Impacts on Behavior		
Preference ⁺	3.18*	3.52*
Availability	.51*	.43*
Budget Allocation		
Large	-.84	
Moderate	-1.94	
Small	—	
Inertia	.99	
Constant	.44*	-.50*
R^2 (adjusted)	.82	.82
Ridership Correlation	.48	.94

*Both regressions are significant at the .01 level. All starred coefficients are significant at the .10 level. Model (1) is not significantly better than Model (2). $F(4, 71) = 1.36$ ($p > .10$)

⁺ Preference is a composite variable created from the measured perceptions with equation 3 and the parameters in table 7.

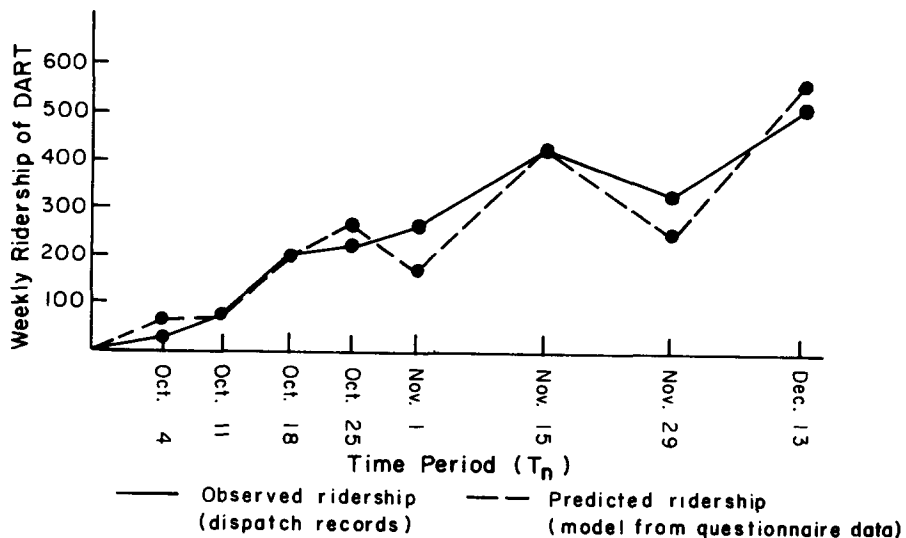


FIGURE 5. Comparison of Actual and Estimated Ridership for the Estimation Period.

the downturn during November 15–29 (due to riders who tried the service but did not repeat), and the upturn during November 30–December 13 (due in part to the direct mail campaign late in that period). Dates in Figure 5 represent when the survey was mailed out. Correlation between predicted and actual ridership was .94. It is interesting to note that the full model, model 1, does not do nearly as well, correlation = .48, in predicting ridership. The direction of this result is as expected; its magnitude is surprising.

Finally, as expected, a model with the same variables but based on the full data including low probability flows (16 flows $\times$ 8 periods) does not do significantly better (adjusted $R^2 = .68$, correlation = .95) and yields similar coefficients. Based on these results and the criterion of parsimony we use the coefficients in model 2 to forecast ridership from December 27 through the period beginning April 17.

Alternative Operationalizations

We used an actionability criterion to select the archival operationalizations of the explanatory variables. However, if the self-reported variables do significantly better we would have to reassess our position and plan future research to further understand how managerial actions impact the intervening self-reported measures. To investigate this issue, we ran regressions with selected models based on alternative operationalizations of the explanatory variables and the dependent measure.

Based on the statistics that measure descriptive fit, these models did not do significantly better than Model 2. See Table 11. Model 2 is the same Model 2 reported in Table 10.

Model 3 breaks the “marketing reach” variable into its component parts. Although its descriptive statistics are comparable to those of Model 2, the marketing variables are now insignificant at the .10 level—probably due to the collinearity among the explanatory variables. Since collinearity blurs the interpretation of Model 3, we retain Model 2.

Model 4 replaces the archival marketing and word of mouth variables with self-reported measures. Again, the descriptive statistics are comparable to those of Model 2, but the variables are all insignificant at the .10 level. Since, in this application, the self-reported variables do not do significantly better than the more actionable archival variables we select Model 2 for its managerial relevance. Other researchers with other goals, e.g., an investigator of cognitive processing, might further investigate Model 4 because it is based on measures of consumers’ subjective evaluations of the impacts of marketing strategies.

Model 5 replaces preference probabilities (equation 2) with a weighted sum of the perceptual dimensions ($\sum_k \beta_k y_{cjk} + \delta_j$). Model 5 does not do as well as Model 2 suggesting that preference probability may be the better measure. We again select Model 2 based on the theoretical justification that, in the absence of other variables, an explanatory variable should be proportional to a transition probability.

Model 6 is based on an alternative operationalization of the dependent variable in which all consumers are weighted equally rather than weighted by

TABLE 11
Alternative Operationalizations

	(2)	(3)	(4)	(5)	(6)
	Selected Model	Marketing Components	Self-Reported Measures	Preference Index	Alternative Dependent Variable
<i>Impacts on Awareness Flows</i>					
Marketing Reach	.62*		2.07	.62*	.64*
Mail Campaign		-.10			
Publicity		.77			
Word of Mouth			-.35		
Constant	.03	.02	.04	.03	.03
<i>Impacts on Behavior</i>					
Preference (Probability)	3.52*	3.52*	3.52*		2.98*
Preference (Index)				.71*	
Availability	.43*	.43*	.43*	.14	.38*
Constant	-.50*	-.50*	-.50*	.35*	-.47*
—	—	—	—	—	—
R ² (adjusted)	.82	.81	.81	.81	.82
Ridership Correlations	.94	.94	.94	.78	.92

* All regressions are significant at the .01 level

All starred coefficients are significant at the .10 level.

the number of trips that they make. Since this model does not do significantly better, we select Model 2 based on our management interest in forecasting ridership rather than the number of users. It is interesting that the $\hat{w}_i$ are similar in both Models 2 and 6.

Finally, we tested three models which each deleted one of the nonconstant variables in Model 2. In each case, Model 2 was significantly better at the .10 level.

Final Selection

Based on the statistical, managerial, and theoretical considerations we select Model 2 for predictive testing.

7. Predictive Test

Model 2 fits the estimation data well. If it is to be useful for managers and planners it must also do reasonably well in predicting consumer behavior under a changing operating environment. We construct the following predictive test:

(1) *marketing strategy* - Use the archival measures of the marketing strategy variables for periods 10 through 16 (percent mailing in each period, newspaper articles weighted by newspaper circulation).

(2) *operating environment* - Use the measured perceptions and availability for periods 10 through 16. These measures account for (unobserved) operating changes.

(3) *preference* - Use the probability equation (equation 3) with the parameters, $(\hat{\beta}_k, \hat{\delta}_j)$ estimated in periods 2 through 9.

(4) *share forecast* - Use the consumer model to forecast share of ridership by DART in periods 10 through 16.

(5) *ridership forecast* - Assume total weekly trip making behavior is the same (except for holidays) throughout the forecast period. DART ridership is then (share) $\times$ (total trips).

(6) *comparison* - Compare the forecast ridership to that measured directly by the archival data for periods 10 through 16.

We feel that this procedure provides a true predictive test because the dynamic consumer model is being used to forecast future behavior as much as 19 weeks in advance—a time period longer than the 11 week period of observation. Not only was the model estimated based on data collected in periods 2 through 9, but its structure and operationalization of explanatory variables were selected based on data collected in periods 2 through 9. Furthermore, as noted above, the predictions that are being tested (ridership in periods 10 through 16) are a structural output of the model, not the dependent measure used in estimation.

The basic output of the predictive test is shown in Figure 6. Overall the model predicts well, but more importantly it correctly forecasts the directional shifts (up or down) in ridership. (Directional shifts are explainable by marketing actions and trial-repeat phenomena.) The correlation of predicted ridership with forecast ridership is .83. Because there are many potential

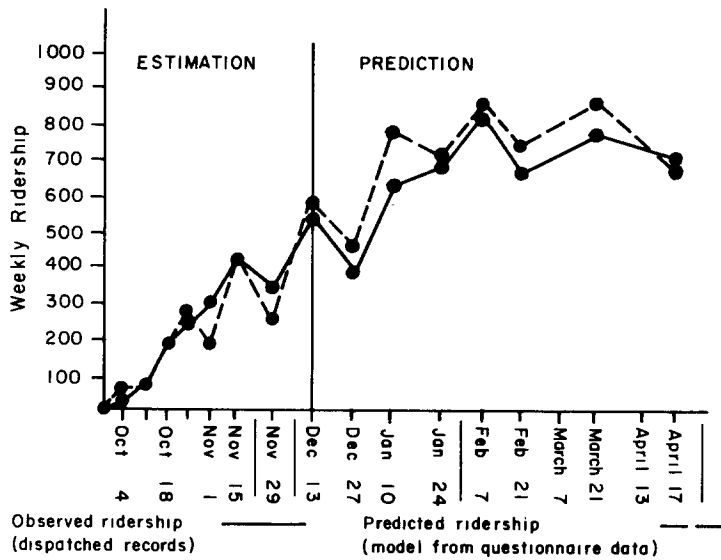


FIGURE 6. Predictive Test for Dynamic Consumer Model Based on Seven Months of DART Service in Schaumburg, IL.

biases¹⁰ in the data, all of which work against successful prediction, we feel the predictions in Figure 6 are quite respectable. Finally, Model 2 outperforms Models 1, 4, 5 and 6 which predict with correlations of .42, .81, .27, and .82 respectively, and performs as well as the collinear model, Model 3, which predicts with correlation .85.

In addition to ridership, the model predicts other statistics of managerial interest. For example, Figure 7 compares forecast cumulative awareness with cumulative awareness as measured by the surveys. The correlation between predicted and actual is .98. (The predictive test based on cumulative awareness is less stringent since the observed variable is a survey measure. The correlation is guaranteed to be high since cumulative awareness is a monotonically increasing measure.)

Based on these results we are encouraged about the accuracy of the dynamic consumer model and its operationalization as presented in this paper. In §9 we discuss future research that has the potential to improve applications of the model.

8. Managerial Recommendations

The consumer model was developed as a forecasting tool, but prior to the predictive test it was an unvalidated forecasting tool. Thus, managerial decisions were not based on the model until the 30th week of service.

¹⁰ Among the potential biases in the data are non-response issues, seasonality not modeled, potentially small sample size, recall measures that require adjustment, measurement error in the survey, collinearity in the archival explanatory variables, and DART having a small share of the market.

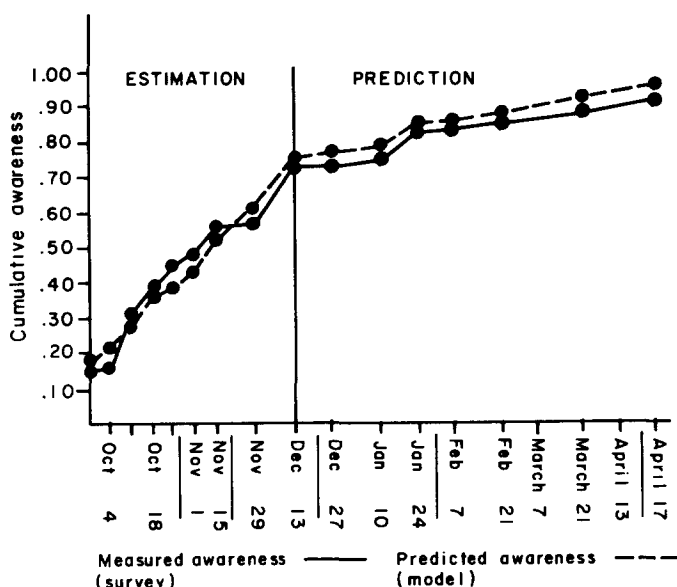


FIGURE 7. Comparison of Predicted and Observed Awareness of DART in Schaumburg, IL.

Before we discuss strategy, we note two additional bits of information: (1) average perceptions of DART were relatively stable throughout the observation periods and (2) the correlations between average perceptions of DART and the archival marketing variables were all insignificant at the .10 level. Interpreting this information within the context of the model, it appears that the aggregate effect of publicity and direct mail for DART in Schaumburg was to create awareness and communicate the characteristics of DART. Thus the effect of such strategies beyond the 30th week would be at most a 10% increase in ridership due to increasing awareness from 90% to almost 100%. We caution the reader that this does not rule out more active media, such as television advertising, which was not tested in Schaumburg.

Once consumers are aware of DART, the model explains the dynamic growth in ridership with a continuous time Markov interpretation of the static multinomial 'Lens' model. Using this model to predict equilibrium ridership, we found our estimates to be below the targeted goals of Schaumburg. Since the transit manager now accepts our model, he decided to take steps to increase perceived 'convenience', 'ease of use', 'safety', 'general opinion', and/or availability. While our model has not yet evolved to explicitly model the link from operating strategies to perceptions and constraints, we can gain some insight through qualitative diagnostic information.

We examine the perceptual map in Figure 8. The points indicate how the average consumer perceives transportation in Schaumburg, the length and direction of the arrows are based on the relative importances of the perceptual dimensions in Table 7. In addition, Figure 8 indicates the relative perceived

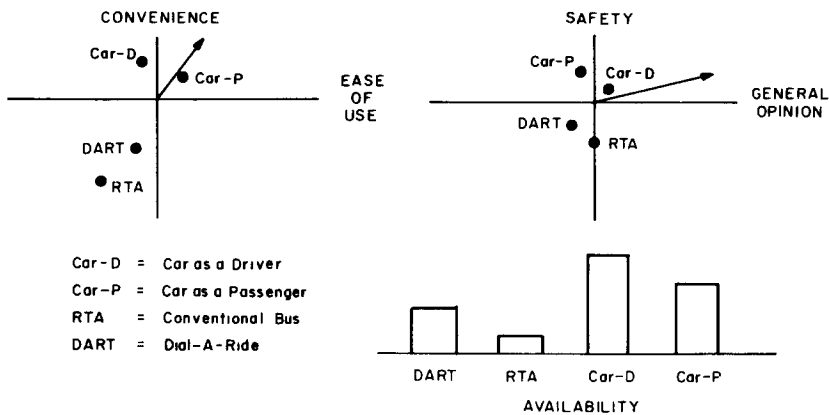


FIGURE 8. Relative Perceptions of the Four Modes of Transportation in Schaumburg, IL. (The Arrows Indicate Relative Importance of the Dimensions. Availability Is Modeled As a Constraint Affecting Behavior Directly.)

availability of the modes of transportation. Figure 8 suggests that the improvements in convenience and ease of use and increased availability would have a major impact on ridership. (The qualitative goal is to improve the relative position of DART in the direction of the arrows.)

Discussions with representatives of the Village indicated that DART was providing as good a service as was possible within the current operating constraints (number of budgeted vehicle hours). The identified need was for more vehicles per hour or extended hours. Both of these strategies translate into more vehicle hours. Increased vehicle hours plus the improved weather in May through August (which affects 'ease of use') should increase ridership beyond the current levels.

These suggestions were made to the Village. Partially as the result of the analysis, the budgeted vehicle hours were increased in May, 1980 from 28 hours per week to 43 hours per week. Ridership was running at approximately 1300 rides per week following the change.

It is interesting to note that had we been willing to accept the model's predictions and diagnostic information prior to predictive testing, many of the managerial recommendations could have been made after the 11th week rather than after the 30th week. Furthermore, the timing and magnitude of publicity and direct mail could have been selected to achieve desired levels of growth in service. (See Horsky [1979] for optimization procedures to select a time stream for advertising expenditures.)

Finally, the predictive success of the simple model (Model 2) encourages us to develop expanded models that include additional phenomena of interest. In particular, the dynamic model in Figure 1 appears to be a useful framework for incorporating explicit submodels linking operating strategies to objectively measured system performance and linking objectively measured system performance to perceived system performance.

9. Discussion

One of our research goals was to critically evaluate the practicality of the semi-Markov methodology and the empirical reasonableness of the consumer model in Figures 1, 2, 3, and 4.

Feasibility

No major difficulties were encountered in implementing the semi-Markov methodology. The required sample sizes per period are reasonable for many consumer goods. The estimation and forecasting were straightforward tasks requiring only existing regression and eigenstructure computer software. The dependent variables are measurable. The bias in recall awareness appears correctable.

Paramorphism

The predictive test was constructed to minimize the likelihood that good results could be spurious. Good results are not guaranteed. Models can be constructed which produce ridership forecasts which are not correlated with actual ridership. (Correlations are insignificant for model 5 in Table 11). The selected model and its predictions have good face validity. For example, the rapid rises in awareness and ridership all appear explainable by either direct mail or publicity. The declines appear explainable by consumers trying DART but only some consumers continuing to ride DART.

Managerial Utility

The model helped a manager make a better decision, but its managerial utility could be improved through evolution. In particular, conjoint models linking physical characteristics to perceptions would enable the manager to optimize over marketing strategies *and* operating decisions rather than simply over direct mail and publicity. We view this evolutionary capability of the model as one of its strengths.

Aggregation Issues

The macro-flow assumption does not appear to greatly impair the model's predictive ability in this application. However, information is lost in aggregation of the explanatory variables. Even though publicity, direct mail and word of mouth do not correlate with average perceptions, they have small, but statistically significant, correlations at the level of the individual consumer. For example, self-reported receipt of newspaper publicity has a .11 correlation with "convenience" and a .12 correlation with "ease of use" *at the disaggregate level*. Both are significant at the .10 level. A fully disaggregate model, which is only possible with full maximum-likelihood computer software, could conceivably increase the diagnostic power of the consumer model. Another benefit would be greater efficiency allowing potentially smaller estimation samples.

Insignificant Variables

Word of mouth, budget allocation, and inertia were not significant in Model 1. There are a number of possible explanations including the following. DART has a small market share and hence the impact of word of mouth may be too small to measure relative to the much larger impacts of the marketing variables. This is not necessarily true for other innovations where word of mouth has proven significant (Bass 1969, Rogers and Shoemaker 1971). Budget allocations are small for transportation and were only measured with a three-level categorical variable. This important economic variable may be significant with improved measures. Preference inertia has been identified for health care with requires a major commitment by the consumer (Neslin 1976), it may be insignificant for DART due to the low commitment required to try DART. It is likely that these and other variables could prove significant in other applications of the model.

10. Conclusions

We are encouraged by the practicality and predictive accuracy of the simple model. It is likely that the model captures many important phenomena. However, the descriptive and predictive tests do not imply that all phenomena are modeled or that other models are not also acceptable. We feel that this paper provides evidence that the methodology is useful in at least one context. We will learn more as the methodology is applied with other sets of behavioral states, other explanatory variables, and other realizations of the dependent measures. We encourage other researchers to use our data to test such models.

Our application was the monitoring of a new transportation service. Based on our one application, we feel the dynamic model methodology is easy to use and holds promise for evolutionary development. We posit that the methodology is useful for other product categories, but this has not yet been fully tested. See Lange (1981) for an initial application of the methodology to the ground coffee category using Universal Product Code (UPC) data.

Appendix 1. Technical Details on the Consumer Model

The following equations briefly summarize the main results from Hauser and Wisniewski (1982).

Estimation

Define $S_i(T_n) = 1$ if the consumer is in state i at time T_n and $S_i(T_n) = 0$ otherwise. The statistic we use to describe the process is the probability, $p_{ij}(t_n)$, that the consumer is in state S_j at time T_n given that he started in state S_i at time T_{n-1} . I.e.,

$$p_{ij}(t_n) = \text{Prob}\{S_j(T_n) = 1 \mid S_i(T_{n-1}) = 1\} \quad (\text{A-1})$$

where $t_n = T_n - T_{n-1}$. Any data collection procedure that provides observations on $S_j(T_n)$ and $S_i(T_{n-1})$ can be used to implement the model.

Let a_{ijn} for $j \neq i$ be the flow rate, i.e., the probability that a consumer flows from S_i to S_j in time Δt is $a_{ijn}\Delta t$. Define $a_{iin} = -\sum_{i \neq j} a_{ijn}$. Define the matrices $P_n(t_n) = \{p_{ij}(t_n)\}$ and $A_n = \{a_{ijn}\}$. Then

$$P_n(t_n) = \exp(A_n t_n) \equiv \sum_{r=0}^{\infty} A_n^r t_n^r / r! \quad (\text{A-2})$$

which is a highly nonlinear system of equations. If θ_n is a matrix with the eigenvalues of A_n on the diagonal and zeros elsewhere and E_n is the matrix of Eigenvectors then $P_n(t_n)$ can be obtained from A_n by $P_n(t_n) = E_n[\exp(\theta_n t_n)] E_n^{-1}$ where the $\exp(\cdot)$ operation applies separately to each eigenvalue.

Let x_{ijln} be the value that the l th explanatory variable takes on for the i and j flow in period n . Let w_l be the (unknown) importance weight of the l th explanatory variable. Assume

$$a_{ijn} = \sum_l w_l x_{ijln}. \quad (\text{A-3})$$

We also define variables x_{iiln}^0 to carry information about flows from S_i and S_i . Let $a_{iin}^0 = \sum_l w_l x_{iiln}^0$. Define the matrix $X_{ln} = \{x_{ijln}\}$.

In the macro-flow version of the theory we observe the number of consumers, C_{ijn} , who are in state S_i at T_{n-1} and in state S_j at T_n . The log-likelihood function, L , is then:

$$L = \sum_n \sum_{(i,j)} C_{ijn} \log \left\{ \exp \left(\sum_l w_l X_{ln} t_n \right) \right\}_{ij} \quad (\text{A-4})$$

where the notation, $\{M\}_{ij}$, indicates the $i-j$ th element of matrix M .

The main estimation result is that an approximation to the maximum likelihood estimators of the w_l 's can be obtained by solving the following regression equation:

$$\tilde{E}_n [\log \tilde{\Lambda}_n] \tilde{E}_n^{-1} \simeq \sum_l w_l X_{ln} t_n \quad \text{for all } n \quad (\text{A-5})$$

where $\tilde{E}_n$ is the matrix of eigenvectors of $\tilde{P}_n(t_n)$ and $[\log \tilde{\Lambda}_n]$ is a matrix with logarithms of the eigenvalues on the principal diagonal and zeros elsewhere. $\tilde{P}_n(t_n)$ is the matrix of $\{C_{ijn}/C_{in}\}$, i.e., the observed percentages of consumers who are in state S_j at T_n given they were in state S_i at T_{n-1} . This is the regression equation described in equation (2). Note that since the diagonal elements of both sides of equation A-5 are functions of the non-diagonal elements, the diagonal equations are deleted from the estimation.

Forecasting Equations

Once we estimate the importance weights, $\hat{w}_i$, we can use equation A-3 to estimate the flow rates, $\hat{a}_{ijn}$, for future periods. This assumes of course we can forecast the explanatory variables, x_{ijn} , for those periods. The estimated flow rates then determine completely the probabilistic system, via equation A-2.

Cumulative statistics. Cumulative awareness, cumulative trial, and other cumulative statistics are simply the total percentage of consumers who flow into a state, say state S_j , by time T_n . We call this statistic penetration.

If we define ${}_1A_n$ such that ${}_1a_{ikn} = a_{ikn}$ for $i \neq j$ and ${}_1a_{jkn} = 0$ then penetration is given by:

$$\text{penetration (into state } S_j) = \sum_i \pi_i(T_{n-1}) {}_1p_{ijn}(t_n). \quad (\text{A-6})$$

where

$${}_1P_n(t_n) = \exp({}_1A_n t_n)$$

and $\pi_i(T_{n-1})$ is the probability that the consumer is in state S_i at time T_{n-1} . Equation A-6 is used recursively when calculating penetration over more than one observation period.

Sales. Expected sales and the variance of sales are computed via moment generating functions. Expected sales are given by:

$$\text{Expected sales} = \sum_i \sum_k^0 \pi_i(T_{n-1}) p_{ikn}(t_n) a_{kjn} t_n \quad (\text{A-7})$$

where $\sum_k^0$ means we use a_{ijn}^0 in the sum. The equation for the variance of sales is given in Hauser and Wisniewski (1982).

Equilibrium statistics. We calculate equilibrium statistics by letting x_{ijn} tend to its long run ($t \rightarrow \infty$) value, x_{ijl} . We use equation A-3 to calculate the long run flow rates, a_{ij} and equilibrium sales are given by:

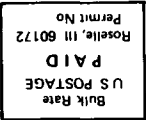
$$\text{Expected equilibrium sales rate} = \sum_k^0 \pi_k a_{kj} = \pi_j (a_{jj}^0 - a_{jj}) \quad (\text{A-8})$$

where the equilibrium π_j are determined by solving the matrix equation $\Pi'A = 0$ subject to $\sum_k \pi_k = 1$.

Simulation Results

Simulation analyses determined whether the model could recover known data under cases of varying sample size and time periods. The key results are (1) that the sample size per time period should be greater than $20 \times$ (number of nonzero flows), and (2) that the length of the observation period should be short relative to the time it takes the process to reach equilibrium.

Appendix 2. Information Brochure Mailed to Schaumburg Residents

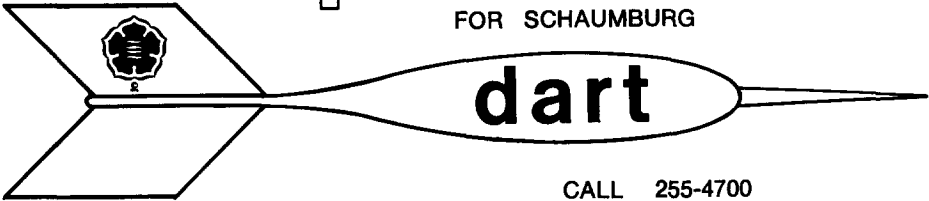


VILLAGE OF SCHAUMBURG
101 Schaumburg Court
Schaumburg, Illinois 60193

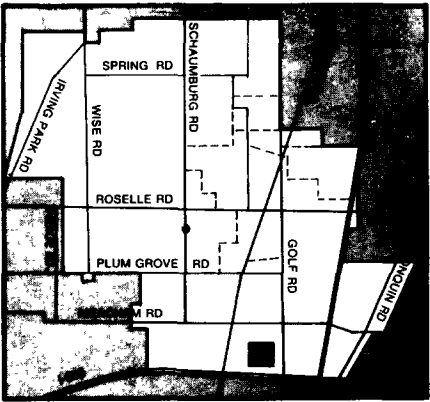


dial-a-ride transportation

FOR SCHAUMBURG



CALL 255-4700



ALL TRIPS ON **dart** MUST BEGIN AND END WITHIN THE BOUNDARIES ABOVE.

TRANSFERS CAN BE MADE TO:

- 209 Woodfield — Des Plaines — Jefferson Park (Chicago)
- 405 Woodfield — Alexan Bros Hospital — Elk Grove
- 693 Harper College — Northwest Hospital — Arlington Heights
- 695 Woodfield — Arlington Heights
- 696 Woodfield — Palatine — Long Grove
- 697 Woodfield — Roselle

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Fares are low on **dart**. For 80¢ you can travel to anywhere in Schaumburg and with a low cost transfer use RTA buses to many parts of the Northwest Suburbs and to Chicago.

HOW TO USE dart

Call **dart** at **255-4700** and tell the dispatcher who answers:

- Where you are
- Where you wish to go
- The time you wish to leave or you wish to arrive.
- Your name
- Your phone number — in case we must call back
- Number in your party

**GIVE dart TIME TO SERVE
YOU BETTER**

If you must meet an appointment give **dart** time to serve you better. It will normally take 30 minutes (sometimes more, sometimes less) to route a bus to pick you up and 30 minutes to make the average trip. Therefore, give **dart** at least 60 minutes or call **dart** in advance and we will reserve a spot for you and call you a few minutes before we pick you up.

BE READY FOR dart

The **dart** dispatcher will tell you when the bus will arrive. Please be ready to catch it a few minutes before, in order to assure speedy service for you and other passengers. The bus will signal its arrival by blowing its horn and will only wait 30 seconds for you to appear.

RETURN TRIPS ON dart

If you know in advance when you want to return, just tell the **dart** dispatcher and he will schedule it. If not, just call 255-4700 when you are ready.

HOURS OF SERVICE

dart buses will begin picking up passengers at 9:00 A.M. Monday thru Friday. Calls will be accepted from 8:30 A.M. The last passengers will be picked up at 5:30 P.M. **dart** will not operate on Saturdays, Sundays or Holidays.

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EXACT CHANGE IS REQUIRED—THE DRIVER DOES NOT CARRY CHANGE

REGULAR FARES

Cash Fare \$.80
All Day Pass — as many rides as you want on **dart** 2.00
Ten Ride Ticket on **dart** 8.00

REDUCED FARES*

Cash Fare .40
All Day Pass — as many rides as you want on **dart** 1.00
Ten Ride Ticket on **dart** 4.00

TRANSFERS

FROM dart TO RTA ROUTES

Regular Fares .10
Reduced Fares* .05

dart ACCEPTS RTA TRANSFERS

Upon payment of surcharge
Regular .20
Reduced* .15

PERSONS ELIGIBLE FOR REDUCED FARES

- Senior Citizens 65 or over with RTA Special Users Card. (Can be obtained at many locations)
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- Students through grade 12—to and from school.
- Children 7-11—Children under 7 with an adult are free

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